Probability and Statistics

Introduction

In our day to day life, we come across many uncertainty ofevents. We wake up in the morning and check the weather report. The statement could be 'there is 60% chance of rain today'. This statement infers that the chance of rain is more than that having a dry weather. We decide upon our breakfast from a statement that "corn flakes might reduce cholesterol". What is the chance of getting a flat tyre on the way to an important apartment? And so on.

How probable an event is? We generally infer by repeated observation of such events in long term patterns.

Probability is the branch of mathematics devoted to the study of such events.

People have always been interested in games of chance and gambling. The existence of games such as dice is evident since 3000 BC. But such games were not treated mathematically till fifteenth century. During this period, the calculation and theory of probability originated in Italy. Later in the seventeenth century, French contributed to this Literature of study. The foundation of modern probability theory is credited to the Russian mathematician, Kolmogorov. In 1993, he proposed the axioms, at which the present subject of probability is based.

Random Experiment and Sample Space

An experiment repeated under essentially homogeneous andsimilar conditions results in an outcome, which is unique or not unique but may be one of the several possible outcomes. When the result is unique then the experiment is called a 'deterministic' experiment.

Example:

While measuring the inner radius of an open tube, using slide callipers, we get the same result by performing repeatedly the same experiment. Many scientific and Engineering experimentsare deterministic.

If the outcome is one of the several possible outcomes, then such an experiment is called a "random experiment" or 'nondeterministic' experiment.

In other words, any experiment whose outcome cannot be predicted in advance, but is one of the set of possible outcomes, is called a random experiment.

If we think an experiment as being performed repeatedly, each repetition is called a trial. We observe an outcome for each trial.

Example:

An experiment consists of 'tossing a die and observing thenumber on the upper-most face'

In such cases, we talk of chance of probability, which numerically measures the degree of chance of the occurrence of events.

Sample Space (S)

The set of all possible outcomes of a random experiment is called the sample space, associated with the random experiment.

Note:

Each element of S denotes a possible outcome. Each element of S is known as sample point.

Any trial results in an outcome and corresponds to one and only one element of the set S.

e.g.,

1. In the experiment of tossing a coin,

S = {H, T}

2. In the experiment of tossing two coins simultaneously,

S = {HH, HT, TH, TT}

3. In the experiment of throwing a pair of dice,

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2),……… (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

EVENTS

An event is the outcome or a combination of outcomes of an experiment. In other words, an event is a subset of the samplespace. e.g.,

- {a head} in the experiment of tossing a coin is an event.

- {a sum equal to 6} in the experiment of throwing a pair of dice is an event. Occurence of an event Suppose we throw a die. Let E be the event of a perfect square number. Then E = {1,4}. Suppose 3 appeared on the upper most face, then we say that the event E has not occurred. E occurs only when 1 or 4 appears on the upper most face. Therefore whenever an outcome satisfies the conditions, given in the event, we say that the event has occurred. In a random experiment, if E is the event of a sample space S and w is the outcome, then we say the event E has occurred if w $\in$ E.

Types of Events S= {1,2,3,4,5,6}] If the event is set of elements less than 2, then E = {1} is a simple event 1)

Simple Event:

If an event has one element of the sample space then it is called a simple or elementary event.

Example:

Consider the experiment of throwing a die.

Compound Event:

If an event has more than one sample points, the event is called a compound event . In the above example, of throwing a die, {1, 4} is a compound event.

Null Event (f):

As null set is a subset of S, it is also an event called the null eventor impossible event. 4)
The sample space S= {1, 2, 3, 4, 5, 6} in the above experiment is a subset of S. The
event represented by S occurs whenever the experiment is performed.
Therefore, the eventrepresented by S is called a **sure event or certain event**.

Complement of an Event:

The complement of an event E with respect to S is the set of all the elements of S which are not in E.

The complement of E is denoted by E' or E^C. **Note:** In an experiment if E has not occurred then E' has occurred. Algebra ofEvents In a random experiment, considering S(the sample space) as the universal set, let A, B and C be the events of S. We can define union, intersection and complement of events and their properties on S, which is similar to those in set theory.

i) $A \cup B, A \cap B$ and A' are events of the random experiment.

ii) A-B is an event, which is same as "A but not B"

iii) $A \cup B = B \cup A, A \cap B = B \cap A$

iv) $(A \cup B)' = A' \cap B', (A \cap B)' = A' \cup B'$

v) $(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C)$

vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example: Consider the same experiment throwing a die, then S= {1, 2, 3, 4, 5} Let A = {1, 2, 3, 4}, B = {2, 3, 4, 5} The following are also events $A \cup B = \{1, 2, 3, 4, 5\}$ $A \cap B = \{2, 3, 4\}$ A - B = {1} A'= {5, 6} Mutually Exclusive Events Two events associated with a random experiment are said to be mutually exclusive, if both cannot occur together in the same trial. In the experiment of throwing a die, the events A = {1, 4} and B = {2, 5, 6} are mutually exclusive events. In the same experiment, the events A = {1, 4} and C = {2, 4, 5, 6} are not mutually exclusive because, if 4 appears on the die, then it is favourable to both events A and C.

The definition of mutually exclusive events can also be extended to more than two events. We say that more than two events are mutually exclusive, if the happening of one of these, rules out thehappening of all other events. The events A = {1, 2}, B = {3} and C={6}, are mutually exclusive in connection with the experiment of throwing a single die. If A and B are two events, then A or B or ($A \cup B$) denotes the event of the occurrence of at least one of theevents A or B. A and B or ($A \cap B$) is the event of the occurrence of both events A and B. If A and B happen to be mutually exclusiveevents, then $P(A \cap B) = 0$. For example, in the experiment of tossing three coins, if A and B be the events of getting at least one head and at most one tail respectively, then S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} A = {HHH, HHT, HTH, THH, HTT, THT, TTH} B = {HHH, HHT, HTH, THH}

Here $A \cup B$ = {HHH, HHT, HTH, THH, HTT, THT, TTH}

and $A \cap B$ = {HHH, HHT, HTH, THH} $E_1, E_2, \ldots, E_n$ are nevents associated with a random experiment are said to be pairwise mutually exclusive, if $E_i \cap E_j = \phi$ for all i, j and $i \neq j$.

