Time, Speed and Distance

The "time, distance and speed" is an important chapter from the CAT perspective and will surely invite one or more questions from this section. The basic concepts are important to understand, all the questions will be solved on the basics. The calculation speed is not important here, the accuracy sure is.

SPEED
The distance covered per unit time is called speed
Speed =
=> Time = distance/speed
=> Speed is directly proportional to distance and inversely to time

Units
1. Time – Seconds, minutes, hours
2. Distance – meter, kilometer
3. Speed - km/hr, m/sec

Conversion
1. Km/hr = 5m/18sec
2. m/s = 18 km/5 hr
3. Km/hr = 5 miles/ 8 hrs
4. miles/hr = 22 ft/15 sec

Average speed: The average speed is given by total distance by total time taken; this is the formula to be remembered all the time.
Average Speed = Total Distance/Total Time = $(d1+d2+d3+.....d_n)/(t1+t2+.....t_{n)}$

The average speed in case of a journey from X to Y at speed of A m/sec and returning back to X at a speed of B m/sec, is [2AB/(A + B)] m/sec.

Example 1: Sunil travels from Delhi to Noida at the speed of 40 km/hr and returns at the speed of 50 km/hr, what is the average speed of the journey?
Using the formula,

2AB/A+B = 2x40x50/40+50 = 4000/90 = 44.44 Km/hr

Always remember the units, in what units you have started and in what units you are ending.
Alternative way is by the basic principle of mechanics, Speed = distance/time, which means Time = Distance/Speed and Average Speed = Total distance / Total time
Suppose distance from Delhi to Noida is A, therefore

Time while going, T1 = A/40
Time while coming T2 = A/50
Total time = a/40+a/50 = 90a/40x50
Total Distance = A + A = 2A
Average speed =Total Distance/Total Time = a/40+a/50 = 90a/40x50 = 44.44 Km/hr

There are various types of problems from this topic and all of them can be easily solved using the basic formulas given above. We will discuss some formulae and results for different kinds of questions but students should always follow the basics in this chapter.

Relative speed: As the name suggests the concept is regarding the relative speeds to two or more objects. The basic concept in relative speed is that speeds get added in case objects are moving from opposite direction, and get subtracted in case objects are moving in same direction. For example if two trains are moving in opposite direction with a speed of X km/hr and Y km/hr respectively, then (X + Y) is their relative speed. In the other case if two trains are moving in same direction with a speed of X km/hr and Y km/hr respectively, then (X – Y) is their relative speed. For the first case the time taken by the trains in passing each other = (L1 + L2)/(X + Y) hours, where L1 and L2 are length of trains. For the second case the time taken by the trains in passing each other = (L1 + L2)/ (X –Y) hours, where L1 and L2 are length of trains.

Example 2: Two trains 100 m and 80 m in length are running in same direction. The first runs at the rate of 51 m/s and the second at the rate of 42 m/s. How long

will they take to cross each other?
Here Length of train I = 100, Length of train II = 80
And Speed of train I = 51 m/s,
Speed of train II = 42 m/s
Relative speed = 51 – 42 = 9 m/s
(sine trains are in opposite direction)
As per the formula L1+L2/X+ Y
=100+80/9= 20 seconds

Example 3: Two trains 100 m and 80 m in length are running in opposite direction. The first runs at the rate of 10 m/s and the second at the rate of 15 m/s. How long will they take to cross each other?
Here Length of train I = 100, Length of train II = 80
And Speed of train I = 10 m/s,
Speed of train II = 15 m/s
Relative speed = 10 + 15 = 25 m/s (since trains are in same direction)
As per the formula L1 + L2 / X+Y =100+80/25 = 7.2 seconds

In case of train questions, students may remember these obvious results:
1. The time taken by train X meters long in passing a signal post is the time taken for the train to cover X.

Example 4: A train 300 meters long has a speed of
10 m/s. How long will it take to pass an electric pole?
Time = Distance/speed, the distance here will be same as the length of the train
This is 300 meters, therefore
Time = 300/10 = 30 seconds

2. The time taken by train X meters long in passing any object which is Y meters long is the time taken for the train to cover X + Y.

Example 5: A train 300 meters long has a speed of 10 m/s. How long will it take to pass a platform of 50 meters?
Time = Distance/speed, the distance here will be same as the length of the train + the length of the platform. This is 300 + 50 = 350.

