Circuits and Measurement Systems

Basic Circuits and Measurement Systems

- AC circuits
- Analogue Circuits and Devices
- Basic electrical theory
- Circuit analysis
- Circuit Analysis Techniques
- Circuit theory

AC Circuits

why study AC circuits? You probably live in a house or apartment with sockets that deliver AC. Your radio, television and portable phone receive it, using (among others) circuits like those below. As for the computer you're using to read this, its signals are not ordinary sinusoidal AC, but, thanks to Fourier's theorem, *any* varying signal may be analyzed in terms of its sinusoidal components. So AC signals are almost everywhere. And you can't escape them, because even the electrical circuits in your brain use capacitors and resistors.

Some terminology

For brevity, we shall refer to electrical potential difference as voltage. Throughout this page, we shall consider voltages and currents that vary sinusoidally with time. We shall use lower case v and i for the voltage and current when we are considering their variation with time explicitly. The**amplitude** or **peak value** of the sinusoidal variation we shall represent by V_m and I_m, and we shall use $V = V_m/2^{1/2}$and $I = I_m/2^{1/2}$ without subscripts to refer to the RMS values. For an explanation of RMS values, see Power and RMS values. For the origin of the sinusoidally varying voltage in the mains supply, see Motors and generators.

So for instance, we shall write:

$v = v(t) = V_m \sin(\omega t + \varphi)$
$i = i(t) = I_m \sin(\omega t)$.

where ω is the **angular frequency**. $\omega = 2\pi f$, where f is theordinary or cyclic frequency. f is the number of complete oscillations per second. φ is the **phase difference** between the voltage and current. We shall meet this and the geometrical significance of ω later.

Resistors and Ohm's law in AC circuits

The voltage v across a resistor is proportional to the current i travelling through it. (See the page on drift velocity and Ohm's law.) Further, this is true at all times: v = Ri. So, if the current in a resistor is

$i = I_m . \sin(\omega t)$, we write:
$v = R.i = R.I_m \sin(\omega t)$
$v = V_m. \sin(\omega t)$ where
$V_m = R.I_m$

So for a resistor, the peak value of voltage is R times the peak value of current. Further, they are in phase: when the current is a maximum, the voltage is also a maximum. (Mathematically, $\varphi = 0$.) The first animation shows the voltage and current in a resistor as a function of time.

The rotating lines in the right hand part of the animation are a very simple case of a **phasor diagram** (named, I suppose, because it is a vector representation of phase). With respect to the x and y axes, radial vectors or phasors representing the current and the voltage across the resistance rotate with angular velocity ω. The lengths of these phasors representthe peak current I_m and voltage V_m. The y components are $I_m \sin(\omega t) = i(t)$ and voltage $V_m \sin(\omega t) = v(t)$. You can compare i(t) and v(t) in the animation with the vertical components of the phasors. The animation and phasor diagram here are simple, but they will become more useful when we consider components with different phases and withfrequency dependent behaviour.

(For a comparison of simple harmonic motion and circularmotion, see Physclips.)

Capacitors and charging

The voltage on a capacitor depends on the amount of charge you store on its plates. The current flowing onto the positive capacitor plate (equal to that flowing off the negative plate) is by definition the rate at which charge is being stored. So the charge Q on the capacitor equals the integral of the current with respect to time. From the definition of the capacitance,

$v_C = q/C$, so

$$v = \frac{q}{C} = \frac{1}{C}\int i.dt$$

Now remembering that the integral is the area under the curve (shaded blue), we can see in the next animation why the current and voltage are out of phase.

Once again we have a sinusoidal current $i = I_m . \sin(\omega t)$, so integration gives

$$v = \frac{1}{C}\int I_m \sin(\omega t)\,dt$$

$$= -\frac{1}{\omega C} I_m \cos(\omega t)$$

$$= \frac{1}{\omega C} I_m \sin(\omega t - \pi/2)$$

$$= X_C . I_m \sin(\omega t - \pi/2)$$

(The constant of integration has been set to zero so that the average charge on the capacitor is 0).

Now we define the **capacitive reactance** X_C as the ratio of the magnitude of the voltage to magnitude of the current in a capacitor. From the equation above, we see that $X_C = 1/\omega C$. Now we can rewrite the equation above to make it look like Ohm's law. The voltage is proportional to the current, and the peak voltage and current are related by

$V_m = X_C.I_m$.

Note the two important differences. First, there is a difference in phase: the integral of the sinusoidal current is a negative cos function: it reaches its maximum (the capacitor has maximum charge) when the current has just finished flowing forwards and is about to start flowing backwards. Run the animation again to make this clear. Looking at the relative phase, the voltage across the capacitor is 90°, or one quarter cycle, behind the current. We can see also see how the $\varphi = 90°$ phase difference affects the phasor diagrams at right. Again, the vertical component of a phasor arrow represents the instantaneous value of its quanitity. The phasors are rotating counter clockwise (the positive direction) so the phasor representing V_C is 90° *behind* the current (90° clockwise from it).

Recall that **reactance** is the name for the ratio of voltage to current when they differ in phase by 90°. (If they are in phase, the ratio is called resistance.) Another difference between reactance and resistance is that the reactance is**frequency dependent**. From the algebra above, we see that the capacitive reactance X_C decreases with frequency . This is shown in the next animation: when the frequency is halved but the current amplitude kept constant, the capacitor has twice as long to charge up, so it generates twice the potential difference. The blue shading shows q, the integral under the current curve (light for positive, dark for negative). The second and fourth curves show $V_C = q/C$. See how the lower frequency leads to a larger charge (bigger shaded area before changing sign) and therefore a larger V_C.

