<u>Monotonic Functions</u>

STRICTLY INCREASING FUNCTION A function f(x) is said to be a strictly increasing function on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ all $x_1, x_2 \in (a, b)$

STRICTLY INCREASING FUNCTION A function f(x) is said to be a strictly decreasing function on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all x_1, $x_2 \in$ (a, b)

NECESSARY CONDITION Let f(x) be a differentiable function defined on (a, b). Then $f'(x) > 0$ or < 0 according as f(x) is increasing or decreasing on (a, b).

SUFFICIENT CONDITION

THEOREM Let f be a differentiable real function defined on an open interval (a, b).

COROLLARY Let f(x) be a function defined on (a, b)

(a) If f'(x) > 0 for all x∈ (a, b) except for a finite number of points, where f'(x)=0, then f(x) is increasing on (a, b).

(b) If f'(x) < 0 for all x∈ (a, b0 except for a finite number of points, where f'(x)=0, then f(x) id decreasing on (a, b).

SOME USEFUL PROPERTIES OF MONOTONIC FUNCTIONS

(1) If f (x) is strictly increasing function on an interval [a, b], then f^{-1} exists and it is also positive.

(2) If f(x) is strictly increasing function on an interval [a, b] such that it is continuous, then f^{-1} I continuous on [f(a), f(b)].

(3) If f(x) is continuous on [a, b] such that f'(c)≥ 0 (f'©>0) for each c∈(a, b), then f(x) is monotonically (strictly) increasing function on [a, b].

(4) If f(x) is continuous on [a, b] such that f' (c)≤ 0 (f' © < 0) for each c∈ (a, b), then f(x) is monotonically (strictly) decreasing function on [a, b]

(5) If f(x) and g(x) are monotonically (or strictly) increasing (or decreasing) functions on [a, b], then gof(x) is a monotonically (or strictly) increasing function on [a, b]

(6) If one of the two functions f(x) and g(x) is strictly (or monotonically) increasing and other a strictly (monotonically) decreasing, then g of (x) is strictly (monotonically) decreasing on [a, b].

(1) Let f(x) = $\begin{cases} xe^{ax} & , x \le 0 \\ x + ax^2 - x^3, & x > 0 \end{cases}$, where a is a positive constant.

Find the intervals in which f' (x) is increasing.

(2) If ϕ (x) =f(x) + f(1-x) and f'' (x) < 0 for all x∈ [0, 1]. Prove that ϕ(x) is increasing in [0, ½]and decreasing in (1/2, 1].

(3) Let g(x) = 2f(x/2) + f(2-x) and f''(x) < 0 for all x∈ (0, 2). Find the intervals of increases and decrease of g(x).