ELECTROMAGNETICS

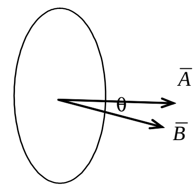
3. ELECTROMANETIC INDUCTION

POINTS TO REMEMBER

1. The magnetic lines of force passing through a normal plane is called magnetic flux.

   $\phi = \bar{B}.\bar{A} = BA\cos\theta$ Where θ is the angle between $\bar{B}$ and $\bar{A}$

   (Vector $\bar{A}$ is perpendicular to its plane)



   a. If θ = 0°, if the plane of the coil is perpendicular to the field ϕ = BA
   b. If θ = 90°, (i.e,) if the plane of the coil is in the direction of the field ϕ = 0.
   c. If θ = 180° ϕ = -BA
2. In 1831 Faraday discovered the converse effect of the previous chapter i.e., production of electric current using magnetic field.
3. **Faraday laws:**
   a. Whenever there is changing in magnetic flux linked with a coil, emf is induced in it.
   b. The induced emf is proportional to the negative rate of change of magnetic flux linked with the coil.

   $$e = -\frac{d\phi}{dt} \quad \text{(or)} \quad e = \frac{-d\phi}{dt}$$

   For N turns $e = -\frac{d(N\phi)}{dt}$ (Proportionality constant K = 1)

   $$e = -\frac{d}{dt}(NBA) \quad \text{(Neumann's Law)}$$

   $$e = -N\frac{d\phi}{dt} = -N\frac{(\phi_2 - \phi_2)}{t}$$

   c. Induced current (i) = $\frac{e}{R} = \frac{-N(\phi_2 - \phi_1)}{tR}$

   d. Induced change (q) = $\frac{-N(\phi_2 - \phi_1)}{R}$ (Induced charge is independent of time)
4. **<u>Lenz's Law</u>** : Induced emf always opposes the change that produces it.Lenz's law obeys the law of conservation of energy.
5. **Motional emf :** If a straight conductor of length l moves with a constant speed v perpendicular to the uniform magnetic field of induction B, then a voltage is developed across ends of the conductor ends which is given by $e = Blv$
6. Induced current (I) = $\frac{e}{R} = \frac{Blv}{R}$
7. **Self Inductance:** If the current in a coil changes, the magnetic flux around the coil changes. Hence emf is induced in the coil called self inductance.

   If i is the current through the coil and ϕ is the flux lines around the coil then $\phi \propto i \Rightarrow \phi = Li$. Where L is the coefficient of self induction.

   $$e = -\frac{d\phi}{dt} = -L\frac{di}{dt}$$

   Unit of L - Henry or wb/amp or volt-sec/amp

8. **Energy stored in an Inductor:**

$$U = \frac{1}{2}Li^2$$

9. Mutual Inductance:

a. When current in a coil changes, the magnetic flux linked with other coil changes and an emf is induced in the secondary coil called mutual induction.

b. If I_P is the current through the primary and ϕ_S is the flux linked with secondary,

$\phi_s \,\alpha\, I_p \Rightarrow \phi_s = M\, I_p$ where M is mutual inductance of the coil

Unit of M : Henry (or) volt - sec. amp (or) weber/ amp.

LONG ANSWER QUESTIONS

1. State Faraday's laws of electromagnetic induction, Lenz's law. Explain self and mutual induction. Derive the equations for L and M.

Ans. **Faraday laws:**

c. Whenever there is changing in magnetic flux linked with a coil, emf is induced in it.

d. The induced emf is proportional to the negative rate of change of magnetic flux linked with the coil.

$e = -\frac{d\phi}{dt}$ (or) $e = \frac{-d\phi}{dt}$ (Proportionality constant K = 1)

For N turns $e = -\frac{d(N\phi)}{dt}$

$e = -\frac{d}{dt}(NBA)$ (Neumann's Law)

$e = -N\frac{d\phi}{dt} = -N\frac{(\phi_2 - \phi_2)}{t}$

Induced current (i) = $\frac{e}{R} = \frac{-N(\phi_2 - \phi_1)}{tR}$

Induced change (q) = $\frac{-N(\phi_2 - \phi_1)}{R}$ (Induced charge is independent of time)

Lenz's Law This law states that 'the direction of an induced emf is always opposes the changes in the magnetic flux that causes it'. Lenz's law obeys the law of conservation of energy.

Self Induction : when a current is passed through a coil of wire, magnetic field is produced in the surrounding space. A proportionate magnetic flux (ϕ) is linked with the coil itself. If the current through the coil is changing with time, the magnetic flux linked with the coil also changes. As a result, an emf and hence a current are induced in that coil itself.

Thus due to a varying current a coil, a current is induced in the coil itself. This is called self induction. The emf developed is called self induced emf (or back emf). The current developed is called self induced current.

From lenz's law, the induced current oppose the main current flowing through coil.

Definition: It is the property of a coil by virtue of which, it opposes the change of current through it.

At any instant, the magnetic flux (ϕ) linked with a coil is directly proportional to the current (i) flowing through it.

$\therefore \phi \propto i \Rightarrow \phi = Li$ (for one turn) ……….(1) or $N\phi = Li$ (for N turns)

Here L is a constant called co-efficient of induction or self inductance of the coil.

Induced emf $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(L.i) = -L\frac{di}{dt}$...............(2)

When $\frac{di}{dt} = 1$, we get L = e

Definition of self inductance (L) of a coil :

It is numerically equal to the em.f induced in the coil due to unit rate of change of current through the coil.

