15. MAXIMA AND MINIMA

Synopsis :

1. A function f(x) is said to be increasing in an interval I if $x_1, x_2 \in I, x_1<x_2 \Rightarrow f(x_1)\le f(x_2)$.
2. A function f(x) is said to be strictly increasing in an interval I if $x_1, x_2 \in I, x_1<x_2 \Rightarrow f(x_1)<f(x_2)$.
3. A function f(x) is said to be decreasing in an interval I if $x_1, x_2 \in I, x_1<x_2 \Rightarrow f(x_1)\ge f(x_2)$.
4. A function f(x) is said to be strictly decreasing in an interval I if $x_1, x_2 \in I, x_1<x_2 \Rightarrow f(x_1)>f(x_2)$.
5. A function f(x) is said to be increasing at x = a

   if $f(a-\delta)<f(a)<f(a+\delta)$ for some $\delta>0$
6. A function f(x) is said to be decreasing at x = a

   if $f(a-\delta)>f(a)>f(a+\delta)$ for some $\delta>0$
7. A function f(x) is said to be monotonic in an interval I if f(x) is either increasing or decreasing in the interval I.
8. A function f(x) is said to be increasing at x = a if $f^{I}(a)>0$
9. A function f(x) is said to be decreasing at x = a if $f^{I}(a)<0$
10. A function f(x) is said to be stationary at x = a if $f^{I}(a) = 0$. Then f(a) is called stationary value of f(x) at x = a and the point (a, f(a)) is called stationary point or turning point of f(x).
11. If f(x) has either maximum or minimum at x = a, then $f^{I}(a) = 0$.
12. If $f^{I}(a) = 0$ then f(x) need not have maximum or minimum at x = a.
13. A function f(x) differentiable at x = a, has maximum at x = a $\Leftrightarrow f^{I}(a) = 0, f^{II}(a)<0$.
14. A function f(x) differentiable at x = a, has minimum at x = a $\Leftrightarrow f^{I}(a) = 0, f^{II}(a)>0$.
15. If f(x) is increasing function in [a, b] then

    i) the minimum value of f(x) is f(a).

    ii) the maximum value of f(x) is f(b).
16. If f(x) is decreasing function [a, b] then

    i) the minimum value of f(x) is f(b).

    ii) the maximum value of f(x) is f(a).
17. The maximum value of f(x) = asinx + bcosx + c is $c + \sqrt{a^2+b^2}$.
18. The minimum value of f(x) = asinx + bcosx + c is $c-\sqrt{a^2+b^2}$.
19. The maximum value of $a\cos^2x + b\sin^2x$ is a and minimum value = b. (if a>b)

20. The minimum value of $f(x) = a\tan x + b\cot x$ is $2\sqrt{ab}$ and attained at $\tan x = \sqrt{b/a}$.
21. The minimum value of $f(x) = a^2\sec^2 x + b^2\mathrm{cosec}^2 x$ is $(a + b)^2$ and attained at $\tan x = \sqrt{b/a}$.
22. The maximum rectangle inscribed in a circle is square.
23. The maximum area of a rectangle in a circle of radius r is $2r^2$.
24. The maximum triangle inscribed in a circle is equilateral.
25. The maximum area of a triangle inscribed in a circle of radius r is $\frac{3\sqrt{3}}{4}r^2$.
26. Perimeter of the sector is given. If the area of the sector is maximum, then the angle of the sector is '2' radians.
27. The semi-vertical angle of a cone of maximum volume with given slant height is $\mathrm{Tan}^{-1}\sqrt{2}$.
28. The semi-vertical angle of a cone of maximum volume with given surface area is $\mathrm{Sin}^{-1}(1/3)$.
29. The semi-vertical angle of a cone of maximum volume with given curved surface area is $\mathrm{Sin}^{-1}(1/\sqrt{3})$
30. The height of a cylinder of maximum volume inscribed in a sphere of radius R is $\frac{2}{\sqrt{3}}R$ and its base radius is $\frac{\sqrt{2}}{\sqrt{3}}R$.
31. The sum of two numbers is 'k'. If the sum of their squares is minimum, then the numbers are k/2, k/2.
32. The sum of two numbers is 'k' and the least sum of their squares is $k^2/2$.
33. The sum of two numbers is 'k'. If their product is maximum, then the numbers are k/2, k/2.
34. The sum of two positive numbers is k. If the sum of their squares is minimum, then the numbers are k/2, k/2.
35. The product of two positive numbers is k. If the sum of their squares is minimum, then the numbers are $\sqrt{k}, \sqrt{k}$.
36. Sum of two numbers is 'k'. If the product of the square of the first and cube of the second is maximum, then the numbers are $\frac{2k}{5}, \frac{3k}{5}$.
37. The hypotenuse of a right angled triangle is 'a'. If the area of the triangle is maximum. Then the sides are $\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}$.

38. Two sides of a triangle are given. If the area of the triangle is maximum, then the angle between sides is $\pi/2$.

39. The sum of the hypotenuse and side of a right angled triangle is given. If the area is maximum, then the angle between the sides is $\pi/3$.

40. If $a>0$, $b>0$, $x>0$, the least value of $f(x) = ax + \frac{b}{x}$ is $2\sqrt{ab}$.

41. The sides of a rectangle, with maximum perimeter, inscribed in a semi circle of radius R are $\frac{4R}{\sqrt{5}}, \frac{R}{\sqrt{5}}$