Therefore Time = 350/10 = 35 seconds
The same concept is used in water with boats and swimmers. As when you move upstream, your speed gets deducted from the speed of the stream and when you move downstream yours and stream's speed gets added:
Let the speed of a boat in still water be A km/hr and the speed of the stream (or current) be B km/hr, then
(a) Speed of boat with the stream = (A + B) km/hr
(b) Speed of boat against the stream = (A – B) km/hr
There are two obvious results in this concept:
1. Boat's speed in still water = speed down stream + Speed upstream/2

Example 6: A boat travels equal distance upstream and downstream, the upstream speed of boat was 10 km/hr, whereas the downstream speed is 20 km/hr, what is the speed of the boat in still water?
Upstream speed = 10 km/hr
Downstream speed = 20 km/hr
As per formula, Boat's speed in still water = speed down stream + Speed upstream/2

Therefore, Boat's speed in still water = = 15
2. Speed of current
= speed down stream - Speed upstream/2

Example 7: A boat travels equal distance upstream and downstream, the upstream speed of boat was 10 km/hr, whereas the downstream speed is 20 km/hr, what is the speed of the current?
Upstream speed = 10 km/hr
Downstream speed = 20 km/hr
As per formula, Speed of current = Speed down stream - upstream / 2
Therefore, Speed of current = 10/2 = 5 km/hr

Concept of Races: Race is a competition between contestants in order to reach a point fastest. There can be many kinds of races. We will study linear and circular races.
Linear races (non-circular): The concepts of time speed and distance are used in

races, which may be linear or circular or other types. Although with the basic concepts given above you will be able to solve the races questions, but here are a few definitions in races,

Imagine, X and Y are two contestants in a race:

1. Before the start of the race, if X is at the starting point and Y is ahead of X by 10 meters, then X is said to give Y a start of 10 meters. 10 meter here can be called as start distance or distance at start
2. In a 100m race, If it is written "X can give Y 20 m start" or "X beats Y by 20 m", it means that in the time X runs 100 m, Y runs 80 m. 20 meter here can be called as beat distance called as beat distance.
3. Similarly, If it is written "X can give Y 20 second start" or "X beats Y by 20 seconds", it means if the given distance is covered by X in a seconds, then Y will take (a – 20)
4. Winner's distance – (start distance + beat distance) = loser's distance
5. Winner's time + (start time + beat time) = loser's time. (Remember here that the winner's time is less than loser's time, so something has to be added to equate)
6. A dead heat means the contestants reached the end point at same time

Example 8: Ram can give Hari 20 m start and Hari can give Ravi 10 m start in a race of 200 m. By how much could Ram beat Hari in the same race?

Ram can give Hari a start of 20 m, that means in 200 meter race ram can cover 200 meters in the same time as Hari covers 180 meters.

Hari can give Ravi a start of 10 m, which means in 200 meter race Ravi can cover 200 meters in the same time as Hari covers 190 meters.

From second point, Ravi can cover 1 meter, when Hari covers 190/200 meters

Also, Ravi can cover 180 meter,

When Hari covers × 180 = 171 meters

Also when Ravi covers 180 meters, Ram covers 200 meters, and in this time Hair covers 171 meters, therefore Ram can give Hari a start distance of 29 meters.

Circular races: In circular races, the race is in a perfect circle. Here are some important points on circular races, with two or more people starting from same starting point and at same time:

1. The time taken by the faster one to gain one full round over the other is (length of race course)/ (relative speed)

Example 9: Ravi and Ram run around a circular path of circumference 1000 meters. Ravi runs at 4 m/s and Ram runs at 2 m/s. If they start from the same point and walk in the same direction, when will they be first together again?
Ravi's speed = 4 m/s, Ram's speed = 2 m/s
Relative speed = 2 m/s
They will be together again when faster gains full circle over the slower
Therefore time to gain full circle = 1000/2 = 500 seconds
2. They will be first together at the starting point again after an interval of time which is the LCM of the times in which each of them makes one complete round.