Thus for a capacitor, the ratio of voltage to current**decreases with frequency**. We shall see later how this can be used for filtering different frequencies.

Inductors and the Farady emf

An inductor is usually a coil of wire. In an ideal inductor, the resistance of this wire is negligibile, as is its capacitance. The voltage that appears across an inductor is due to its own magnetic field and Faraday's law of electromagnetic induction. The current i(t) in the coil sets up a magnetic field, whose magnetic flux φ_B is proportional to the field strength, which is proportional to the current flowing. (Do not confuse the phase φ with the flux φ_B.) So we define the (self) inductance of the coil thus:

$\varphi_B(t) = L.i(t)$

Faraday's law gives the emf $E_L = - d\varphi_B/dt$. Now this emf is a voltage rise, so for the voltage drop v_L across the inductor, we have:

$$v_L(t) = -E_L = \frac{d\phi_B}{dt} = \frac{d}{dt}(Li)$$

$$= L\frac{d}{dt} I_m \sin(\omega t)$$

$$= \omega L I_m \cos(\omega t)$$

$$= \omega L I_m \sin(\omega t + \pi/2)$$

$$= X_L I_m \sin(\omega t + \pi/2)$$

Again we define the **inductive reactance** X_L as the ratio of the magnitudes of the voltage and current, and from the equation above we see that $X_L = \omega L$. Again we note the analogy to Ohm's law: the voltage is proportional to the current, and the peak voltage and currents are related by

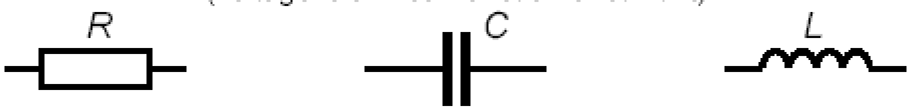
$V_m = X_L.I_m$.

Remembering that the derivative is the local slope of the curve (the purple line), we can see in the next animation why voltage and current are out of phase in an inductor.

Analogue Circuits

PROGRAM

Linear elements in electric circuits
(voltage is a linear function of current)



Resistor Capacitor Inductor (coil)

1. Effect of the electromagnetic Induction
2. Capacitance and inductance in a d.c. circuit
3. Alternating current (a.c.)
4. Elementary a.c. circuits
5. Analysis of a.c. circuits: phasor diagrams
6. Analysis of a.c. circuits: complex notations
7. Frequency response of a.c. circuits
8. Power in a.c. circuits
9. General network theorems

1. ELECTROMAGNETIC INDUCTION

- Magnetic field flux
- Lenz's rule
- Law of the electromagnetic induction
- Self-inductance

Electric current causes magnetic field

Is it possible the opposite?

Can magnetic field cause electric current?

In an immobile circuit, a **constant magnetic field cannot** cause any current

Current is induced by varying magnetic field

If a magnetic field through a circuit varies, the electric current is induced

- Current in a circuit is proportional to the rate with which the **magnetic field flux through the circuit** is varied

Basic Electrical Theory

- Thevenin's Theorem

- Millman's Theorem
- Alternating Current
- RMS Voltage

Thevenin's Theorem

In the first few parts of this series, we've used "differential Ohm's Law," Kirchoff's Current Law, Kirchoff's Voltage Law, node voltage analysis, and mesh analysis to analyze a simple three resistor circuit (figure 1). This time, we'll use *Thevenin's Theorem* to simplify the circuit, which cannot be analyzed by Ohm's Law by itself, down to something that **can** be analyzed using Ohm's Law. We'll also look at a couple ways to analyze a Wheatstone Bridge circuit using Thevenin's Theorem.

In doing research for this article, I found hundreds of books that discussed the application of Thevenin's Theorem, but very little about its origin. I *did* find that Thevenin's Theorem was apparently first published by Herman Von Helmholtz (1821 - 1834) in 1853. Helmhotz was a German scientist who worked in a wide variety of sciences. He was originally trained as a doctor, since the German government would pay for his training if we would then serve in the military. One reference stated "Helmholtz was the last scholar whose work, in the tradition of Leibniz, embraced all the sciences, as well as philosophy and the fine arts." I was unable to find a translation of his original stating of this theorem.

M. L. Thevenin (1857 - 1926), a French telegraph engineer, again discovered the theorem in 1883 (see *rendus hebdomadaires de seances de L'Academie des sciences, XCVII, 159 (1883)*, excuse my French!).

Finally, I also found a reference to it as the Thevenin-Pollard theorem. And, in 1926, E. L. Norton of Bell Telephone Laboratories replaced the voltage sources in the Thevenin (or whoever) equivalent with current sources to come up with the Norton equivalent. We'll look at Thevenin equivalents this time and Norton equivalents in a later article.

Thevenin Equivalent

A circuit containing linear devices (whose current is proportional to the applied voltage) and independent sources can be replaced with a single voltage source with a single series impedance for analysis. This *Thevenin equivalent* can replace the original complicated subcircuit in the original circuit. The use of the equivalent circuit should simplify (or make possible) the analysis of the original circuit. It should be noted that the equivalent circuit is **only** equivalent *outside* the equivalent. You cannot make efficiency calculations of the original subcircuit based on its equivalent.

