To find the number of factors of a given number, express the number as a product of powers of prime numbers.

In this case, 48 can be written as $16 * 3=\left(2^{4} * 3\right)$
Now, increment the power of each of the prime numbers by 1 and multiply the result.
In this case it will be $(4+1) *(1+1)=5 * 2=10$ (the power of 2 is 4 and the power of 3 is 1)

Therefore, there will 10 factors including 1 and 48. Excluding, these two numbers, you will have $10-2=8$ factors.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
The sum of first n natural numbers $=\mathrm{n}(\mathrm{n}+1) / 2$
The sum of squares of first $n$ natural numbers is $n(n+1)(2 n+1) / 6$
The sum of first $n$ even numbers $=n(n+1)$
The sum of first n odd numbers $=\mathrm{n} \wedge 2$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ To find the squares of numbers near numbers of which squares are known

To find $41 \wedge 2$, Add $40+41$ to $1600=1681$
To find $59^{\wedge} 2$, Subtract $60^{\wedge} 2-(60+59)=3481$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ If an equation (i:e $f(x)=0$ ) contains all positive co-efficient of any powers of $x$, it has no positive roots then.
eg: $x^{\wedge} 4+3 x^{\wedge} 2+2 x+6=0$ has no positive roots .
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ For an equation $f(x)=0$, the maximum number of positive roots it can have is the number of sign changes in $f(x)$; and the maximum number of negative roots it can have is the number of sign changes in $f(-x)$.
Hence the remaining are the minimum number of imaginary roots of the equation(Since we also know that the index of the maximum power of $x$ is the number of roots of an equation.)
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

For a cubic equation $a x^{\wedge} 3+b x^{\wedge} 2+c x+d=0$
sum of the roots $=-b / a$
sum of the product of the roots taken two at a time $=c / a$
product of the roots $=-\mathrm{d} / \mathrm{a}$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
For a biquadratic equation $a x^{\wedge} 4+b x^{\wedge} 3+c x^{\wedge} 2+d x+e=0$
sum of the roots $=-b / a$
sum of the product of the roots taken three at a time $=c / a$
sum of the product of the roots taken two at a time $=-d / a$
product of the roots $=\mathrm{e} / \mathrm{a}$
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
If for two numbers $x+y=k$ (=constant), then their PRODUCT is MAXIMUM if
$x=y(=k / 2)$. The maximum product is then $\left(k^{\wedge} 2\right) / 4$

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+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
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If for two numbers $x^{*} y=k$ (=constant), then their SUM is MINIMUM if
$x=y(=\operatorname{root}(k))$. The minimum sum is then $2 * \operatorname{root}(k)$.
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
$|x|+|y|>=|x+y|$ ( $|\mid$ stands for absolute value or modulus )
(Useful in solving some inequations)
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

Product of any two numbers = Product of their HCF and LCM .
Hence product of two numbers = LCM of the numbers if they are prime to each other
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
For any regular polygon, the sum of the exterior angles is equal to 360 degrees
hence measure of any external angle is equal to $360 / \mathrm{n}$. ( where n is the number of sides)
For any regular polygon, the sum of interior angles $=(n-2) 180$ degrees

So measure of one angle in

| Square | $=90$ |
| :--- | :--- |
| Pentagon | $=108$ |
| Hexagon | $=120$ |
| Heptagon | $=128.5$ |
| Octagon | $=135$ |
| Nonagon | $=140$ |
| Decagon | $=144$ |

$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ If any parallelogram can be inscribed in a circle, it must be a rectangle.
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
If a trapezium can be inscribed in a circle it must be an isosceles trapezium (i:e oblique sides equal).
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
For an isosceles trapezium, sum of a pair of opposite sides is equal in length to the sum of the other pair of opposite sides . (i:e $A B+C D=A D+B C$, taken in order) .
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
Area of a regular hexagon : $\operatorname{root}(3) * 3 / 2 *($ side $) *($ side $)$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ For any 2 numbers $a>b$
$a>A M>G M>H M>b$ (where $A M, G M, H M$ stand for arithmetic, geometric, harmonic menasa respectively)

$$
(\mathrm{GM})^{\wedge} 2=A M * H M
$$

For three positive numbers $a, b$, $c$

```
(a+b+c)*(1/a+1/b+1/c)>=9
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

For any positive integer $n$
$2<=(1+1 / n)^{\wedge} n<=3$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
$a^{\wedge} 2+b^{\wedge} 2+c^{\wedge} 2>=a b+b c+c a$
If $a=b=c$, then the equality holds in the above.

