Numbers

1) 2^2n-1 is always divisible by 3
2^2n-1 = (3-1)^2n -1
= 3M +1 -1
= 3M, thus divisible by 3

2) What is the sum of the divisors of 2^5.3^7.5^3.7^2?
ANS : (2^6-1)(3^8-1)(5^4-1)(7^3-1)/2.4.6
Funda : if a number 'n' is represented as
a^x * b^y * c^z ....
where, {a,b,c,.. } are prime numbers then

> **Quote:**
>
> **(a) the total number of factors is (x+1)(y+1)(z+1) ....**
> **(b) the total number of relatively prime numbers less than the number is n * (1-1/a) * (1-1/b) * (1-1/c)....**
> **(c) the sum of relatively prime numbers less than the number is n/2 * n * (1-1/a) * (1-1/b) * (1-1/c)....**
> **(d) the sum of factors of the number is {a^(x+1)} * {b^(y+1)} * ...../(x*y*...)**

3) what is the highest power of 10 in 203!ANS : express 10 as product of primes; 10 = 2*5

divide 203 with 2 and 5 individually
203/2 = 101
101/2 = 50
50/2 = 25
25/2 = 12
12/2 = 6
6/2 = 3
3/2 = 1
thus power of 2 in 203! is, 101 + 50 + 25 + 12 + 6 + 3 + 1 = 198

divide 203 with 5
203/5 = 40
40/5 = 8
8/5 = 1

thus power of 5 in 203! is, 49

so the power of 10 in 203! factorial is 49

4) x + y + z = 7 and xy + yz + zx = 10, then what is the maximum value of x? (CAT

2002 has similar question)
ANS: 49-20 = 29, now if one of the y,z is zero, then the sum of other 2 squares shud be equal to 29, which means, x can take a max value at 5

5) In how many ways can 2310 be expressed as a product of 3 factors?
ANS: 2310 = 2*3*5*7*11
When a number can be expressed as a product of n distinct primes,
then it can be **expressed as a product of 3 numbers in (3^(n+1) + 1)/2 ways**

6) In how many ways, 729 can be expressed as a difference of 2 squares?
ANS: 729 = a^2 - b^2
= (a-b)(a+b),
since 729 = 3^5,
total ways of getting 729 are, 1*729, 3*243, 9*81, 27*27.
So 4 ways
Funda is that, all four ways of expressing can be used to findout distinct a,b values,
for example take 9*81
now since 9*81 = (a-b)(a+b) by solving the system a-b = 9 and a+b = 81 we can have 45,36 as soln.

7) How many times the digit 0 will appear from 1 to 10000
ANS: In 2 digit numbers : 9,
In 3 digit numbers : 18 + 162 = 180,
In 4 digit numbers : 2187 + 486 + 27 = 2700,
total = 9 + 180 + 2700 + 4 = 2893

8) What is the sum of all irreducible factors between 10 and 20 with denominator as 3?
ANS :
sum = 10.33 + 10.66 + 11.33 + 11.66 + 12.33 + 12.66 + 13.33 + 13.66…….
= 21 + 23 + ……
= 300

9) if n = 1+x where x is the product of 4 consecutive number then n is,
1) an odd number,
2) is a perfect square
SOLN : (1) is clearly evident
(2) let the 4 numbers be n-2,n-1,n and n+1 then by multing the whole thing and adding 1 we will have a perfect square

10) When 987 and 643 are divided by same number 'n' the reminder is also same, what is that number if the number is a odd prime number?
ANS : since both leave the same reminder, let the reminder be 'r',
then, 987 = an + r
and 643 = bn + r and thus
987 - 643 is divisible by 'r' and

987 - 643 = 344 = 86 * 4 = 43 * 8 and thus the prime is 43
hence 'r' is 43

11) when a number is divided by 11,7,4 the reminders are 5,6,3 respectively. what would be the reminders when the same number is divided by 4,7,11 respectively?
ANS : whenever such problem is given,
we need to write the numbers in top row and rems in the bottom row like this

11 7 4
| \ \
5 6 3

(coudnt express here properly)
now the number is of the form, LCM (11,7,4) + 11*(3*7 + 6) + 5
that is 302 + LCM(11,7,4) and thus the rems when the same number is divided by 4,7,11 respectively are,

302 mod 4 = 2
75 mod 7 = 5
10 mod 11 = 10

12) a^n - b^n is always divisible by a-b

13) if a+b+c = 0 then a^3 + b^3 + c^3 = 3abc
EXAMPLE: 40^3-17^3-23^3 is divisble by
since 40-23-17 = 0, 40^3-17^3-23^3 = 3*40*23*17 and thus, the number is divisible by 3,5,8,17,23 etc.