For example, let a pair of dice be thrown and let A, B, C be the events"the sum is 7", sum is 8, sum is greater than 10, respectively. \ A = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)} B = {(2,6), (3,5), (4,4), (5,3), (6,2)} and C = {(5,6), (6,5), (6,6)} The events A, B and C are pairwise mutually exclusive. Two events A and B are said to be mutually exclusive if the occurrence of A prevents the occurrence of B and vice versa.

Exhaustive Events For a random experiment, let $E_1, E_2, E_3, \ldots E_n$be the subsets of the sample space S. $E_1, E_2, E_3, \ldots E_n$ form a set of Exhaustive events if $E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$ A set ofevents $E_1, E_2, E_3, \ldots E_n$ of S are said to mutually exclusive and exhaustive events if $E_1 \cup E_2 \cup E_3, \ldots E_n = S$ and $E_i \cap E_j = \phi$ for $i \neq j$

Example:

- In tossing of a coin, there are two exhaustive cases, {H}, {T}.
- In throwing of a dice, there are 6 exhaustive cases, {1}, {2}, {3}, {4}, {5}, {6}.
- In throwing of a pair of dice, there are 36 exhaustive cases. Example of an event which is exhaustive, but not mutually exclusive. In throwing a die experiment, let E_1 represent the outcomes which are less than 4, E_1= {1,2,3}.

Let E_2 represent the outcomes which are greater than 2, E_2= {3,4,5,6}.

Clearly $E_1 \cup E_2 = S$, this implies E_1 and E_2 are exhaustive.

But $E_1$and E_2 are not mutually exclusive since $E_1 \cap E_2 = \{3\} \neq \phi$.

Let E_3 represent the outcomes which are greater than 4, $E_3 = \{5, 6\}$. Clearly E_1 and E_3 are mutually exclusive, but not exhaustive. Let $E_4 = \{2, 4, 6\}$ and $E_5 = \{1, 2, 5\}$ Then E_4 and E_5 are mutually exclusive and exhaustive since.

$E_4 \cup E_5 = S$ and $E_4 \cap E_5 = \phi$

Equally Likely Outcomes The outcomes of a random experiment are said to be equally likely, if each one of them has equal chance of occurrence. Example: The outcomes of an unbiased coin are equally likely.

Probability of an Event

So far, we have introduced the sample of an experiment and used it to describe events. In this section, we introduceprobabilities associated to the events. If a trial results in n-exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the occurrence of an event A, then the probability of the happening of A, denoted by P(A), is given by

$$P(A) = \frac{m}{n} \ldots\ldots \text{(i)}$$

Note 1: $0 \le P(A) \le 1$ as $0 \le m \le n$.

Note 2:

If P(A) = 0 then A is called a null event, or impossible event.

Note 3:

If P(A) = 1 then A is called a sure event.

Note 4:

If m is the number of cases favourable to A. Then m - n is favourable to "non occurrence of A

$$\therefore \quad P(A^c) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

Note 5: If the odds are a:b in favour of A then

$$P(A) = \frac{a}{a+b} \quad P(B) = \frac{b}{a+b}$$

This is the same as odds are b:a against the event A. Statistical or Empirical Probability If a trial is repeated Nnumber of times under essential homogeneous

and identical conditions and the event A occurs f times then

$$P(A) = \lim_{N \to \infty} \frac{f}{N}.$$

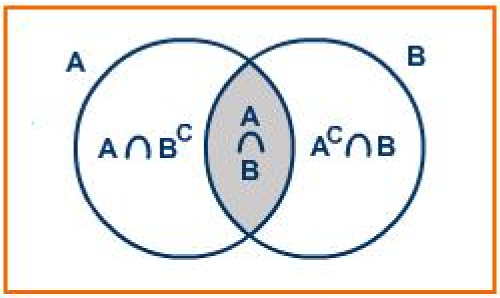
Axiomatic Approach to Probability Axiomatic approach toprobability closely relates the theory of probability to set theory. Let S be the sample space of an experiment.Probability is a function, which associates a non-negative realnumber to every event A of the sample space denoted by P(A) satisfying the following axioms. For every event A in S, P(A)³ 0. P(S) = 1. If A_1, A_2, A_3,….A_n are mutually exclusiveevents in S, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

Theorems of Probability

Theorem 1:(Addition Rule of Probability)

If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Proof:



We have $A \cup B = A \cup (A^C \cap B)$ and $A \cap (A^C \cap B) = \phi$ (From the Venn diagram) $\therefore P(A \cup B) = P(A) + P(A^C \cap B)$ ...(1)

($\because$ A and $A^C \cap B$ are mutually exclusive) We have $B = (A \cap B) \cup (A^C \cap B)$ and $(A \cap B) \cap (A^C \cap B) = \phi$ $\therefore P(B) = P(A \cap B) + P(A^C \cap B)$

$\therefore P(A^C \cap B) = P(B) - P(A \cap B)$ ...(2)

Substituting for $P(A^C \cap B)$ in (1) We get $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note 1:

If $A \cap B = \phi$ then $P(A \cup B) = P(A) + P(B)$ $\Rightarrow$
If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

Note 2:

If A, B, C are any three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Example:

In tossing a fair die, what is the probability that the outcome is odd or grater than 4? **Suggested answer:** Let E_1 be the event that the outcomes are odd. $E_1 = \{1,3,5\}$ Let E_2 be the event that the outcomes are greater than 4. $E_2 = \{5,6\}$ $P(E_1) = \frac{3}{6} = \frac{1}{2}$ $P(E_2) = \frac{2}{6} = \frac{1}{3}$

$P(E_1 \cap E_2) = P\{5\} = \frac{1}{6}$ $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{3}{6} = \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Theorem 2:

$P(A^C) = 1 - P(A)$ Proof: We have $A \cap A^C = \phi$ and $A \cup A^C = S$ $P(A \cup A^C) = P(S)$