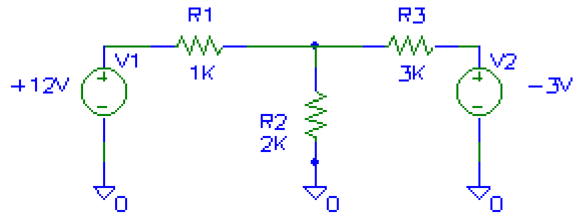


Figure 1 - Circuit to be analyzed.

Let's find the Thevenin equivalent of the "left half" of the circuit (replacing V1, R1 and R2). Other equivalents are possible, but care must be taken that you don't replace too much of the circuit with the equivalent, leaving points of interest "inside" the equivalent where they are not available for analysis. Figure 2 shows the circuit we wish to "Thevenize". Figure 3 shows how it has been replaced by a single "Thevenin voltage" and a single "Thevenin resistance".

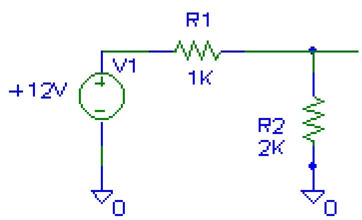


Figure 2 - Circuit to be *Thevenized.*

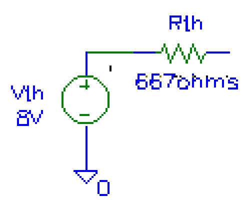


Figure 3 - Thevenin equivalent of figure 2

To determine the *Thevenin voltage* take the original circuit for which the equivalent is desired (figure 2), and place an "imaginary voltmeter" across the two terminals that connected this subcircuit to the remainder of the circuit. In this case, we measure the voltage at the junction of R1 and R2 with respect to ground. This "open circuit voltage" is the Thevenin voltage. In this circuit, it was easily determined using Ohm's Law or using the voltage divider formula (12V*(2K/3K)=8V).

To determine the *Thevenin resistance* (or, in AC circuits, the*Thevenin impedance*), replace all independent voltagesources with zero volt sources (a short) and replace all independent current sources with zero amp sources (an open). Place an "imaginary ohmmeter" the same place we put the voltmeter above. In this case, the shorting of V1 puts R1 in parallel with R2, yielding a *Thevenin resistance* of 667 ohms. The Thevenin equivalent of figure 2 is shown in figure 3. Figure 4 shows the original circuit with V1, R1 and R2 replaced by their Thevenin equivalent.

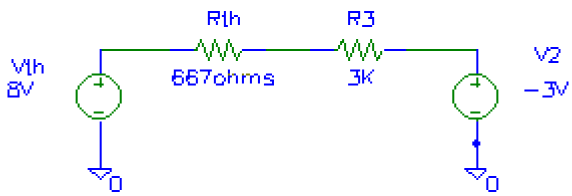


Figure 4 - Original circuit with equivalent.

Differential Ohm's Law, Again!

We can apply "differential Ohm's Law" to the circuit of figure 4 to determine circuit characteristics. First, we establish the direction of *conventional* current, which "flows downhill," from high (more positive) voltage to low (less positive or more negative). Here, the conventional current flows to the right. We can then determine that current using I=(Vtail-Vtip)/R. Vtail is +8 volts, while Vtip is -3 volts. R is the series combination of Rth and R2: 3.667K. The current is 3mA.

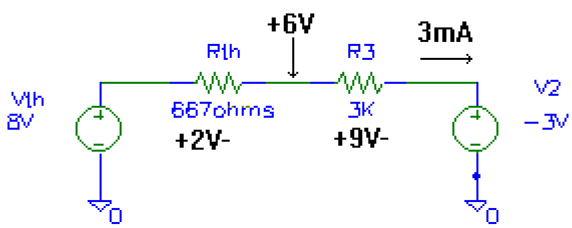


Figure 5 - Analyzing the equivalent circuit.

Using Ohm's Law, we can determine the voltages across Rth and R2, as shown in figure 5. To determine the voltage at the junction of Rth and R3, we can start at the right side of R3, which is at -3 volts due to V2, then "go up" 9 volts as we pass through R3 to the left. This puts the junction at +6 volts, as shown. We could also start at the top of Vth (+8 volts) and go down 2 volts as we go to the right through Rth. Again, we get +6 volts.

Back to the original circuit!

Once we know the voltage at the left end of R3, we can drop this information into figure 1 and determine any other desired currents. Note that this determination agrees with our previous calculations using node voltage analysis, mesh analysis, and superposition.

Next time, we'll try applying *two* Thevenin equivalents to the Wheatstone bridge. Your homework assignment is to find out who Wheatstone was, and why he got a bridge named after him.

Millman's Theorem

San Luis Obispo, Calif. In the last article in this series, we applied Norton equivalents to a simple three resistor circuit. Let's try generalizing the analysis to come up with Millman's Theorem.

Millman's Theorem is named after Jacob Millman. Mr. Millman was born in Russia in 1911 received a Ph.D. from MIT in 1935. He went on to write eight textbooks on electronics between 1941 and 1987 and was a professor of electrical engineering at Columbia University.

