

Ratio and proportion

Ratio and proportion is the continuation of the basic concepts for the entrance tests. The application of the topics is far and wide, a direct application which will be covered here is “Allegation and Mixtures”.

Ratio:

The comparison of two quantities of the same unit is the ratio of one quantity to another. The ratio of A and B is written as A : B or A / B , where A is called the antecedent and B the consequent.

Example: The ratio of 10 kg to 20 kg is 10:20 or $10/20$, which is 1:2 or $\frac{1}{2}$, where 1 is called the antecedent and 2 the consequent.

Properties of Ratio

1. $a : b = ma : mb$, where m is a constant

2. $a : b : c = A : B : C$ is equivalent to $a / A = b / B = c / C$, this is an important property and has to be used in ratio of three things.

3. If $a / b = c / d$, then

$(a+b) / b = (c+d) / d$ – Componendo

Example: $\frac{1}{2} = \frac{2}{4}$, so $(1+2)/2 = (2+4)/4 \Rightarrow \frac{3}{2} = \frac{6}{4} \Rightarrow \frac{3}{2} = \frac{3}{2}$

$(a-b) / b = (c-d) / d$ – Dividendo

Example: $\frac{10}{4} = \frac{20}{8}$, so $(10-4)/4 = (20-8)/8 \Rightarrow \frac{6}{4} = \frac{12}{8} \Rightarrow \frac{3}{2} = \frac{3}{2}$

$(a+b) / (a-b) = (c+d) / (c-d)$ – Componendo and Dividendo

Example: $\frac{10}{4} = \frac{20}{8}$, so $(10+4)/(10-4) = (20+8)/(20-8) \Rightarrow \frac{14}{6} = \frac{28}{12} \Rightarrow \frac{7}{3} = \frac{7}{3}$

Application: These properties have to be used with quick mental calculations; one has to see a ratio and quickly get to results with mental calculations.

Example: $\frac{10}{4} = \frac{20}{8}$, should quickly tell us that $\frac{14}{4}(\frac{7}{2}) = \frac{28}{8}(\frac{7}{2})$, $\frac{6}{4}(\frac{3}{2}) = \frac{12}{8}(\frac{3}{2})$ and $\frac{14}{6}(\frac{7}{3}) = \frac{28}{12}(\frac{7}{3})$

4. If $a / b = c / d = e / f = \dots$, then $(a+c+e+\dots)/(b+d+f+\dots) =$ each of the individual ratio

Example: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, there fore $(1+2+4)/(2+4+8) = \frac{7}{14} = \frac{1}{2}$

5. If $A > B$ then $(A+C)/(B+C) < A/B$ Where A, B and C are natural numbers

Example: $3 > 2$, then $(3+4)/(2+4) = \frac{7}{6} (1.16)$

6. If $A < B$ then $(A+C)/(B+C) > A/B$ Where A, B and C are natural numbers.

Proportion is an expression in which two ratios are equal, that is $A/B = C/D$. Here $AD = BC$

If four numbers a, b, c and d are in proportion then we can say that:

$$a / b = c / d$$

Example: The ratio of 10 kg to 20 kg is 10:20 or $\frac{10}{20}$ or $\frac{1}{2}$, and the ratio of 30 kg to 60 kg is 30:60, or $\frac{30}{60}$ or $\frac{1}{2}$, so $\frac{10}{20} = \frac{30}{60}$, they are in proportion

If $a:b = b:c = c:d$ then a, b, c, d are in continued proportion. Here $a/b = b/c = c/d$.

Also a, b, c, d are in geometric progression

Example: 2, 4, 8, 16 or 3, 9, 27, 81, Here $\frac{2}{4} = \frac{4}{8} = \frac{8}{16}$, and $\frac{3}{9} = \frac{9}{27} = \frac{27}{81}$.

So they are in continued proportion and also in geometric proportion.

Continued Proportion

If three numbers a, b and c are in continued proportion, then:

We can say that a, b, b and c are in proportion.

i.e. $a / b = b / c$

$$b^2 = a c$$

Here we can say that

a is called first proportion, c is called third proportion and b is called mean proportion.

Types of proportion

1. DIRECT PROPORTION:

If X is directly proportional to Y, that means any increase or decrease in any of two quantities will have proportionate effect on the other quantity. If X increases then Y will also increase and vice-versa.

When X is directly proportional to Y, it is written as $X \propto Y$, to bring in an equality sign, you have to introduce a constant, say k. so $X = k Y$. From here X/Y is a constant, so $X/Y = k$.

2. INVERSE PROPORTION:

If X is inversely proportional to Y, that means any increase or decrease in any of two quantities will have inverse proportionate effect on the other quantity. This means if X increases, then Y decreases and if X decreases the Y increases and vice-versa for Y.

When X is inversely proportional to Y, it is written as $X \propto 1/Y$, to bring in an equality sign, you have to introduce a constant, say k. so $X = k/Y$. From here XY is a constant, so $XY = k$.

Both these proportions have wide applications in many subjects, especially sciences and economics, where many factors are directly and inversely proportional to each other.