$\Rightarrow \quad P(A) + P(A^C) = 1 \qquad (\because A \text{ and } A^C \text{ are mutually exclusive})$ \P(A) = 1 - P(A^C)

Example:

In tossing a die experiment, what is the probability of getting at least 2. **Suggested answer:** Let E be the event that the outcome is at least 2, then $E = \{2,3,4,5,6\}$ $E^C = \{1\}$

$P(E) = \frac{5}{6}, P(E^C) = \frac{1}{6}, \Rightarrow P(E^C) = 1 - P(E) = 1 - \frac{5}{6} = \frac{1}{6}$

Theorem 3:

P(f) = 0 Proof: The proof follows from theorem 2, P(f)^C = 1 - P(f) $\Rightarrow P(\phi) = 1 - P(\phi^C)$

$= 1 - P(S) \qquad (\because \phi^C = S)$ = 1 - 1 = 0 $\therefore \qquad P(\phi) = 0$

Example:

In throwing a die experiment, what is the probability of occuring a number greater than 8 ? **Suggested answer:** Let E be the event where the outcome is greater than 8. E = f P(f) = 0

Random Variables and Probability Distributions

It is often very important to allocate a numerical value to an outcome of a random experiment. For example, consider an experiment of tossing a coin twice and note the number of heads (x) obtained. Outcome HH HT TH TT No. of heads (x) 2 1 1 0 x is called a random variable, which can assume the values 0, 1 and 2.

Thus, random variable is a function that associates a realnumber to each element in the sample space. Random variable(r.v) Let S be a sample space associated with a given randomexperiment.

A real valued function X which assigns to each w_i Î S, a unique real number, $X(w_i) = x_i$ is called a random variable.

Note:

There can be several r.v's associated with an experiment. Arandom variable which can assume only a finite number of values or countably infinite values is called a discrete randomvariable. e.g., Consider a random experiment of tossing three coins simultaneously. Let X denote the number of heads obtained. Then, X is a r.v which can take values 0, 1, 2, 3.Continuous random variable A random variable which can assume all possible values between certain limits is called acontinuous random variable. Discrete probability distribution A discrete random variable assumes each of its values with a certain probability.

Let X be a discrete random variable which takes values x_1, x_2, x_3,…x_n where $p_i = P\{X = x_i\}$ Then X : x_1 x_2 x_3 …x_n P(X) : $p_1 p_2$ p_3 … p_n is called the probability distribution of x, If in theprobability distribution of x, (i) $P_i \geq 0 \ \forall i$ and (ii) $\sum_{i=1}^{n} P_i = 1$

Note 1 :

P{X = x} is called probability mass function.

Note 2:

Although the probability distribution of a continuous r.v cannot be presented in tabular forms, we can have a formula in the form of a function represented by f(x) usually called theprobability density function.

Multiplication Rule of Probability

We have already proved that if two events A and B from a sample S of a random experiment are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$.

In this section, we examine whether such a rule exists, if '$\cup$' isreplaced by '$\cap$' and '+' is replaced by 'x' in the above addition rule. If it does exist, what are the particular conditions restricted on the events A and B.

This leads us to understand the dependency and independency of the events.

Example:

A bag contains 5 white and 8 black balls, 2 balls are drawn at random. Find

a) The probability of getting both the balls white, when the first ball drawn, is replaced.

b) The probability of getting both the balls white, when the first ball is not replaced.

Suggested answer:

a) The probability of drawing a white ball in the first draw is $\frac{5}{13}$. Since the ball is replaced, the probability of getting white ball in the second draw is also $\frac{5}{13}$.

Probability of getting both the balls white $= \frac{5}{13} \cdot \frac{5}{13} = \frac{25}{169}$

b) The probability of drawing a white ball in the first draw is $\frac{5}{13}$. If the first ball drawn is white and if it is not replaced in the bag, then there are 4 white balls and 8 black balls. Therefore, the probability of drawing a white ball in the second draw = $\frac{4}{12}$.

In this case, the probability of drawing a white ball in the 2^{nd}draw depends on the occurrence and non-occurrence of the event in the first draw.

Probability of both the balls white $= \frac{5}{13} \cdot \frac{4}{12} = \frac{20}{156} = \frac{5}{39}$Independent Events

Events are said to be independent if the occurrence of one event does not affect the occurrence of others.

Observe in case(a) of above example,

The probability of getting a white ball in the second draw does not depend on the occurrence of the event on the first draw.

However in case(b), the probability of getting a white ball in the second draw depends on the occurrence and non - occurrence of the event in the first draw.

It can be verified by different example.

If A and B are two independent events, then

$P(A \cap B) = P(A) \times P(B)$

This is known as **Multiplication Rule of Probability**.

The converse is also true, that is if two events A and B associated with a random experiment are such that

$P(A \cap B) = P(A) \times P(B)$,

then the two events are independent.

Two events are said to be dependent if the occurrence of one affects the occurrence of the other. In this case,

$$P(A \cap B) \neq P(A).P(B)$$

In the above example,

let

A = event that the outcome is a white ball in the first draw

B = event that the out come is a white ball in the second draw

In case (a),

$P(A) = P(B) = \frac{5}{13}$ (since the ball after the first draw is replaced)

\ The probability that both the balls drawn is white

$$= P(A \cap B) = P(A).P(B) = \frac{5}{13}.\frac{5}{13}$$

$$= \frac{25}{169}$$

In case (b),

$$P(A) = \frac{5}{13}$$

(Since the ball after the first draw is not replaced)

P(B) = P (first draw is white and second draw is white)

+ P (first draw is black and second draw is white)

$$= \frac{5}{13} \times \frac{4}{12} + \frac{8}{13}.\frac{5}{12} = \frac{60}{156}$$

$P(A \cap B) = P$ (both the balls are white)

$$= \frac{5}{13}.\frac{4}{12} = \frac{20}{156}$$

Here $P(A \cap B) \neq P(A). P(B)$ since the events are notindependent. Independent Experiment

Two random experiments are said to be independent if for every pair of events E and F, where E is associated with the first experiment and F is associated with the second experiment, the probability of simultaneous occurrence of E and F, when the two experiments are performed, is the product of the probabilities P(E) and P(F), calculated separately on the basis of the two experiments.

i.e,. $P(E \cap F) = P(E).P(F)$

Example:

Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problems be solved.