Example 10: Ravi and Ram run around a circular path of circumference 1000 meters. Ravi runs at 4 m/s and Ram runs at 2 m/s. If they start from the same point and walk in the same direction. When will they be first together again at the starting point?
Ravi's speed = 4 m/s, Ram's speed = 2 m/s
Time taken for Ravi to complete one lap
= 1000/4 = 250 seconds
Time taken for Ravi to complete one lap
= 1000/2 = 500 seconds
They will be together again at the starting point
= LCM of 250 and 500
This is 500 seconds.
3. The persons will be together again for the first time at the time which is the LCM of the times taken by the fastest to gain a lap over the others. This is a universal formula for any number of people running.

Example 11: Ravi, Bhuvan and Ram run around a circular path of circumference 1000 meters. Ravi runs at 4 m/s, Bhuvan runs at 6 m/s and Ram runs at 2 m/s. If they start from the same point and walk in the same direction. When will they be first together again?
Ravi's speed = 4 m/s, Bhuvan's speed = 6 m/s, Ram's speed = 2 m/s
Time taken for Bhuvan to gain a lap over Ravi
= 1000 /6 - 4 = 500 seconds
Time taken for Bhuvan to gain a lap over Ram

= 1000/6 - 2= 250 seconds
Time taken for Ravi to gain a lap over Ram
= 1000/4 - 2 = 500 seconds
They will be together again = LCM of 500, 250 and 500, which is 500

Clocks or Watches

The dial of a clock or watch is a circle whose circumference is divided into 60 equal parts (called minute spaces), all the hands of clock which are second, minute and hour hands travel along this circumference. Here are some important learning's on clocks:

1. In the clock, the larger hand (minute hand) moves 60 minute spaces, the smaller hand (hour hand) moves only 5 minute spaces, which is completion of an hour.
2. In each minute space the hour hand travels, the minute hand travels 12 minutes; this is the most important concept in clocks.
3. When the two hands are at right angles, they are 15 minute spaces apart. This happens twice every hour.
4. When the hands are in opposite directions, they are 30 minute spaces apart. This happens once every hour.
5. The hands are in the same straight line when they are coincident or opposite to each other.
6. If both the hands start moving together from the same position, both the hands will coincide after
= 65 minutes.
7. In case of incorrect clocks, the clocks either gain time or loose time

Example 12: The time in a clock is 20 minute past 2. Find the angle between the hands of the clock.

Here the hour hand is close to 2 and the minute hand is at 4, the angle between 2 and 4 is × 2 = 60 degrees. Angle made by the hour hand in one hour is 360/12 = 30 degrees, Angle made in 20 minutes = 30/3 = 10.
Therefore, required angle = 60 – 10 = 50 degrees

SOLVED EXAMPLES

Q1. Ram and Ravi are 100 km apart and started to walk towards each other at 10

am. Ram walked at the rate of 5 km/hr and Ravi at 10 km/hr. at what time will they meet?
Ans. Relative speed = 5 + 10 = 15 km/hr

Time to walk 100 km = 100/15 = 6.66 hours, which is 6 hours and 40 min (approx)
As they started at 10 am, therefore they meet after
10 + 6 hours and 40 min = 4:40 pm

Q2. Hari runs after Sam who is 200 m ahead of him. If speed of Hari is 15 km/hr and that of Sam is 10 km/hr, what will be the distance covered by Hari when he catches Sam?
Ans. Relative speed = 15 × 10 = 5 km/hr
Distance to be covered = 200 m = .2 km
Time taken = = 0.04 hrs
Distance covered = 15 × 0.04 = 0.6 km = 600 meters

Q3. Two trains, 200 and 160 meters long take a minute to cross each other while traveling in the same direction and take only 10 seconds when they cross in opposite directions. What are the speeds at which the trains are traveling?
Ans. Distance covered (sum of lengths of the train)
= 200 + 160 = 360 meters.
Let Train 1 be traveling at X m/sec and Train 2 be traveling at Y m/sec. (considering X > Y)
Now while traveling in same direction Time =
Therefore 60 = , 60X – 60Y = 360
Now while traveling in opposite direction Time
=
Therefore 10 = , 10X +10Y = 360
Solving the two equations X = 21 m/sec
Y = 15 m/sec

Q4. A train traveling at 72 km/hr crosses a platform in 30 seconds and a man standing on the platform in 18 seconds. What is the length of the platform in meters?
Ans. Students may recall from the theory of the chapter, When the train crosses a

man standing on a platform, the distance covered by the train is equal to the length of the train and when the same train crosses a platform, the distance covered by the train is equal to the length of the train plus the length of the platform.
Train takes 18 seconds to cross a man, which is basically to cover length of the train
Train takes 30 seconds to cross the platform, which is to cover length of the train and length of the platform. Therefore the extra 12 seconds are to cover the length of the platform. Therefore Length of platform
=72x12/3600 = 0.24 km = 240 meter.