Applications of Ratio and Proportion

Partnership

Partnership, as the name suggests, its more than one person investing in something to increase resources, reduce risk etc.

Important point: The partners are liable for income/expenditure/profit/loss as per their percentage holding of the business or their partnership percentage. This is defined by their capital/work/other things contribution in terms of time. The partnership terms can of many types, in questions you may encounter partners putting monies for different time periods, which can be calculated by multiplying the money with the time in months the money has been invested for.

Example: Ram partnered with Rohan and Ravi (equal partners) in a business and added capital of Rs. 10000 for six months in a year. The capital for those six months became Rs. 25000 for the year, if the end of year profit is 2500, what is Ram's net profit?

Ram's capital = 10000 for six months

Total capital = 25000 for six months

Ravi and Rohan's capital = 25000 – 10000 = 15000 for 12 months

Ravi's Capital = 7500 for 12 months Rohan's capital = 7500 for 12 months

Ram's share = $(10000 \times 6) / (10000 \times 6) + (7500 \times 12) + (7500 \times 12)$
 $= 60000/24000 = \frac{1}{4}$

Ram's Profit = $\frac{1}{4} \times 2500 = \text{Rs. } 625$

Mixtures and Allegation

“Mixtures and allegations” is about mixing different objects in order to get desired levels/percentage/concentration of different objects. Ratio and proportion has direct application in “mixtures and allegations”. The concept of weighted average is also used to solve mixture questions, so it is suggested that students brush up the “Averages” chapter before starting off with this chapter.

Example: If 100 ml water is mixed 1000 ml of milk, what is the ratio of the mixture solution?

Using basic percentage, total solution = 1100 ml, so $100/1100 = 1/11$

Water: milk = 1:10,

This can also be read as milk solution 10:11, where if milk is 10, water is 1 and total solution is 11.

This lingo will be used extensively in this chapter; it may be for milk, alcohol, spirit etc.

Example: Two bottles contain mixture of milk and water in the ratio of 8:3 and 5:1. In what ratio must liquid be drawn from each bottle to give a mixture in the ratio of 4:1?

Here using basic logic, suppose we take x units from bottle one and y units from bottle two to make a mixture of 4:1, bottle with ratio 8:3 will contribute $\frac{8}{11}x$ milk and $\frac{3}{11}x$ water, similarly other bottle will contribute $\frac{5}{6}y$ milk and $\frac{1}{6}y$ water

Total milk = $\frac{8}{11}x + \frac{5}{6}y$

Total Water = $\frac{3}{11}x + \frac{1}{6}y$

Now total milk/ total water = 4/1, so $[\frac{8}{11}x + \frac{5}{6}y] / [\frac{3}{11}x + \frac{1}{6}y] = 4/1$

Solving we get $x/y = 11/24$, so $x:y = 11:24$

We will study a “Rule of allegation” to solve the questions on mixing products like tea etc where different varieties (with different costs) are mixed to get desired Variety, but students should know that formula is just an aid, another way of solving the question. Students should know how to solve the question without the formula. The allegation uses simple logic, but some students become slave of the allegation formula, which may lead to lot of errors.

Rule of allegation

This rule helps us in solving questions where two varieties (of different prices) are mixed to get a new variety with a new Average price.

Quantity of cheaper = Price of Dearer - Average price

 Quantity of dearer Average price – Price of cheaper

Example: In what ratio should tea at the rate Rs. 40/kg be mixed with tea at the rate Rs. 27/kg, so that mixture may cost Rs 30/kg?

Using the above formula

Quantity of cheaper / quantity of dearer = $(40 - 30) / (30 - 27) = 10/3$

So, the two should be mixed in the ratio 10:3.

In case of liquids, there is another formula which can be used in various questions. In a vessel containing x litre of a one liquid (say pure milk), if y litres is withdrawn and replaced by another liquid (say pure water), and this is repeated n times, then:

Milk left in vessel after nth operation = $x [1 - y/x]^n$

Example: In a vessel full of 5 litres of milk, five times 250 ml is taken and replaced with water, what is the concentration of milk left in the vessel

As per the formula, Milk left in vessel after nth operation = $x [1 - y/x]^n$

Milk left = $5(1 - 0.25/5)^5$ (remember to covert ml into litres)

Solving, Milk left = 3.86 litres

The concentration is $3.86/5 \times 100 = 77.3 \%$

Miscellaneous Examples:

Q1. If two numbers are in the ratio of 10:3 and 2 is subtracted from each, the resulting numbers are in the ratio 9:2. Find the numbers

Ans1. Since the ratio is 10:3, let the numbers be 10X and 3X

Therefore $(10X - 2) / (3X - 2) = 9/2$

$7X = 14, X = 2$

Therefore numbers are 20 and 6

Q2. A common foodstuff is found to contain 2.5% iron. The serving size is 90.0 grams. If the recommended daily allowance is 18 gm of iron, how many servings would a person have to eat to get 100% of the daily allowance of iron?

Ans2. Iron percentage = $2.5/100 \times 90 = 2.25$ grams

Total grams required = 18

Servings required = $18/2.25 = 8$ servings in a day

Q3. A substance is 99% water. Some water evaporates, leaving a substance that is 98% water. How much of the water evaporated?