Suggested answer:

Let A be the event of A solving the problem.

Let B be the event of B solving the problem.

Then,

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}$$

P (A not solving the problem)

$$= P(A^c) = 1 - \frac{1}{2} = \frac{1}{2}$$

P (B not solving the problem)

$$= P(B^c) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(A^c \cap B^c) = P(A^c).P(B^c)$$

(Considering the experiments as independent, because A and B solve the problem independently)

$\Rightarrow$ P (both not solving the problem)

$$= \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$\Rightarrow$ Probability that the problem can be solved

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Note:

If A and B are independent, then

i) A^c and B^c are independent

ii) A^c and B are independent

iii) A and B^c are independent

Conditional Probability

Let us consider the random experiment of throwing a die. Let A be the event of getting an odd number on the die.

\ S = {1, 2, 3, 4, 5, 6} and A = {1, 3, 5}

$$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2}$$

Let B = {2, 3, 4, 5, 6}. If, after the die is thrown, we are given the information, that the event B has occurred, then theprobability of event A will no more be 1/2, because in this case, the favourable cases are two and the total number of possible outcomes will be five and not six.

The probability of event A, with the condition that event B has happened will 2/5. This conditional probability is denoted as P(A/B). Let us define the concept of conditional probability in a formal manner.

Let A and B be any two events associated with a randomexperiment. The probability of occurrence of event A when the event B has already occurred is called the conditionalprobability of A when B is given and is denoted as P(A/B). The conditional probability P(A/B) is meaningful only when $P(B) \neq 0$, i.e., when B is not an impossible event.

P(A/B) =Probability of occurrence of event A when the event B as already occurred.

$$= \frac{\text{Number of cases favourable to B which are also favourable to A}}{\text{Number of cases favourable to B}}$$

$$\therefore \quad P(A/B) = \frac{\text{Number of cases favourable to } A \cap B}{\text{Number of cases favourable to B}}$$

Also, $P(A/B) = \dfrac{\dfrac{\text{Number of cases favourable to } A \cap B}{\text{Number of cases in the sample space}}}{\dfrac{\text{Number of cases favourable to } B}{\text{Number of cases in the sample space}}}$ $\therefore P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ ....(1)

Remark 1:

$$\text{If } P(A) \neq 0, \text{ the } P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

Remark 2:

If A and B are mutually exclusive events, then

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0 \text{ and}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0}{P(A)} = 0$$

\ If A and B are mutually exclusive events, then A/B and B/A are impossible events.

For an illustration, let us consider the random experiment of throwing two coins.

\ S = {HH, HT, TH, TT}

Let A = {HH, HT}, B = {HH, TH} and C = {HH, HT, TH}

$$\therefore \quad P(A) = \frac{2}{4} = \frac{1}{2}, P(B) = \frac{2}{4} = \frac{1}{2}, P(C) = \frac{3}{4}$$

A/B is the event of getting A with the condition that B has occurred.

$$\therefore \quad P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{n\{HH\}}{n\{HH, TH\}} = \frac{1}{2},$$

$$\text{Similarly, } P(A/C) = \frac{n\{HH, HT\}}{n\{HH, HT, TH\}} = \frac{2}{3}, \quad P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{2}{3}$$

We observe that $P(A/C) \neq P(A)$, $P(B/C) \neq P(B)$.

Remark 3:

We know that for the events A and B,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

If B = S then

$$P(A/S) = \frac{P(A \cap S)}{P(S)}$$

$$= \frac{P(A)}{1}$$

= P (A) $(\text{sin ce } A \cap S = A \text{ and } P(S) = 1)$

Remark 4:

If A = B

$$P(A/B) = P(B/B)$$

$$= \frac{P(B \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$= 1 \ (\because B \cap B = B)$$

Remark 5:

From the formula of conditional probabilities, we have

$$P(A \cap B) = P(A/B).P(B) \quad ....(2)$$

$$P(A \cap B) = P(B/A).P(A) \quad ....(3)$$

Equation (2) and equation (3) are known as multiplication rules of probability for any two events A and B of the same sample space.

Remark 6:

We know that two events are independent if the occurrence of one does not effect the occurrence of other. If A and B areindependent events

P (A/B) = P (A) and P (B/A) = P (B)

$$\therefore P(A \cap B) = P(A/B).P(B)$$

$$= P(A).P(B)$$

\ The multiplication rule for the independent events A and B is given by

$$P(A \cap B) = P(A).P(B)$$

Remark 7:

So far, we have assumed that the elementary events are equally likely and we have used the corresponding definition ofprobability. However the same definition of conditionalprobability can also be used when the elementary events are not equally likely. This will be clear from the following example.

Suppose a die is tossed. Let B be the event of getting a perfect square.

The die is so constructed that the event numbers are twice as likely to occur as the odd numbers.

Let us find the probability of B given A, where A is the eventgetting a number greater than 3 while tossing the die.

S = {1, 2, 3, 4, 5, 6}

If probability of getting an odd number is x, the probability ofgetting an even number is 2x.

Since P (S) = 1

$x + 2x + x + 2x + x + 2x = 1$

$9x = 1$

$$x = \frac{1}{9}$$

$$2x = \frac{2}{9}$$

$$\Rightarrow P(\text{even number}) = 2x = \frac{2}{9}$$

$$P(\text{odd number}) = x = \frac{1}{9}$$

A = {4, 5, 6}

$$P(A) = \frac{2}{9} + \frac{1}{9} + \frac{2}{9} = \frac{5}{9}$$

$$A \cap B = \{4\}$$

$$P(A \cap B) = \frac{2}{9}$$

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{2/9}{5/9}$$

$$= \frac{2}{5}$$

Example 1:

A card is drawn from an ordinary deck and we are told that it is red, what is the probability that the card is greater than 2 but less than 9.