```
a^4+b^4+c^4+d^4 >=4abcd
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
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$(\mathrm{n}!)^{\wedge} 2>\mathrm{n}^{\wedge} \mathrm{n}$ (! for factorial)
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++{ }_{+}^{+}$

If $a+b+c+d=$ constant, then the product $a \wedge p * b^{\wedge} q{ }^{*} c^{\wedge} r^{*} d \wedge s$ will be maximum if $a / p=b / q=c / r=d / s$.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

## Consider the two equations

$a 1 x+b 1 y=c 1$
$a 2 x+b 2 y=c 2$

Then,
If $a 1 / a 2=b 1 / b 2=c 1 / c 2$, then we have infinite solutions for these equations.
If $a 1 / a 2=b 1 / b 2<>c 1 / c 2$, then we have no solution for these equations.( $<>$ means not equal to )
If $a 1 / a 2<>b 1 / b 2$, then we have a unique solutions for these equations..
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ For any quadrilateral whose diagonals intersect at right angles, the area of the quadrilateral is $0.5^{*} \mathrm{~d} 1 * \mathrm{~d} 2$, where $\mathrm{d} 1, \mathrm{~d} 2$ are the lenghts of the diagonals.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ Problems on clocks can be tackled as assuming two runners going round a circle , one 12 times as fast as the other. That is,
the minute hand describes 6 degrees /minute
the hour hand describes $1 / 2$ degrees /minute .
Thus the minute hand describes $5(1 / 2)$ degrees more than the hour hand per minute .
The hour and the minute hand meet each other after every 65(5/11) minutes after being together at midnight.
(This can be derived from the above).
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ If $n$ is even, $n(n+1)(n+2)$ is divisible by 24

If $n$ is any integer, $\mathrm{n}^{\wedge} 2+4$ is not divisible by 4
meeting point of the diagonals can be found out by solving for $[(a+e) / 2,(b+f) / 2]=[(c+g) / 2,(d+h) / 2]$

Area of a triangle
$1 / 2 *$ base*altitude $=1 / 2 * a * b * \sin C=1 / 2 * b * c^{*} \sin A=1 / 2 * c^{*} a * \sin B=\operatorname{root}\left(s^{*}(s-a) *(s-b) *(s-\right.$ c)) where $s=a+b+c / 2$
$=a^{*} b^{*} c /(4 * R)$ where $R$ is the CIRCUMRADIUS of the triangle $=r^{*} s$, where $r$ is the inradius of the triangle .

In any triangle
$a=b^{*} \operatorname{Cos} C+c^{*} \cos B$
$b=c^{*} \cos A+a * \cos C$
$c=a * \cos B+b^{*} \operatorname{Cos} A$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
If $a 1 / b 1=a 2 / b 2=a 3 / b 3=$ $\qquad$ then each ratio is equal to
(k1*a1+ k2*a2+k3*a3+ $\qquad$ / (k1*b1+ k2*b2+k3*b3+ $\qquad$ .), which is also equal to
(a1+a2+a3+ $\qquad$ /b1+b2+b3+ $\qquad$ ..)
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
(7)In any triangle
$a / \operatorname{Sin} A=b / \operatorname{Sin} B=c / \operatorname{Sin} C=2 R$, where $R$ is the circumradius
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ $x^{\wedge} n-a^{\wedge} n=(x-a)\left(x^{\wedge}(n-1)+x^{\wedge}(n-2)+\ldots . . .+a^{\wedge}(n-1)\right) \ldots .$. Very useful for finding multiples .For example ( $17-14=3$ will be a multiple of $17 \wedge 3-14 \wedge 3$ )
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
$e^{\wedge} x=1+(x) / 1!+\left(x^{\wedge} 2\right) / 2!+\left(x^{\wedge} 3\right) / 3!+$ $\qquad$ to infinity
$2<e<3$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
$\log (1+x)=x-\left(x^{\wedge} 2\right) / 2+\left(x^{\wedge} 3\right) / 3-\left(x^{\wedge} 4\right) / 4$ $\qquad$ .to infinity [ Note the alternating sign .
.Also note that the ogarithm is with respect to base e ]
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
In a GP the product of any two terms equidistant from a term is always constant .
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
For a cyclic quadrilateral, area $=\operatorname{root}((s-a) *(s-b) *(s-c) *(s-d))$, where $s=(a+b+c+d) / 2$ $++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

For a cyclic quadrilateral, the measure of an external angle is equal to the measure of the internal opposite angle.
$(m+n)!$ is divisible by $m!* n!$.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ $+++++$
If a quadrilateral circumscribes a circle, the sum of a pair of opposite sides is equal to the sum of the other pair.