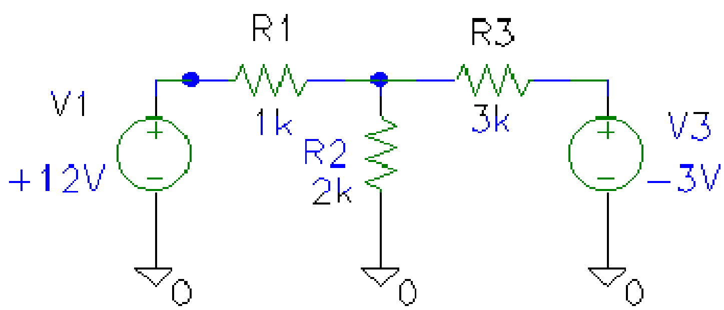


Figure 1 — Circuit to be analyzed

As you may recall from the last article in the series, we took the circuit of figure 1, reconfigured it as shown in figure 2, then converted the voltage sources with series resistances to current sources with parallel resistances (Norton equivalents). Note that the ground at the bottom of R_2 has been replaced with a zero volt voltage source (V_2) to ground. Recall that a "piece of wire" can be replaced with a zero volt voltage source. Adding thisvoltage source makes each branch of the circuit identical (avoltage source with a series resistance). Let's generalize the analysis by keeping the V's and R's, instead of substituting values.

Each voltage source with a series resistance is converted to a current source with a parallel resistance. The current is the "short circuit current" of the circuit which we are "Nortonizing". We find that $I_1=V_1/R_1$, $I_2=V_2/R_2$, $I_3=V_3/R_3$. As the circuit is expanded, it's fairly obvious that $I_N=V_N/R_N$. Further, as we determined last time, the Norton resistance (that resistance we will place across the current source) is the same as the Thevenin resistance (that resistance in series with the voltagesource). We determine the Thevenin resistance by shorting out the voltage sources and opening any current sources in the circuit we are trying to simplify, then measure the resulting resistance. For example, in the left portion of figure 2, we short out V_1, setting it to zero volts. We then measure the resistancefrom the top of R_1 to ground, getting (surprise!) R_1 ohms. Figure 3 shows the circuit of figure 2 reconfigured to use Norton equivalents for each section.

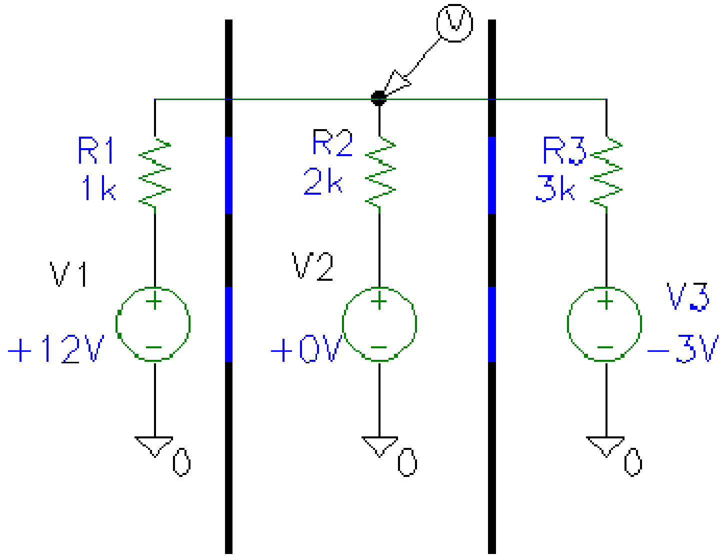


Figure 2 — Reconfigured circuit

This is pretty much where we left off last time. We added the parallel current sources to get the total current and applied this to the combined parallel resistance to find the voltage at the junction of the resistances. Let's generalize it! The total current is:

$I_1 + I_2 + I_3$

$(V_1/R_1) + (V_2/R_2) + (V_3/R_3)$

Further, the equivalent parallel resistance is:

$1/((1/R_1)+(1/R_2)+(1/R_3))$

Finally, the voltage at the resistor junctions is determined by multiplying the total current by the parallel resistance (Ohm's Law):

$V=IR$

$V=((V_1/R_1)+(V_2/R_2)+(V_3/R_3)) * (1/((1/R_1)+(1/R_2)+(1/R_3))$

$V=((V_1/R_1)+(V_2/R_2)+(V_3/R_3))/((1/R_1)+(1/R_2)+(1/R_3))$

The last equation is Millman's theorem. It might be more easily remembered by considering its components. If we replace the R in V=IR with 1/G (where G is conductance measured in Siemens) and replace V/R with I, the equation becomes:

$V=(I_1+I_2+I_3)/(G_1+G_2+G_3)$

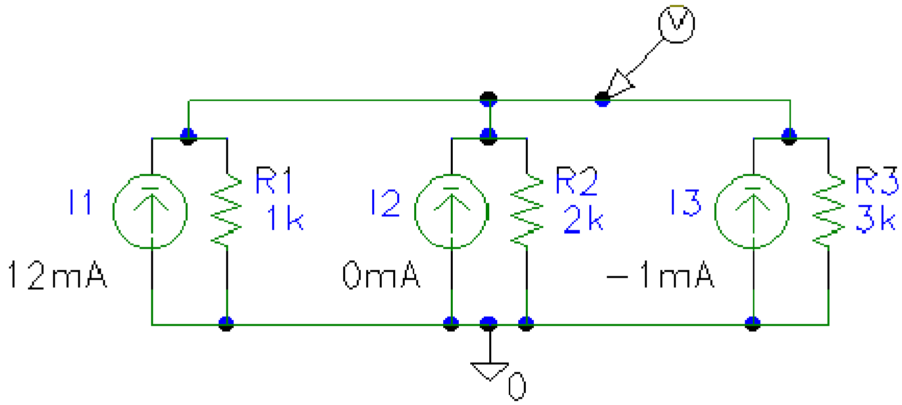


Figure 3 – Nortonized circuit

Generalizing for any number of sources and resistances,

$V=(I_1+I_2+I_3+...I_N)/(G_1+G_2+G_3+...G_N)$

It would be interesting to try to derive Millman's theorem using other circuit analysis technqiues. Let's see if we can derive it using superposition next month!