Suggested answer:

Let A be the event of getting a card greater than 2 but less than 9.

B be the event of getting a red card. We have to find theprobability of A given that B has occurred. That is, we have to find P (A/B).

In a deck of cards, there are 26 red cards and 26 black cards.

\ n(B) = 26

Among the red cards, the number of outcomes which are favourable to A are 12. That is $n(A \cap B) = 12$

$$\therefore P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{12}{26} = \frac{6}{13}$$

Example 2:

A pair of dice is thrown. If it is known that one die shows a 4, what is the probability that

a) the other die shows a 5

b) the total of both the die is greater than 7

Suggested answer:

Let A be the event that one die shows up 4. Then the outcomes which are favourable to A are

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) (1, 4), (2, 4), (3, 4),

(5, 4), (6, 4)

$\Rightarrow n(A) = 11$

(a) Let B be the event of getting a 5 in one of the dies. Then the outcomes which are favourable to both A and B are (4, 5), (5, 4)

$\Rightarrow n(A \cap B) = 2$

$$\therefore P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{2}{11}$$

(b) Let C be the event of getting a total of both the die greater than 7.

The out-comes which are favourable to both C and A.

(4, 4), (4, 5), (4, 6), (5, 4), (6, 4)

\ n (C) = 5

$$P(C/A) = \frac{n(A \cap C)}{n(A)} = \frac{5}{11}$$

Note that in the above example P (B) and P (B/A) are different.

$P(B/A)$ is $\frac{2}{11}$ where as $P(B) = \frac{11}{36}$.

Similarly P (C) and P (C/A) are different.

Baye's Theorem

In the previous section, we have learnt that

i) If A and B are two mutually exclusive events, then

$P(A \cup B) = P(A) + P(B)$

ii) $P(A \cap B) = P(A).P(B/A) = P(B).P(B/A)$

Before we state and prove Baye's Theorem, we use the above two rules to state the law of total probability. The law of totalprobability is useful in proving Baye's theorem and in solvingprobability problems. Following is an example which explainsthis law.

Example:

Let S is the sample space which is the population of adults in asmall town who have completed the requirement for a college degree. The population is categorized according to sex and employment status as follows

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

\ One of these individual is to be selected for a tour throughout the country. Knowing that the individual chosen is employed, what is the probability that the individual is a man?

Suggested answer:

Let M be the event that a man is selected, E be the event thatthe individual selected, is employed. Using the reduced samplespace, we have

$$P(M/E) = \frac{460}{600} \quad \text{(from the table)} = \frac{23}{30}$$

Also, we have $P(M/E) = \dfrac{P(M \cap E)}{P(E)}$

From the original sample space, we have

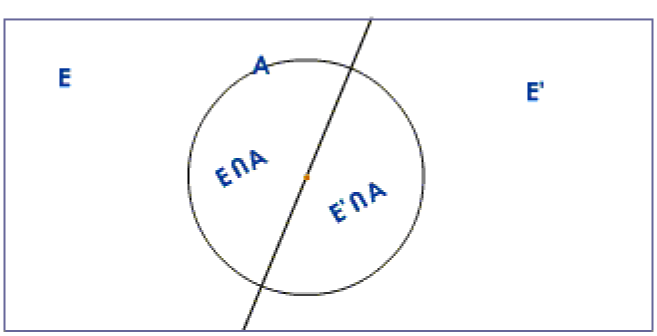
$$P(E) = \frac{600}{900} = \frac{2}{3} \quad P(M \cap E) = \frac{460}{900} = \frac{23}{45}$$

$$\Rightarrow P(M/E) = \frac{P(M \cap E)}{P(E)} = \frac{23}{45} \times \frac{3}{2} = \frac{23}{30}$$

Suppose that we are now given the additional information that 36 of those employed and 12 of these unemployed are the members of the rotary club. What is the probability of the event A that the selected individual is a member of the rotary club?

Suggested answer:

A is the event that the selected individual is a member of the rotary club.



From the figure, it is clear that

$$A = (E \cap A) \cup (E' \cap A) \Rightarrow P(A) = P(E \cap A) + P(E' \cap A)$$

Since $E \cap A$ and $E' \cap A$ are mutually exclusive

$$= P(E)\ P(A/E) + P(E')\ P(A/E') \quad ...(1)$$

$$P(E) = \frac{2}{3}$$

$$P(E') = 1 - P(E)$$

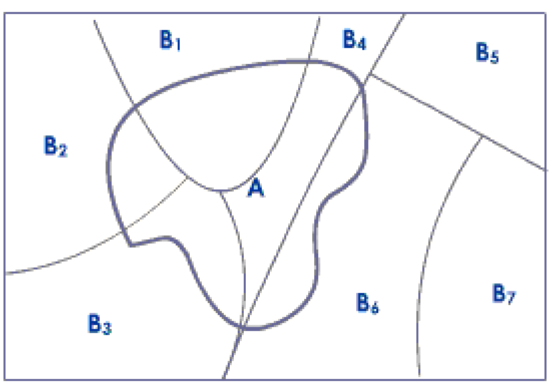
$$= \frac{1}{3} \quad P(A/E) = \frac{36}{600} \quad = \frac{3}{50} \quad P(A/E') = \frac{12}{300} \text{ or } P(A/E') = \frac{1}{25}$$

$$\therefore P(A) = \frac{2}{3} \times \frac{3}{50} + \frac{1}{3} \times \frac{1}{25} \qquad \text{(From (1))} = \frac{4}{75}$$

The generalization of the foregoing example, where the samplespace is partitioned into n subsets is known as Law of TotalProbability. Theorem: (Law of Total Probability) If B_1, B_2, B_3, ….., B_n are mutually exclusive and exhaustive events of thesample space S, then for any event A of S.

$$P(A) = P(B_1).P(A/B_1) + P(B_2).P(A/B_2) + P(B_3).P(A/B_3) + ... + P(B_n).P(A/B_n) \quad = \sum_{i=1}^{n} P(B_i)\ P(A/B_i)$$

Proof:



From the venn diagram, we have

$$A = (B_1 \cap A) + (B_2 \cap A) + ... + (B_n \cap A)$$

where $B_1 \cap A,\ B_2 \cap A, ...,\ B_n \cap A$ are mutually exclusive events.

$$\Rightarrow P(A) = P(B_1 \cap A) + P(B_2 \cap A) + ... + P(B_n \cap A)$$

$= P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + \ldots + P(B_n)\,P(A/B_n)$

$$= \sum_{i=1}^{n} P(B_i)\,P(A/B_i)$$

Baye's Theorem Let S be a sample space.