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++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
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The sum of an infinite GP $=a /(1-r)$, where $a$ and $r$ are resp. the first term and common ratio of the GP .
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ The equation whose roots are the reciprocal of the roots of the equation $a x^{\wedge} 2+b x+c$ is $c x^{\wedge} 2+b x+a$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ The coordinates of the centroid of a triangle with vertices $(a, b)(c, d)(e, f)$ is $((a+c+e) / 3,(b+d+f) / 3)$.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ The ratio of the radii of the circumcircle and incircle of an equilateral triangle is $2: 1$.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
Area of a parallelogram $=$ base $*$ height
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

## APPOLLONIUS THEOREM:

In a triangle, if $A D$ be the median to the side $B C$, then
$A B^{\wedge} 2+A C^{\wedge} 2=2\left(A D^{\wedge} 2+B D^{\wedge} 2\right)$ or $2\left(A D^{\wedge} 2+D C^{\wedge} 2\right)$.

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for similar cones , ratio of radii $=$ ratio of their bases.
The HCF and LCM of two nos. are equal when they are equal .

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+++++++++++++++++++++++++++++++++++++++++++++++++++++
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Volume of a pyramid $=1 / 3$ * base area $*$ height

In an isosceles triangle, the perpendicular from the vertex to the base or the angular bisector from vertex to base bisects the base.
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
In any triangle the angular bisector of an angle bisects the base in the ratio of the other two sides.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$ The quadrilateral formed by joining the angular bisectors of another quadrilateral is always a rectangle.
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
Roots of $x^{\wedge} 2+x+1=0$ are $1, w, w^{\wedge} 2$ where $1+w+w^{\wedge} 2=0$ and $w^{\wedge} 3=1$
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
$|a|+|b|$
$=$
$|a+b|$
if
$a * b>=0$
else $|a|+|b|>=|a+b|$
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

$$
2<=(1+1 / n)^{\wedge} n<=3
$$


when a three digit number is reversed and the difference of these two numbers is taken, the middle number is always 9 and the sum of the other two numbers is always 9 .
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
ANy function of the type $y=f(x)=(a x-b) /(b x-a)$ is always of the form $x=f(y)$.
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$

| Let | W | be | any | point | inside | a | rectangle | ABCD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Then |  |  |  |  |  |  |  |  |
| WD^2 |  | + |  |  | = |  | $+$ |  |

Let a be the side of an equilateral triangle . then if three circles be drawn inside this triangle touching each other then each's radius $=a /\left(2^{*}(\operatorname{root}(3)+1)\right)$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
Let ' $x$ ' be certain base in which the representation of a number is 'abcd', then the decimal value of this number is $a^{*} x^{\wedge} 3+b^{*} x^{\wedge} 2+c^{*} x+d$
$+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
when you multiply each side of the inequality by $\mathbf{- 1}$, you have to reverse the direction of the inequality.
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
To find the squares of numbers from 50 to 59
For $5 X^{\wedge} 2$, use the formulae
$(5 X)^{\wedge} 2=5^{\wedge} 2+X / X^{\wedge} 2$
$\mathrm{Eg} ;\left(55^{\wedge} 2\right)=25+5 / 25$
$=3025$
$(56)^{\wedge} 2=25+6 / 36$
= 3136
$(59)^{\wedge} 2=25+9 / 81$
$=3481$
$++++++++++++++++++++++++++++++++++++++++++++++++++++++++++$
many of $u$ must $b$ aware of this formula, but the ppl who don't know it must b useful for them.
$\mathrm{a}+\mathrm{b}+(\mathrm{ab} / 100)$
this is used for succesive discounts types of sums. like 1999 population increses by $10 \%$ and then in 2000 by $5 \%$ so the population in 2000 now is $10+5+(50 / 100)=+15.5 \%$ more that was in 1999
and if there is a decrease then it will be preceeded by a -ve sign and likeiwse