Q5. Two trains starting from the same station and traveling in opposite directions are 228 km apart in 3 hours. Had they been traveling in same direction they would have been 33 km apart in the same time. What are their speeds?
Ans. They are 228 km apart in 3 hrs traveling in opposite direction
In one hour they will be 228/3 = 76 km = Sum of speeds of two trains
Assume speeds to be X km/hr and Y km/hr
Therefore X + Y = 76 km/hr
In the other case X – Y = 33/3 = 11 km/hr
Solving X = 43.5 km/hr, Y = 32.5 Km/hr

Q6. Two trains traveling in the opposite directions pass each other in 8 seconds. But when they travel in same direction at the same rates, the man in the faster train passes the other in 30 seconds. Find the lengths of the trains when their speeds are 45 km/hr and 35 km/hr.
Ans. When trains travel in opposite direction,
Time = L1+L2 /X+Y
Therefore, 8/3600 = L1+L2/45+35
Lengths of train, L1 + L2 = 0.17778 km
= 177.78 meter
When trains travel in same direction, the man in the faster train passes the other in 30 seconds, the man only passes the length of the smaller train.
Therefore distance traveled by trains in 30 seconds = length of the smaller train
Relative speed = 45 – 35 = 10 km/hr
Distance traveled = × 10
= 0.08334 km = 83.34 meter

Therefore length of longer train
= 177.78 – 83.34
= 94.44 meters

Q7. Two trains 150 miles distant travel towards each other along the same track, the first train at 60 km/hr, the second at 90 km/hr. A fly buzzes back and forth between the two trains until they collide. If the fly's speed is 110km/hr, how far will it travel and how many rounds will it take?
Ans. Since the trains start 150 km apart and have a relative speed of 90 + 60 = 150 km/hr, they will meet in exactly 1 hour. The bird is flying at a speed of 110 km/hr, so in 1 hour it flies 110 km. The interesting part remains the number of rounds, which actually is an interesting physics problem with a infinite series being formed and the answer is infinity.

Q8. Two trains A and B each of length 100m travel in opposite directions in parallel tracks. The speeds are 20m/s and 30m/s respectively. A boy sitting in the front end of train A throws a ball to a boy sitting in the front end of train B when they are at the closest distance. The speed of the ball is 2m/s. The ball, instead of reaching the boy, hits the rear end of the train. Find the distance between the parallel tracks.
Ans. The boy throws the ball when trains are at closest distance, which is when front ends of trains meet; therefore boy throws perpendicular to the train, in line with the distance between the tracks.
Suppose from the time ball leaves boy's hands to hitting the train's rear is T
In this time T, Train A has traveled 20T
Train B has traveled 30T
Since ball hits the rear of train B, with length of trains being 30T + 20 T = 100, therefore T = 2 seconds
Since the ball travels at 2 m/s, and ball hitting the other train means that ball has traveled the distance between two tracks, assuming distance between tracks is D
Therefore 2T = D, D = 4 meters

Q9. Ram and Mohan start at the same time from A to B to go to B and A, a distance of 42 km at the rates of 4 km/hr and 3 km/hr. They meet at X, then go to B and A and return immediately and meet again at Y. find the distance XY

Ans. First Ram and Mohan meet at X, Distance = 42 km
First Ram and Mohan meet at Y,
Distance = 42 × 3 = 126 km
Speed of Ram = 4 km/hr,
Speed of Mohan = 3 km/hr
Ram travels AX distance and Mohan BX
Relative speed = 4 + 3 = 7 km/hr
Time when they meet first = 42/7 = 6 hrs
In 6 hrs Ram travels 6 × 4 = 24 km = AX
Therefore Mohan travels = 42 – 24 = 18 = BX
Time when they meet second = 126/7 = 18 hrs
In 18 hrs Ram travels 18 × 4 = 72 km
= AX + XB + BY
Since AX + XB = 42, BY = 30 and AY = 12
Since AX = 24, and AY = 12. XY = 12 km