If A_1, A, A_3 ... A_n are mutually exclusive and exhaustiveevents such that $P(A_i) \neq 0$ for all i.

Then for any event A which is a subset of $S = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$ and $P(A) > 0$

We have,

$$P(A_i/A) = \frac{P(A_i).P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)} \text{ for all } i = 1,2,3,\ldots,n$$

Proof:

We have $A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n = S$ and $A_i \cap A_j = \phi$ for $i \neq j$

Since $A \subseteq S$

$$\Rightarrow \quad A = A \cap S = A \cap (A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n)$$

$$= (A \cap A_1) \cup (A \cap A_2) \cup (A \cap A_3) \cup \ldots \cup (A \cap A_n)$$

$$\therefore \quad P(A) = P(A \cap A_1) + P(A \cap A_2) + P(A \cap A_3) + \ldots + P(A \cap A_n)$$

$[\because (A \cap A_i) \cap (A \cap A_j) = \phi,\ \text{for } i \neq j]$

$P(A) = P(A_1)\,P(A/A_1) + P(A_2)P(A/A_2) + \ldots + P(A_n)\,P(A/A_n)$ ….(1)

Also, $P(A \cap A_i) = P(A)\ P(A_i/A)$ ....(2)

$$P(A_i/A) = \frac{P(A \cap A_i)}{P(A)}$$

$$\Rightarrow \quad P(A_i/A) = \frac{P(A_i)P(A/A_i)}{\sum_{i=1}^{n} P(A_i)P(A/A_i)} \qquad \text{(From (1) and (2))}$$

Example:

In a bolt factory 25%, 35% and 40% of the total is manufactured by machines A, B and C, out of which 5%, 4% and 2% are respectively defective. If the bolt drawn is found to be defective, what is the probability it is manufactured by the machine A?

Suggested answer:

Given P (A) = 0.25, P (B) = 0.35 and P (C) = 0.4 Let D be the event of getting a defective bolt.

P (D/A) = 0.05 P (D/B) = 0.04 P (D/C) = 0.02

$$\therefore P(A/D) = \frac{P(A)\,P(D/A)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{(0.25)(0.05)}{(0.25)(0.05) + (0.35)(0.04) + (0.4)(0.02)}$$

$$= \frac{0.0125}{0.0345}$$

$$= \frac{125}{345}$$

Random Variable and Probability Distribution

If is often very important to allocate a numerical value to an outcome of a random experiment. For example consider an experiment of tossing a coin twice and note the number of heads (x) obtained. Outcome : HH HT TH TT No. of heads (x) : 2 1 1 0 x is called a random variable, which can assume the values 0, 1 and 2. Thus random variable is a function that associates a real number to each element in the sample space.

Random variable (r.v) Let S be a sample space associated with a given random experiment. A real valued function X which assigns to each wi Î S, a unique real number.

$X(\omega_i) = x_i$ is called a random variable.

Note:

There can be several r.v's associated with an experiment. A random variable which can assume only a finite number of values or countably infinite values is called a discrete randomvariable. e.g., Consider a random experiment of tossing three coins simultaneously. Let X denote the number of heads then X is a random variable which can take values 0, 1, 2, 3.

Continuous random variable

A random variable which can assume all possible values between certain limits is called a continuous random variable.

Discrete Probability Distribution

A discrete random variable assumes each of its values with a certain probability, Let X be a discrete random variable which takes values $x_1, x_2, x_3, \ldots x_n$ where $p_i = P\{X = x_i\}$ Then X : $x_1 x_2 x_3 .. x_n$ P(X): $p_1\ p_2\ p_3 \ldots\ p_n$ is called the probabilitydistribution of x.

Note 1:

$$(a)\ p_i \geq 0 \quad \forall i \quad \text{and}$$

In the probability distribution of x $$(b)\ \sum_{i=1}^{n} p_i = 1$$

Note 2:

P{X = x} is called probability mass function.

Note 3: Although the probability distribution of a continuous random variable cannot be presented in tabular form, it can have a formula in the form of a function represented by f(x) usually called the probability density function.

Probability distribution of a continuous random variable Let X be continuous random variable which can assume values in the interval [a,b]. A function f(x) on [a,b] is called the probability density function if

$$(a)\ f(x) \geq 0,\ \forall\, x \in [a, b] \text{ and } \quad (b)\ \int_a^b f(x)\, dx = 1$$

Mean and Variance of a Discrete Random Variable Let X be a discrete random variable which can assume values $x_1, x_2, x_3, \ldots x_n$ with probabilities $p_1, p_2, p_3 \ldots\ p_n$ respectively then (a) Mean of X or expectation of X denoted by E(X) or m is given by

$$E(X) = \mu = \sum_{i=1}^{n} x_i\ p_i$$

(b) Variance of X denoted by s^2 is given by

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2\ p_i \ = \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2x_i\, \mu) p_i \ = \sum_{i=1}^{n} x_i^2\ p_i + \mu^2 \sum_{i=1}^{n} p_i - 2\mu \sum_{i=1}^{n} x_i\ p_i$$

$$= \sum_{i=1}^{n} x_i^2 \; p_i + \mu^2 - 2\mu^2 \quad = \sum_{i=1}^{n} x_i^2 \; p_i - \mu^2 \quad \sigma^2 = \sum_{i=1}^{n} x_i^2 \, p_i - \left(\sum_{i=1}^{n} p_i \, x_i \right)^2 = E(X^2) - [E(X)]^2$$

Note : σ is called the standard deviation and is equal to $\sqrt{\text{variance}}$.