Ans3. The substance has 99% water and 1 % other substance

Let the amount of water be X and the other substance be Y

Now $X/(X+Y) = 99/100, Y = X/99$

After some evaporation suppose water left = Z

Therefore $Z/(Z+Y) = 98/100, Y = 2Z/98$

Equating Y, $X/99 = 2Z/98$

$Z = 0.495X$

Therefore Current water level is 0.495 for earlier water level, so 50.5 % water has been evaporated.

Q4. If I clean a 3200 square foot building five nights per week for a sum of Rs. 575 per month, what is the cost per square foot?

Ans4. Total square foot = 3200

Total sum = Rs. 575

Cost per square foot = $575/3200 = \text{Rs. } 0.18$ per square foot

Q5. A and B started a business by investing Rs 50000 and Rs 25000. What is the share of each if yearly profit is Rs 2000?

Ans5. Total investment = 50000 + 25000 = Rs.75000

A's share = $50000/75000 \times 2000 = \text{Rs.}1333.34$

B's share = $25000/75000 \times 2000 = \text{Rs.} 666.66$

Q6. The salary of Ravi, Ajay and Bhuvan is Rs 350000. If they spend 70%, 75%, and 80% of their salaries respectively, their savings are in ratio of 15:10:25. Find their salaries.

Ans 6. Total Salary of the three = 350000

Ravi's spent = 70%, therefore Ravi's saving = 30%

Ajay's spent = 75%, therefore Ajay's saving = 25%

Bhuvan's spent = 80%, therefore Bhuvan's saving = 20%

30 % of Ravi's Salary: 25 % of Ajay's Salary: 20 % of Bhuvan's Salary =
15:10:25

$30/100 \text{ R} : 25/100 \text{ A} : 20/100 \text{ B} = 15:10:25$

$30\text{R}:25\text{A}:20\text{B} = 15:10:25$

From here $30\text{R}/25\text{A} = 15/10$, $\text{R}/\text{A} = 375/300 = 17/12 = 34/24$

Also $25\text{A}/20\text{B} = 10/25$, $\text{A}/\text{B} = 8/25 = 24/75$

Now $\text{R} : \text{A} : \text{B} = 34:24:75$

Ravi's Salary = $34/133 \times 350000 = 89474$

Ajay's Salary = $24/133 \times 350000 = 63158$

Bhuvan's Salary = $75/133 \times 350000 = 197368$

Q7. Divide Rs 435 among A, B and C so that if Rs 9, Rs 4 Rs 2 be subtracted from their respective shares, the shares left may be in the ratio 6:4:5.

Ans7. Here the ratio of shares is given 3:4:5

The total is 435, and $9+4+2 = 15$ needs to be subtracted from it
 $= 435 - 15 = 420$

Now diving 420 in ratio of 6:4:5

A's Share = $6/15 \times 420 = 168$, $168 + 9 = 177$

B's Share = $4/15 \times 420 = 112$, $112 + 4 = 116$

C's Share = $5/15 \times 420 = 140$, $140 + 2 = 142$

Q8. A, B and C partnered in a business. A contributed Rs 12000, B Rs 10000 and C Rs 8000 and their profit was Rs. 2400. What is the share of each?

Ans8. A's Contribution = 12000

B's Contribution = 10000

C's Contribution = 8000

Total = 30000

A's share = $12000/30000 \times 2400 = \text{Rs } 960$

B's share = $10000/30000 \times 2400 = \text{Rs } 800$

C's share = $8000/30000 \times 2400 = \text{Rs } 640$

Q9. A, B and C partnered in a business for a year. A contributed Rs 12000 for 6 months, B Rs 10000 for 8 months and C Rs 8000 for the entire year and their profit was Rs. 2728. What is the share of each?

Ans9. A's Contribution = 12000 for 6 months = $12000 \times 6 = 72000$

B's Contribution = 10000 for 8 months = $10000 \times 8 = 80000$

C's Contribution = 8000 for 12 months = $8000 \times 12 = 96000$

Total = 248000

A's share = $72000/248000 \times 2728 = \text{Rs } 792$

B's share = $80000/248000 \times 2728 = \text{Rs } 880$

C's share = $96000/248000 \times 2728 = \text{Rs } 1056$

Q10. Ravi and Mayank enter into a partnership by investing Rs. 7000 and Rs. 3000 respectively. At the end of one year, they divided their profits such that $1/3$ of the profit is divided equally for the efforts they have put into the business and the remaining amount of profit is divided in the ratio of the investments they made in the business. If Ravi received Rs. 8000 more than Mayank did, what was the profit made by their business in that year?

Ans10. Let the profit made during the year be X

The Profit to be divided equally = $X/3$

The profit to be divided as per contribution = $2X/3$

The diving ratio is $7000:3000 = 7:3$

Ravi shall get more profit in $2X/3$, by $70-30 = 40\%$ more

Therefore $40/100 \times 2X/3 = 8000$

The profit, $X = \text{Rs. } 30000$