Q10. There are two boats that start out on opposite sides of a river at the same time. Each one is heading across the river to the other side. They each go a constant speed throughout the entire problem (so ignore having to slow down to turn around, and ignore current, etc.), but they are not necessarily the same speed as each other. When each boat reaches its opposite bank, it immediately turns around and heads back to where it started. The boats thus pass each other twice. The first time they pass, they are 700 yards from one of the banks of the river. The second time they pass, they have each turned around after reaching their respective opposite shores and have started back toward where they each began. When they pass the second time, they are 300 yards from the other bank of the river. How wide is the river?
Ans. Assume total distance = X, and speeds of boats being S1 and S2 when the two boats meet for the first time,
Distance travel by boat I, D1 = 700 yards
Distance travel by boat II, D2 = X – 700 yards
Here Time is the same, therefore
T = D1/S1 and T = D2/S2
Therefore D1/S1 = D2/S2
AlsoD1/S1 = D2/S2 = 700 /700-x

Now, Boat I then continues on to the bank, which is (X – 700) yards away, and then it turns around and goes back 300 yards. After the first meeting, boat I travels (X – 700) + 300, which is (X – 400) yards. After the first meeting Boat II travels 700 yards to the bank and then turns around and travels back (X – 300) yards, which is (X + 400) yards, where it then meets Boat I again.
When the two boats meet for the second time after first meeting,
Distance travel by boat I, D1 = X – 400 yards
Distance travel by boat II, D2 = X + 400 yards
Here Time is the same, therefore
T = D1/S1 and T = D2/S2
Therefore D1/S1 = D2/S2
Also = X - 400/X+400=

Solving X = 1800 meters

Q11. Ravi can swim with the stream at the rate of 10 km/hr and 5 km/hr against the stream; find his speed in still water.
Ans. Upstream speed = 10 km/hr, Downstream speed is 5 km/hr
Using direct formula 1/2 (10 + 5) = 7.5 km/hr

Q12. Hari swims 20 km downstream a river in 5 hours and returns in 10 hours. What is his speed and speed of the stream?
Ans. Speed downstream
= 20/5 = 4 km/hr
= Speed of Hari + Speed of stream
Speed downstream
= 20/10 = 2 km/hr
= Speed of Hari – Speed of stream
From the two equations speed of Hari = 3 km/hr
Therefore speed of stream = 4 – 3 = 1 km/hr

Q13. A boat travels from point A to point B upstream and returns from point B to point A downstream. If the round trip takes the boat 5 hours and the distance between point A and point B is 120 km and the speed of the stream is 10 km/hr,

what is the speed of the boat?
Ans. Total distance = 120 × 2 = 240, Total time = 5hrs
Average speed = 240/5 = 48 km/hr
Assuming speed of boat = X
Speed downstream
= Speed of Boat + Speed of stream
= X + 10
Speed downstream
= Speed of Boat – Speed of stream
= X – 10
Since average speed = 48, and using formula of average speed
48 =
Solving for X, X = 50 km/hr

Q14. Mohan can beat Ravi by 50 m in a 1700 m race; Ravi can beat Shyam by 20 m in a 1700 m race. If Mohan and Shyam run 1700 m, by how much will Mohan win?
Ans. The students should be able to do this faster using simple logic:
Mohan can beat Ravi by 50 m, Mohan travels 1700, when Ravi travels 1650
Ravi can beat Shyam by 20m, Ravi travels 1700, when Shyam travels 1680
Therefore when Ravi travels 1 Shyam travels
Therefore when Ravi travels 1650 Shyam travels 1680/1700
1680/1700 × 1650 = 1630.5
Mohan beats Shyam by 1700 – 1630.5 = 69.5 meters

Q15. In a kilometer race, A can give B a start of 100 m or 15 seconds. How long does A take to complete the race?
Ans. In a 1000 meter race A gives B a start of 100 m or 15 seconds. This means that B takes 15 seconds to run 100 m. Therefore, B will take 150 seconds to run the stretch of 1000 meters. As A takes 15 seconds less than B, he will take 135 seconds to run the 1000 m.