14) There is a seller of cigerette and match boxes who sits in the narrow lanes of cochin. He prices the cigerattes at 85 p, but found that there are no takers. So he reduced the price of cigarette and managed to sell all the cigerattes, realising Rs. 77.28 in all. What is the number of cigerattes?

a) 49
b) 81
c) 84
d) 92

ANS : (d)
since 77.28 = 92 * 84, and since price of cigarette is less than 85, we have (d) as answer

> **Quote:**
>
> i have given this question to make the funda clear

15) What does 100 stand for if 5 X 6 = 33
ANS : 81
SOLN : this is a number system question,
30 in decimal system is 33 in some base 'n', by solving we will have n as 9
and thus, 100 will be 9^2 = 81

16) In any number system 121 is a perfect square,
SOLN: let the base be 'n'
then 121 can be written as n^2 + 2*n + 1 = (n+1)^2
hence proved

17) Most of you ppl know these, anyways, just in case

> **Quote:**
>
> **(a) sum of first 'n' natural numbers - n*(n+1)/2**
> **(b) sum of the squares of first 'n' natural numbers - n*(n+1)*(2n+1)/6**
> **(c) sum of the cubes of first 'n' natural numbers - n^2*(n+1)^2/4**
> **(d) total number of primes between 1 and 100 - 25**

18) See Attachment to know how to find LCM, GCF of Fractions

> **Quote:**
>
> **CAT 2002 has 2 questions on the above simple concept**

19) Converting Recurring Decimals to Fractions

let the number x be 0.23434343434........

thus 1000 x = 234.3434343434......
and 10 x = 2.3434343434.........
thus, 990 x = 232
and hence, x = 232/990

20) **Reminder Funda**

(a) (a + b + c) % n = (a%n + b%n + c%n) %n
EXAMPLE: The reminders when 3 numbers 1221, 1331, 1441 are divided by certain number 9 are 6, 8, 1 respectively. What would be the reminder when you divide 3993 with

9? (never seen such question though)
the reminder would be (6 + 8 + 1) % 9 = 6

(b) (a*b*c) % n = (a%n * b%n * c%n) %n

EXAMPLE: What is the remainder left when 1073 * 1079 * 1087 is divided by 119 ? (seen this kinda questions alot)
1073 % 119 = ?
since 1190 is divisible by 119, 1073 mod 119 is 2
and thus, "the remainder left when 1073 * 1079 * 1087 is divided by 119 " is 2*8*16 mod 119 and that is 256 mod 119 and that is (238 + 18) mod 119 and that is 18

Glossary : % stands for reminder operation

find the number of zeroes in 1^1* 2^2* 3^3* 4^4............. 98^98* 99^99* 100^100

the expresion can be rewritten as (100!)^100 / 0!* 1!* 2!* 3!....99!

Now the numerator has 2400 zeros

the formular for finding number of zeros in n! is

[n/5]+[n/5^2]...[n/5^r]
where r is such that 5^r<=n<5^(r+1)

and [..] is the grestest integer function

for the numerator find the number of zeros using the above formulae..

for 0!...4! number of zeros ..0
5!...9!.number os zeros ..1
9!...14!... 2
15!..19!..................3
20!..24!..................4!
now at 25! the series makes a jump to 6
25!...29!.................6
30!...34!.................7
this goes on and again makes a jump at 50!
and then at 75!

so the number of zeros is...

5(1+2....19) + 25+ 50+ 75

the last 3 terms 25 50 and 75 are because of the jumps..

this gives numerator has 1100 zeros

now total number of zeros in expression is no of zeros in denominator - no of zeros in numerator
2400 - 1100

the Answer 1300