Alternating Current

San Luis Obispo, Calif. So far in this series, we've analyzed DC circuits with a few voltage or current sources and a few resistances. Let's start looking at alternating current. After getting a few definitions and derivations out of the way, we'll be able to analyze circuits using the same techniques we've used before, but use complex numbers instead of realnumbers. We'll use complex numbers to represent the magnitude and phase of voltages, currents, and impedances.

We'll start with a brief review of trigonometry. Figure 1 shows a right triangle on the x-axis of the x-y plane. Relative to angle *a*, the three sides can be identified as the *adjacent*, marked 'x', the *opposite*, marked 'y', and the *hypotenuse*, marked 'r'. The adjacent side is marked *x*, since it is along the x-axis. The opposite side is marked *y*, since it is in the same direction as the y-axis. Finally, the hypotenuse is marked *r*, since it can be thought of as the radius of a circle centered at the origin (0,0) as the angle a is varied. Further, a point at the far end of the hypotenuse has coordinates of (x,y) and is r units from the origin.

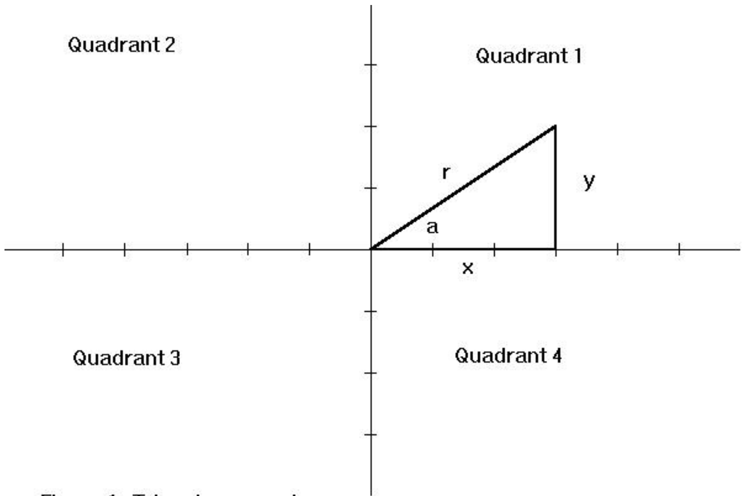


Figure 1. Triangle on x-y plane

The three basic trig functions are shown in table 1. The first column gives the English spelling of the function, the second gives the mathematical function notation, and the third column shows how to calculate the function based on thetriangle shown in figure 1.

sine	sin(a)	y/r
cosine	cos(a)	x/r
tangent	tan(a)	y/x

Table 1 - Basic trig functions.

In the triangle of figure 1, we can determine **r** as the square root of $\mathbf{(x^2+y^2)}$ or about 3.606. This gives makes sin(a)=2/3.606=0.555; cos(a)=3/3.606=0.832; and tan(a)=2/3=0.667. We can use these values and the inverse trig function to determine the angle. For example, the arctangent ($\tan^{-1}$) of 0.667 is 33.7 degrees. Similarly, the arcsine ($\sin^{-1}$) of 0.555 and the arccosine ($\cos^{-1}$) of 0.832 are both about 33.7 degrees.

If we set **r** = 1, then as **a** varies from 0 to 360 degrees, it scribes a *unit circle*, a circle with a radius of 1. This simplifies the sine and cosine as being the **y** and **x** values of the point on the unit circle.

Above, we referred to the angle **a** as being measured in*degrees*. A degree is 1/360 of the full cycle formed by the angle **a** before the point on the unit circle returns to the point it started. We generally measure this angle from the positive x-axis. If the point on the unit circle is in quadrant 4 (below the positive x-axis), we can either say the angle is negative, or say it is somewhere between 270 and 360 degrees.

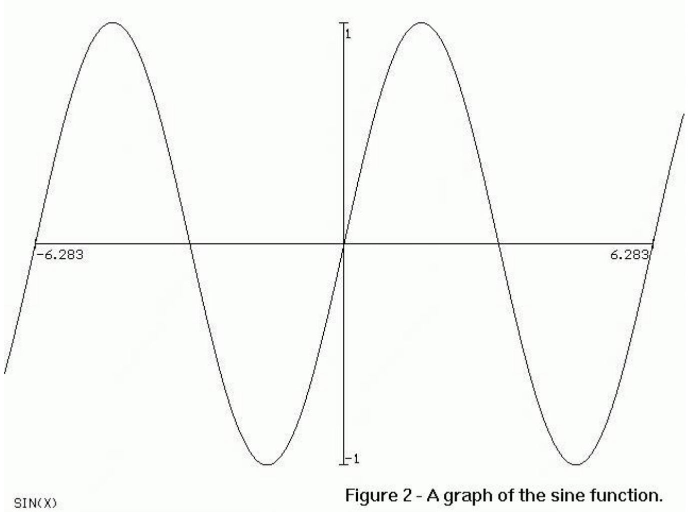


Figure 2 - A graph of the sine function.

