Differential Equations

DIFFERENTIAL EQUATION: An equation containing an independent variable, dependent variable and differential coefficients of dependent variable with respect to independent variable is called a differential equation.

GENERAL SOLUTION: The solution which contains as many as arbitary constants as the order of the differential equation is called the general solution of the differential equation.

PARTICULAR SOLUTION: Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular r solution.

ILLUSTRATIVE EXAMPLES

(1) Obtain the differential equation of all circles of radius r.

(2) find the differential equation of all the circles in the first quadrant which touch the coordinate axes.

(3) Find the differential equation of all conics whose center lies at the origin.

Solve the following differential equations by inspection method

(4) $(ydx - xdy)\cos\left(\frac{x}{y}\right) = xy^3(xdy + ydx)$

(5) $\frac{xdx + ydy}{ydx - xdy} = \frac{x\cos^2(x^2 + y^2)}{y^3}$

(6) $xdy - ydx = (x^2 + y^2)\ dx$

LINEAR DIFFERENTIAL EQUATIONS OF THE FORM

$$\frac{dx}{dy} + Rx = S$$

Sometimes a linear differential equation can be put in the form

$$\frac{dx}{dy} + Rx = S,$$

Where R and S are functions of y or constants.

Note that y is independent variable and x is a dependent variable

The following algorithm is used to solve these types of equations

ALGORITHM

STEP I Write the differential equation in the form $\frac{dx}{dy} + Rx = S$ and obtain R and S.

STEP II Find I.F by using I.F. = $e^{\int R\,dy}$

STEP III Multiply both sides of the differential equation in step I by I. F.

STEP IV Integrate both sides of the equation obtained is step III w.r.t y to obtain the solution given by $x(\text{I.F.}) = \int S(\text{I.F})\,dy + C$

Where C is the constant of integration. Following examples illustrate the procedure.

(1) Solve $y\,dx - (x + 2y^2)dy = 0$

(2) If y_1 and y_2 are the solutions of the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone and $y_2 = y_1 z$, then prove that $z = 1 + C\, e\int\frac{-Q}{y_1}dx$, where C is an arbitrary constant.

(3) Let u(x) and v(x) satisfy the differential equation $\frac{da}{dx} + P(x)$. u=f(x) and $\frac{dv}{dx}$+p(x). v=g(x) respectively where p(x), f(x) and g(x) are continuous functions. If $u(x_1) > v(x_1)$ for some x_1 and f(x) > g(x) for all $x > x_1$, prove that any point (x, y), where $x > x_1$, does not satisfy the equation y=u(x) and y = v(x).

EQUATIONS REDUCIBLE TO LINEA FORM BERNOULLI'S DIFFERENTIAL EQUATIONS

The equations of the form $\frac{dy}{dx} + Py = Qy^n$ Where P and Q are constants or functions of x alone and n is a non-zero constant other than unity, are known as Bernoulli's equations.

(1) Solve $\frac{dy}{dx}$+xsin2y = $x^3 \cos^2 y$ (2) Solve $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

Solve each of the following differential equations:

(3) $\frac{dy}{dx} + \frac{y}{x} y^3$ (4) $2\frac{dy}{dx} - y\sec x = y^3 \tan x$

(5) $\frac{dy}{dx} + \frac{y}{x} = xe^x y^2$ (6) $(xy^2 - e^{1/x^3})dx - x^2 y\ dy = 0$

EQUATIONS SOLVABLE FOR Y

If the given differential equation is expressible in the form $y = f(x, p)$ then we say that it is solvable for y.

Differentiating (i) with respect to x, we get $\frac{dy}{dx} = f\left(x, p\frac{dp}{dx}\right) or\ \ p = f\left(x, p, \frac{dp}{dx}\right)$

This equation contain two variables x and p. Solving this equation, we obtain

$$\phi(x, p, c) = 0$$

The solution of differential equation (i) is obtained by eliminating p between (i) and (iii).

Following examples will illustrate the above procedure.

(1) Solve the differential equation $y=(1+p)x+ap^2$, where P= $\frac{dy}{dx}$.

(2) Solve the differential equation $x^2p^2 + xyp - 6y^2 = 0$

(3) A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionally constant = k > 0).