14. TANGENT AND NORMALS

Synopsis :

1. The gradient of the curve $y = f(x)$ at $P(x_1, y_1)$ is $\left(\frac{dy}{dx}\right)_P$.

2. The equation of the tangent at $P(x_1, y_1)$ to the curve $y = f(x)$ is $y-y_1 = m(x-x_1)$ where $m = \left(\frac{dy}{dx}\right)_P$.

3. The equation of the normal at $P(x_1, y_1)$ to the curve $y = f(x)$ is $y-y_1 = -\frac{1}{m}(x-x_1)$ where $m = \left(\frac{dy}{dx}\right)_P$.

4. Let θ be the angle between two curves $y = f(x)$, $y = g(x)$ at their point of intersection P.

   i) The two curves are said to touch each other at P if $\theta = 0$.

   ii) The two curves are said to cut orthogonally at P if $\theta = \pi/2$.

5. Let m_1, m_2 be the gradients of two curves at their point of intersection P. If θ is the acute angle between the curves at P, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

6. Let m_1, m_2 be the gradients of two curves at their point of intersection P. Then

   i) The two curves touch each other at P $\Leftrightarrow m_1 = m_2$.

   ii) The two curves cut each other orthogonally $\Leftrightarrow m_1 m_2 = -1$.

7. If two curves touch each other at a point P, then the two curves have a common tangent and common normal at P.

8. Let $y = f(x)$ be a curve and P be a point on the curve. Let the tangent at P to the curve meet x-axis at T and the normal at P to the curve meet x-axis at N. Let Q be the projection of P on x-axis. Then (i) PT is called length of tangent (ii) PN is called length of normal (iii) QT is called subtangent (iv) QN is called subnormal of $y = f(x)$ at P.

9. Let $P(x_1, y_1)$ be a point on the curve $y = f(x)$ and let $\left(\frac{dy}{dx}\right)_P = m$. Then

   i) the length of the tangent to the curve at P is $\left|\frac{y_1\sqrt{1+m^2}}{m}\right|$

   ii) the length of the normal to the curve at P is $\left|y_1\sqrt{1+m^2}\right|$.

   iii) the subtangent to the curve at P is $|y_1/m|$.

   iv) the subnormal to the curve at P is $|y_1 m|$.

10. To any curve $y = f(x)$, length of S.T. and ordinate, length of S.N. are in G.P., whose common ratio is the slope of the tangent m.

11. The angle between the curve $y^2 = 4ax$ and $x^2 = 4by$
    i) at the origin is □/2
    ii) at the other point is $\text{Tan}^{-1}\left[\dfrac{3a^{1/3}b^{1/3}}{2(a^{2/3}+b^{2/3})}\right]$.
12. The angle between the curves $y^2 = 4ax$, $x^2 = 4ay$ is □/2 at the origin, $\text{Tan}^{-1}(3/4)$ at (4a, 4a).
13. The angle of intersection of the curves $xy = a^2$, $x^2 + y^2 = 2a^2$ is zero or □.
14. The angle of intersection of curves $y = a^x$ and $y = b^x$ is $\text{Tan}^{-1}\left[\dfrac{\log a - \log b}{1+\log a \log b}\right]$.
15. Angle between the curves y = sinx and y = cosx at the common point of intersection is $\text{Tan}^{-1}(2\sqrt{2})$.
16. The condition that the curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ may intersect orthogonally is $\dfrac{1}{a_1}-\dfrac{1}{a_2}=\dfrac{1}{b_1}-\dfrac{1}{b_2}$.
17. The angle of intersection of curves $\dfrac{x^2}{a^2+k_1}+\dfrac{y^2}{b^2+k_1}=1$ and $\dfrac{x^2}{a^2+k_2}+\dfrac{y^2}{b^2+k_2}=1$ is □/2.
18. If the curves $y^2 = 4ax$ and $xy = c^2$ cut orthogonally, then $c^4 = 32a^4$.
19. If the curves $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ and $xy = c^2$ intersect orghogonally then $a^2 = b^2$.
20. If the curves xy = k and $y^2 = x$ are orthogonally, then $8k^2 = 1$.
21. The slope of the tangent to the curve $\left(\dfrac{x}{a}\right)^n+\left(\dfrac{y}{b}\right)^n=2$ at (a, b) is $-b/a$.
22. The slope of the tangent to the curve $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at □ point is $-\dfrac{b}{a}\cot\theta$.
23. The slope of the tangent to the curve $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ at □ point is $\dfrac{b}{a}\text{cosec}\,\theta$.
24. Equation of the tangent to the curve $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at □ point is $\dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1$.
25. Equation of the normal at □ point is $\dfrac{ax}{\cos\theta}+\dfrac{by}{\sin\theta}=a^2-b^2$.
26. Equation of the tangent to the curve $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ at □ point is $\dfrac{x\sec\theta}{a}-\dfrac{y\tan\theta}{b}=1$
27. The sum of the intercepts on the axes made by the tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at any point is 'a'.
28. At any point on the curve y = f(x) if the subnormal is constant, then the curve is a parabola.
29. The length of the intercept of the tangent between the co-ordinate axes to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at any point is 'a' (constant).
30. For the curve $x^m.y^n = a^{m+n}$, the portion of the tangent intercepted between the axes is divided at its point of contact in the ratio AP:PB = n:m.
31. At any point on the curve $xy = c^2$, the sub normal varies as cube of the ordinate of the point.

32. For the curve $y^2 = 4ax$ the ratio of the sub tangent to the abscissa of the point is 2:1.
33. Area of the triangle formed by the tangent at any point on the curve $xy = c^2$ and the co-ordinate axes is $2c^2$ sq.units.