Example :

Two cards are drawn successively without replacement, from a well shuffled deck of cards. Find the mean and standard deviation of the random variable X, where X is the number of aces.

Suggested answer:

X is the number of aces drawn while drawing two cards from a pack of cards. The total ways of drawing two cards ${}^{52}C_2$. Out of 52 cards these are 4 aces. The numbers of ways of notdrawing an Ace $= {}^{48}C_2$. The number of ways of drawing an ace is ${}^{4}C_1$ x ${}^{48}C_1$ and two aces ${}^{4}C_2$. Therefore the r.v. X can take the values 0, 1, 2.

X	x = 0	x = 1	x = 2
P (X = x)	$\frac{{}^{48}C_2}{{}^{52}C_2}$	$\frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}$	$\frac{{}^{4}C_2}{{}^{52}C_2}$

$$\text{MEAN} = E(X) = \sum_{x_i=0}^{2} x_i \; P(X = x_i)$$

$$= 0\left(\frac{{}^{48}C_2}{{}^{52}C_2}\right) + (1)\left(\frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}\right) + (2)\left(\frac{{}^{4}C_2}{{}^{52}C_2}\right) = \frac{{}^{4}C_1 \times {}^{48}C_1 + (2)\,{}^{4}C_2}{{}^{52}C_2}$$

$$= \frac{(4 \times 48) + 2(6)}{\frac{52 \times 51}{2}} = \frac{204}{26 \times 51} = \frac{2}{13} = 0.1538 \qquad E(X^2) = \sum_{x_i=0}^{2} (x_i^2)\, P(X = x_i)$$

$$= (0)\left(\frac{{}^{48}C_2}{{}^{52}C_2}\right) + 1^2\left(\frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}\right) + 2^2\left(\frac{{}^{4}C_2}{{}^{52}C_2}\right) = \frac{192 + 24}{{}^{52}C_2} = \frac{216}{26 \times 51} = 0.1629$$

$\text{Variance} = E(X^2) - [E(X)]^2 = 0.1629 - 0.0236 = 0.13925$ $\text{S.D} = \sqrt{\text{Variance}} = 0.373$ Let X be a continuousrandom variable which can assume values in (a, b) and f(x) be the probability density of x then (a) Mean of X or Expectation of X is given by $\mu = \int_a^b x \; f(x)\; dx$

(b) Variance of x is given by $\sigma^2 = \int_a^b (x - \mu)^2 \; f(x)\,dx = \int_a^b x^2 \; f(x)dx - \left\{\int_a^b x \; f(x)\; dx\right\}^2$

Binomial Distribution

A trial, which has only two outcomes i.e., "a success" or "a failure", is called a Bernoulli trial.

Let X be the number of successes in a Bernoulli trial, then X can take 0 or 1 and

$P(X = 1) = p$ = "probability of a success"

$P(X = 0) = 1 - p = q$ = "probability of failure".

Consider a random experiment of performing n independentBernoulli trials.

Let p be the probability of success, $q = 1 - p$ be the probability of failure.

The probability of x successes and consequently (n - x) failures in n independent trials in a specified order say SSSFFSSFF….FSF is given by

P(SSSFFSSFF…FSF)

= P(S) P(S) P(S) P(F) P(F) P(S) P(S) P(F)….P(F) P(S) P(F)

= p.p.p.qq.ppq….qpq

$= p^x q^{n-x}$

But x successes can occur in ${}^nC_x$ ways.

\ $P(X = x) = {}^nC_x\, p^x\, q^{n-x}$ is the probability mass function of exactly x successes.

The probability distribution of the number of successes, so obtained is called the binomial distribution.

X P[X = x]

0 q^n

1 ${}^nC_1\, pq^{n-1}$

2 ${}^nC_2\, p^2\, q^{n-2}$

3 ${}^nC_3\, p^3\, q^{n-3}$

n p^n

Note 1:

n and p are called the parameters of the binomial distribution.

Note 2:

If x is a binomial variate with parameters n and p then it is denoted by x = b(n, p).

Note 3:

$$\sum_{x=0}^{n} P(X = x) = \sum_{x=0}^{n} {}^{n}C_{x}\, p^{x}\ q^{n-x} = (p + q)^{n} = 1$$

Example:

5 cards are drawn successively with replacement from well shuffled deck of 52 cards. What is the probability that

i) all the five cards are spades

ii) only 3 cards are spades

iii) none is a spade.

Suggested answer:

Let X be the random variable for the number of spade cards drawn.

p = probability of drawing a spade card

$$= \frac{13}{52}$$

$$= \frac{1}{4}$$

q = 1 - p

$$= \frac{3}{4}$$

n = 5

$$\text{i) } P(X = 5) = {}^{5}C_{5}\left(\frac{1}{4}\right)^{5} \times \left(\frac{3}{4}\right)^{0}$$

$$\Rightarrow P(X = 5) = \left(\frac{1}{4}\right)^{5}$$

$$\text{ii) } P(X = 3) = {}^{5}C_{3}\left(\frac{1}{4}\right)^{3} \times \left(\frac{3}{4}\right)^{2}$$

$$= 10 \times \frac{1}{4^3} \times \frac{3}{4} \times \frac{3}{4}$$

$$= 90 \times \left(\frac{1}{4}\right)^5$$

$$P(X = 0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \times \left(\frac{3}{4}\right)^5$$

$$= \left(\frac{3}{4}\right)^5$$

Recurrence Relation for the Binomial Distribution

We have

$P(X = x + 1) = {}^nC_{x+1}\, p^{x+1}\, q^{n-x-1}$

$$\frac{P\{X = x + 1\}}{P\{X = x\}} = \frac{\dfrac{n!}{(x+1)!\,(n-x-1)!}\, p^{x+1}\, q^{n-x-1}}{\dfrac{n!}{x!\,(n-x)!}\, p^x\, q^{n-x}}$$

$$= \frac{n - x}{x + 1} \cdot \frac{p}{q}$$

$$\therefore \quad P(X = x + 1) = \frac{(n - x)}{(x + 1)} \frac{p}{q} P(X = x)$$

Mean and variance

$$E(x) = \sum_{x=0}^{n} x P\{x = x\}$$

$$= \sum_{x=0}^{n} x\ {}^nC_x\, p^x\, q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x\, n!}{x!\,(n-x)i}\, p^x\, q^{n-x}$$

$$= np \sum_{x=0}^{n-1} \frac{(n-1)!}{(x-1)!\,\{(n-1)-(x-1)\}!}\, p^{x-1}\, q^{(n-1)-(x-1)}$$

$$= np \sum_{x=0}^{n-1} {}^{n-1}C_x \; p^{x-1} \; q^{n-x}$$

$= np\ (p+q)^{n-1}$

$$E(x) = np \qquad (\because p + q = 1)$$

To find the variance:

We have

$$E(x^2) = \sum_{x=0}^{n} x^2 p\{X = x\}$$

$$= \sum_{x=0}^{n} (x(x-1) + x)\ p(X = x)$$

$$= \sum_{x=0}^{n} [x(x-1)]P(X = x) + \sum_{x=0}^{n} x P(X = x)$$

$$= \sum_{x=0}^{n} \frac{x(x-1)n!}{x!\ (n-x)!}\ p^x\ q^{n-x} + np$$

$$= p^2 n(n-1) \sum_{x=0}^{n} \frac{(n-2)!}{(x-2)!\ \{(n-2)-(x-2)\}!}\ p^{x-2}\ q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \sum_{x=0}^{n} {}^{(n-2)}C_{x-2}\ p^{x-2}\ q^{(n-2)-(x-2)} + np$$

$= n(n-1)\ p^2\ (p+q)^{n-2} + np$

$= n^2p^2 - np^2 + np$ $(\because p + q = 1)$

$= n^2p^2 + np(1-p)$

$= n^2p^2 + npq$

Now $V(x) = E(x^2) - [E(x)]^2$

$= n^2p^2 + npq - n^2\ p^2$

$= npq$

Example:

If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.

Suggested answer:

Mean of a binomial distribution = np = 9

Variance of a binomial distribution = npq = 6

$$\Rightarrow q = \frac{6}{np}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

$$\Rightarrow p = 1 - q = \frac{1}{3}$$

$$\Rightarrow n = \frac{9}{p} = 27$$

Binomial distribution is given by

$$P(X = x) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$$

Applications

After the elementary study in probability, let us see how we can utilise this basic knowledge in solving problems of different areas.

Example 1:

In a housing colony 70% of the houses are well planned and 60% of the houses are well planned and well built. Find theprobability that an arbitrarily chosen house in this colony is well built given that it is well planned.

Suggested answer:

Let A be the event that the house is well planned. B be theevent that the house is well built. P (A) = 0.7 $P(A \cap B) = 0.6$ Probability that a house, selected is well built given that it is well planned.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.6}{0.7}$$

$$= \frac{6}{7}$$

Example 2:

In a binary communication channel, A is the input and B is the output. Let P (A) = 0.4, P (B/A) = 0.9 and $P(\bar{B}/\bar{A}) = 0.6$. Find P (A/B) and $P(A/\bar{B})$.

Suggested answer:

We know that

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$0.9 = \frac{P(A \cap B)}{0.4}$$

$\Rightarrow P(A \cap B) = 0.36$ $P(\bar{B}/\bar{A}) = 0.6$ (given)

$$\Rightarrow \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = 0.6$$

$\Rightarrow P(\bar{A} \cap \bar{B}) = (0.6)(0.6)$ $(P(\bar{A}) = 0.6)$

$P(\bar{A} \cap \bar{B}) = 0.36$ $\Rightarrow P(A \cup B)' = 0.36$ $(\because \bar{A} \cap \bar{B} = (A \cup B)')$ $\Rightarrow P(A \cup B) = 0.64$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.64 = 0.4 + P(B) - 0.36$$

$$\Rightarrow P(B) = 0.6$$

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.36}{0.6}$$

$$= 0.6$$

By Law of total probability

$$P(A) = P(A/B)\,P(B) + P(A/\bar{B})\,P(\bar{B})$$

$$\Rightarrow 0.4 = (0.6)(0.6) + P(A/\bar{B})(0.4)$$

$$\Rightarrow 0.4 - 0.36 = (0.4)\,P\,(A/\bar{B})$$

$$\Rightarrow P(A/\bar{B}) = \frac{0.04}{0.4} = 0.1$$

Example 3:

A manufacture ships his products in boxes of 10. He guarantees that not more than 2 out of 10 items are defective. If the probability that an item selected at random from his production line will be defective is 0.1, what is the probabilitythat the guarantee is satisfied.

Suggested answer:

Let X be the random variable which represents number of defective items selected which has a binomial distribution with

n = 10, p = 0.1, q = 0.9

Probability that the guarantee is satisfied

$$= P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^{10}C_0\,(0.1)^0(0.9)^{10} + {}^{10}C_1(0.1)^1(0.9)^9 + {}^{10}C_2(0.1)^2(0.9)^8$$

$$= (0.9)^8\left[{}^{10}C_0(0.9)^2 + {}^{10}C_1(0.1)(0.9) + {}^{10}C_2(0.1)^2\right]$$

$$= (0.9)^8\,[0.81 + 0.9 + 0.45]$$

$$= (0.4305)(2.16) = 0.929$$

Conclusion

In this chapter we have studied the method of evaluating probabilities of events relating toindependent events and conditional events. We have also studied about random variables and their probability distributions, namely binomialdistribution and Poisson distribution.

The binomial distribution is defined on a random variable which takes finite discrete values whereas the Poisson distribution is defined on a random variable which takes infinite discrete values such as 0, 1, 2, 3, ….

There are few more discrete probabilitydistribution which will be discussed in higher classes. The most important continuous probability distribution in the entire field of statistics is normal distribution.

This bell shaped probability distribution function is also an approximate to binomial distribution. We will be learning the importance of thesedistribution function in later classes.