We can also measure angles in *radians*. A *radian* is that angle formed when the circle scribed by the increasing angle draws an arc length equal to the radius of the circle. Since we know the circumference of a circle is pi times its diameter, and the diameter is two times the radius, a the circumference of the full circle is 2 pi r. Therefore, the full circle of 360 degrees is equivalent to 2 pi radians. Figure 2 shows how the sin(x) varies as x varies from -2 pi to +2 pi radians.

Note also that the sine varies between -1 and +1. We can multiply this by a constant and end up with a sine wave whose peak amplitude is equal to the constant. Further, we can substitute 360*f*t for **a** and get a sine wave that progresses with time, assuming the sine function expects an argument (input) in degrees and that **f** is the frequency in Hz (1/seconds) and **t** is time in seconds. If our sine function expects an argument in radians, we can use **wt** (where w is the Greek letter omega) as the argument (the input) to the sine function. Here, **w** represents the frequency in radians per second.

In figure 3, we've set **f** to 1,000, and **w** = 2 pi f . Further, we've multiplied the sine function by 2, giving peak amplitude of 2. The resulting waveform is a 2 volt peak sine wave. The amplitude (which could represent a voltage or current) can be determined at any instant in time using the function V(t)=2*sin(w*t) where w=2*pi*1000. This is assuming the sine function accepts an argument in radians. If the sine function expects an argument in degrees, the function becomes V(t)=2*sin(360*1000*t) . You might try both of these on your calculator, then compare the results to the instantaneous voltage indicated on an oscilloscope displaying a 2 volt peak 1 KHz sine wave.

Next time, we'll look at other characteristics of the sine wave. These include its average value, RMS value, period and wavelength when it is propagated. Stay tuned!

RMS Voltage

San Luis Obispo, Calif. In the last article in this series, we discovered that the average voltage of a full cycle of a sinewaveform is 0 volts (since the positive side "cancels out" the negative side), while the average of a *half* cycle of a sinewaveform is the peak voltage times 2/pi. This was determined first by taking the average of several instantaneous voltages through the half-cycle, then by increasing the number of samples towards infinity by applying a little calculus. This time, we'll look at the RMS voltage of asine wave.

Root Mean Square

We've heard that RMS stands for "Root Mean Square", but that expression may not be the model of clarity. If we recall that the *arithmetic mean* of two numbers is just the average of those two numbers, the expression *starts* to make a little sense. What we are doing is taking the square root of the average of the squares of the instantaneous voltages.

But he *means* well...

As a side note, I found this summer while tutoring algebra that the *mean* is a number that is part of a sequence ofnumbers that is between two other numbers. If the sequence of numbers is an arithemtic sequence (where each successivenumber is the previous number plus some constant), each of the elements of the sequence between two other elements of the sequence are *means* of the other two. For example, in the below sequence of numbers, each successive number is determined by adding three to the previous number.

1, 4, 7, 10, 13, 16

In this sequence, the *means* of 4 and 13 are 7 and 10. How about if there is only a single *mean*? That single mean is then*the arithmetic mean*. If we try a couple examples, we find that 7 is the arithmetic mean of 4 and 10. Further, 7 is the*average* of 4 and 10 (since (4+10)/2=7). So, to find thearithmetic mean of a couple numbers, just take the average.

Another kind of sequence is a *geometric* sequence. In a geometric sequence, each successive number is determined by multiplying the previous number by a constant (as opposed to *adding* a constant, which we did before). The below geometric sequence is formed using a constant of 3.

1, 3, 9, 27, 81, 243

9 and 27 are geometric means of 3 and 81 in this sequence. 9 is *the geometric mean* of 3 and 27 in this sequence. We normally find the geometric mean by taking the squre root of the product of the two numbers. Here, 3*27=81, and the square root of 81 is indeed 9. How about 27 and 243? 27*43=6,561 and the square root of 6,561 is 81.

Back on RMS, we'll take the square root of the *arithmetic*mean of the squares of the instantaneous voltages. Why? It has something to do with power. Let's see what the average power delivered by a 1 volt peak sine wave is into a 1 ohm resistor. Starting with the formula for power,

P=IV

and substituting Ohm's Law's I=V/R for I, we get

$P=V^2/R$

Further, the instantaneous voltage of the sine wave is

$V(a)=V_P*\sin(a)$

where V_P is the peak voltage and *a* is how far we are in to the waveform in degrees or radians (depending upon which sine function we're using). Table 1 shows the voltage at various points through a single cycle of a 1 volt peak sine wave.

Table 1 - Instantaneous voltage and power.

Radians	Degrees	Volts	Power (W)
0	0	0	0
0.392699	22.5	0.382683	0.1464463
0.785398	45	0.707107	0.5
1.178097	67.5	0.92388	0.8535543
1.570796	90	1	1
1.963495	112.5	0.92388	0.8535543
2.356194	135	0.707107	0.5
2.748894	157.5	0.382683	0.1464463
3.141593	180	0	0
3.534292	202.5	-0.382683	0.1464463

3.926991	225	-0.707107	0.5
4.31969	247.5	-0.92388	0.8535543
4.712389	270	-1	1
5.105088	292.5	-0.92388	0.8535543
5.497787	315	-0.707107	0.5
5.890486	337.5	-0.382683	0.1464463

To find the average of these instantaneous powers, we can merely add them up and divide by the number of samples. My calculator shows the sum of the powers to be 8.0000. Dividing by the number of samples (16), we find the average power is 1/2 watt (or 500 mW). This continuously varying voltage dissipates 500 mW in a 1 ohm resistor. What DC voltage would dissipate that same power?

Above, we found that $P=V^2/R$. Solving for V, we get V=sqrt(P*R). Since, in this case, R=1 ohm, we can find the voltage by taking the square root of the power. So... A DC voltage source of sqrt(500mW) would deliver the same power to a 1 ohm load as a 1 volt peak sine wave does. Doing the square root (more popular than the Macarena?), we find the equivalent DC voltage is 707 mV. What did we do? We took the square root of the mean of the squares of the instantaneous voltages, hence RMS.

If we have a higher peak voltage, all the voltages in the volt column of table 1 would be multiplied by a constant (the peak voltage). All the powers in the power column would be multiplied by the peak voltage squared, which would result inan average of $V_P^2/2$. Taking the square root of the average, we get V_P/sqrt(2) as the RMS value of *any* sine wave. Messing around with the equation, we find that $V_P=V_{RMS}$*sqrt(2). Hence, a 117 VAC power line has a peak voltage of 165.463 volts. The instantaneous voltage varies between 165.463 and -165.463 volts.

Calculus

We were lucky that our average turned out as well as it did. Choosing 16 samples in a cycle gave us the exact relationship between RMS and peak voltage. Recall from our discussion of average voltage last month that we can keep increasing thenumber of samples towards infinity and find the true average of a continuoously varying waveform. Figure 1 shows how calculus can demonstrate the relationship between peak voltage and RMS voltage.

Circuit Analysis Techniques

Fundamentals

Ohm's Law states the voltage across a resistor, *R* (or impedance, **Z**) is directly proportional to the current passing through it (the resistance/impedance is the proportionality constant)

dc: $v(t) = i(t)\, R$ ac: $\mathbf{V} = \mathbf{I}\,\mathbf{Z}$

Kirchhoff's Voltage Law (KVL): the algebraic sum of thevoltages around any loop of *N* elements is zero (like pressure drops through a closed pipe loop)

$$\sum_{j=1}^{N} v_j(t) = 0$$

Kirchhoff's Current Law (KCL): the algebraic sum of the currents entering any node is zero, *i.e.*, **sum of currents entering** equals **sum of currents leaving** (like mass flow at a junction in a pipe)

$$\sum_{j=1}^{N} i_j(t) = 0$$

Nodal Analysis

Nodal analysis is generally best in the case of several voltage sources. In nodal analysis, the variables (unknowns) are the "node voltages."

<u>Nodal Analysis Procedure</u>:

1. Label the *N* node voltages. The node voltages are defined positive with respect to a common point (*i.e.*, the reference node) in the circuit generally designated as the *ground* ($V = 0$).

2. Apply KCL at each node in terms of node voltages.

    a. Use KCL to write a current balance at *N*-1 of the*N* nodes of the circuit using assumed current directions, as necessary. This will create *N*-1 linearly independent equations.

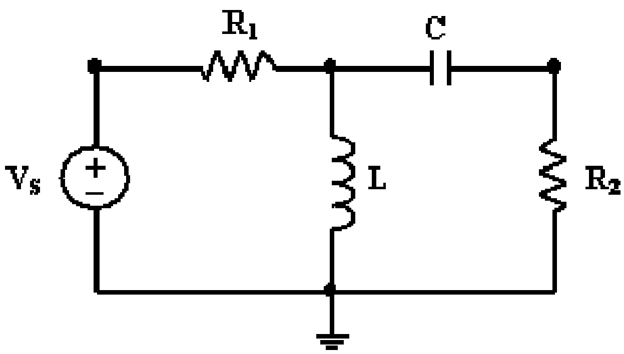
    b. Take advantage of *supernodes*, which create constraint equations. For circuits containingindependent voltage sources, a supernode is generally used when two nodes of interest are separated by a voltage source instead of aresistor or current source. Since the current (*i*) is unknown through the voltage source, this extra constraint equation is needed.

    c. Compute the currents based on voltage differences between nodes. Each resistive element in the circuit is connected between two nodes; the current in this branch is obtained via Ohm's Law where V_m is the positive side and current flows from node *m* to *n* (that is, *I* is *m* --> *n*).

$$\text{dc: } i = \frac{V_{mn}}{R} = \frac{V_m - V_n}{R} \qquad \text{ac: } I = \frac{V_{mn}}{Z} = \frac{V_m - V_n}{Z}$$

3. Determine the unknown node voltages; that is, solve the *N*-1 simultaneous equations for the unknowns, for example using Gaussian elimination or matrix solution methods.

Nodal Analysis Example



1. Label the nodal voltages.

2. Apply KCL.

   a. KCL at top node gives $I_S = I_L + I_C$

   b. Supernode constraint eq. of $V_L = V_S$

   c. $$\frac{V_L - V_T}{R_1} = \frac{V_T - 0}{Z_L} + \frac{V_T - 0}{Z_C + R_2}$$

3. Solve for V_T for instance.

Loop or Mesh Analysis

Mesh (loop) analysis is generally best in the case of several current sources. In loop analysis, the unknowns are the loop currents. Mesh analysis means that we choose loops that have no loops inside them.

Loop Analysis Procedure:

1. Label each of the loop/mesh currents.

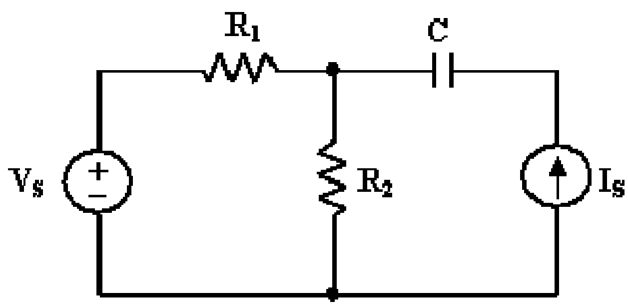
2. Apply KVL to loops/meshes to form equations with current variables.

a. For *N* independent loops, we may write *N* total equations using KVL around each loop. *Loop currents* are those currents flowing in a loop; they are used to define *branch currents*.

b. Current sources provide constraint equations.

3. Solve the equations to determine the user defined loop currents.

Mesh Analysis Example:



1. Label mesh currents.

2. Apply KVL.

   a. Left loop KVL:

$V_S = R_1 I_1 + R_2(I_1 - I_2)$

   b. Constraint equation $I_2 = -I_S$.

3. Solve for I_1 and I_2. Note: Branch current from mesh currents: $I_M = I_1 - I_2$

Superposition

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

Procedure:

1. For each independent voltage and current source (repeat the following):

   a. Replace the other independent voltage sources with a *short circuit* (*i.e.*, $v = 0$).

   b. Replace the other independent current sources with an *open circuit* (*i.e.*, $i = 0$).

Note: Dependent sources are not changed!

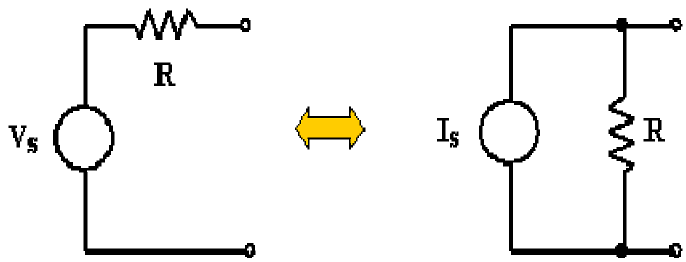
c. Calculate the contribution of this particular voltage or current source to the desired output parameter.

2. Algebraically sum the individual contributions (current and/or voltage) from each independent source.

Source Transformation

An ac voltage source **V** in series with an impedance **Z** can be replaced with an ac current source of value **I**=**V**/**Z** in parallel with the impedance **Z**.
An ac current source **I** in parallel with an impedance **Z** can be replaced with an ac voltage source of value **V**=**IZ** in series with the impedance **Z**.

Likewise, a dc voltage source *V* in series with a resistor *R* can be replaced with a dc current source of value *i* = *v*/*R* in parallel with the resistor *R*; and vice versa.

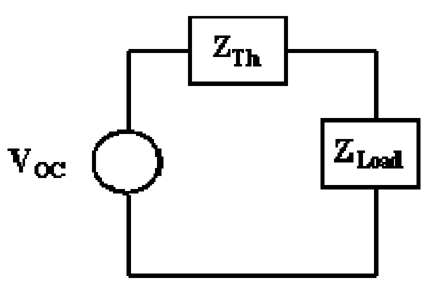


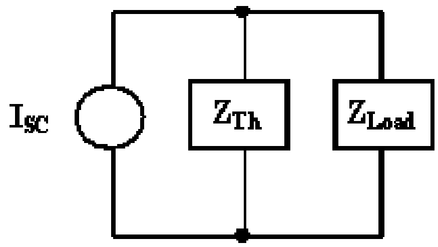
Thévenin's and Norton's Theorems

Thévenin's Theorem states that we can replace entire network, exclusive of the load, by an equivalent circuit that contains only an independent voltage source in series with an impedance (resistance) such that the current-voltage relationship at the load is unchanged.

Norton's Thereom is identical to Thévenin's Theorem except that the equivalent circuit is an independent current source in parallel with an impedance (resistor). Hence, the Norton equivalent circuit is a source transformation of the Thévenin equivalent circuit.

Thévenin Equivalent Circuit Norton Equivalent Circuit





Procedure:

1. Pick a good breaking point in the circuit (cannot split a dependent source and its control variable).

2. **Thevenin**: Compute the open circuit voltage, $\mathbf{V}_{OC}$.
   Norton: Compute the short circuit current, $\mathbf{I}_{SC}$.

3. Compute the Thevenin equivalent resistance, R_{Th} (or impedance, $\mathbf{Z}_{Th}$).

   a. If there are only independent sources, then short circuit all the voltage sources and open circuit the current sources (just like superposition).

   b. If there are only dependent sources, then must use a test voltage or current source in order tocalculate $R_{Th}= v_{Test}/i_{Test}$ (or$\mathbf{Z}_{Th}=\mathbf{V}_{Test}/\mathbf{I}_{Test}$).

   c. If there are both independent and dependent sources, then compute R_{Th} (or $\mathbf{Z}_{Th}$) from $R_{Th}=v_{OC}/i_{SC}$ (or $\mathbf{Z}_{Th}=\mathbf{V}_{OC}/\mathbf{I}_{SC}$).

4. Replace circuit with Thevenin/Norton equivalent.
   Thevenin: $\mathbf{V}_{OC}$ in series with R_{Th} (or $\mathbf{Z}_{Th}$).
   Norton: $\mathbf{I}_{SC}$ in parallel with R_{Th} (or $\mathbf{Z}_{Th}$).

Note: for 3(b) the equivalent network is merelyR_{Th} (or $\mathbf{Z}_{Th}$), that is, no current or voltage sources.