SI.I unit of L = henry $= \frac{volt - \sec}{ampere}$

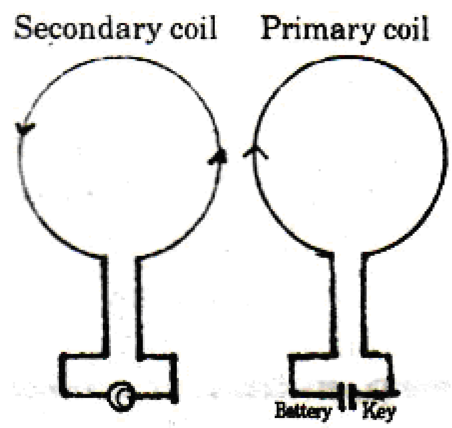
b) Mutual induction: Two coils P and S are placed close to each other and electrically insulated from each other. The primary coil P is connected to a battery B and a tap key K. The secondary coil S is connected to a galvanometer (G).

When a current is passed through the primary coil, a magnetic field is produced in the surrounding space. This magnetic field links both primary and secondary. Hence, a proportional magnetic flux (ϕ) is linked with the secondary. If the current through the primary coil is changing with time, the magnetic flux linked with the secondary coil also changes. As a result, an emf and hence a current are induced in the secondary coil.

Thus, due to a varying current in one coil, a current is induced in the neighbouring coil. This is called mutual induction. The emf induced is called mutual induced emf and current induced is called mutual induced current.

Definition: It is property, of a pair of coils by virtue of which, a current is induced in one coil, when the current is changing in the neighbouring coil.

Let a current (i) be flowing in one coil (P). Then the magnetic flux (ϕ) linked with the other coil (S) is



$\phi \propto i \Rightarrow \phi = Mi$...........(1) for N turns, $N\phi = Mi$

Here, M is a constant called coefficient of mutual induction between the two coils.

Induced emf $e = \frac{d\phi}{dt} = -\frac{d}{dt}(M.i) = -M.\frac{di}{dt}$...............(2)

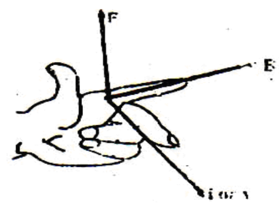
When $\frac{di}{dt} = 1$, we get M = e

Definition of M:- It is numerically equal to the emf induced in one coil due to unit rate of change of current in the other coil. S.I. unit of M = Henry= $\frac{volt-\sec}{ampere}$

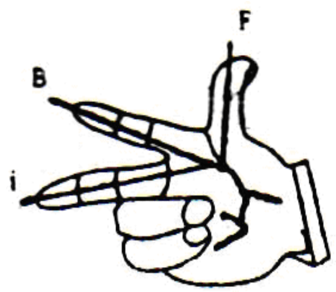
SHORT ANSWER QUESTIONS

1. What is the difference between Fleming's right hand and Fleming's left hand rule?

Ans. **<u>Fleming's left hand rule:</u>** Stretch fore finger, middle finger and thumb of the left hand in mutually perpendicular directions. If the fore finger represents the direction of magnetic field, middle finger represents electric field and then the thumb represents the direction of force (or) motion of the conductor.



Fleming's Right hand Rule: When a conductor is moving in a magnetic field, and induced current flows through it. The direction of induced current in relation to the direction of magnetic field and the force acting on the conductor is given by Fleming's right hand rule.



Stretch fore-finger, central finger and thumb of right hand mutually perpendicular to each other. If fore-finger represents the direction of magnetic field, the thumb represents the direction of motion of the conductor, and then the central finger indicates the direction of induced current.

2. State and explain Lenz's law with examples.

Sol: **Lenz's law statement:** The direction of an induced E.m.f. is always such that it tends to oppose the change in the magnetic flux that caused it.

Explanation: Consider a circular coil into which a magnet with its north pole is inserted.

The changing magnetic field induces electric current in a coil. This induced current itself creates magnetic field and hence magnetic flux around the coil. If the induced current in the coil is due to increase in flux, then the direction of induced flux is so oriented as to decrease the external flux.

If the decreasing flux induces current in coil then the direction of induced flux is such that the external field is increased. Therefore, the change in the external magnetic field is always field is increased. Therefore, the change in the external magnetic field is always opposed. Due to this opposition some work must be done to move the magnet. This work is liberated in the form of heat in coil.

VERY SHORT ANSWER QUESTIONS

1. Define self induction and mutual induction. (March2010)

Ans. **<u>Self induction of a coil:</u>** When the current in a coil changes, then an e.m.f is induced in the coil opposing the change in the current through the coil. This phenomenon is called self induction.

Mutual Induction: Consider two coils P and Q placed close to each other. If current i_1 flows in P, a magnetic flux is set up. When current in the coil P changes, the magnetic flux linked with other coil changes inducing e.m.f.This induced emf is known as mutually induced e.m.f and this phenomenon is known as mutual induction.

2. Define coefficient of self induction, coefficient of mutual induction.

Ans. **Coefficient of self induction:** Self inductance of a coil is numerically equal to induced e.m.f produced in the coil when the rate of change of current is unity. $L = \frac{-E}{\left(\frac{di}{dt}\right)} = -\frac{Edt}{di}$

SI unit of coefficient of self induction or self inductance is Henry (H).

Coefficient of mutual induction : The coefficient of mutual induction or mutual inductance of two coils is numerically equal to the induced e.m.f. produced in one coil when the rate of change of current in the other coil is unity.

$$M = \frac{-E}{\left(\frac{dI_p}{dt}\right)} = -\frac{Edt}{dI_p}$$

SI unit of mutual inductance is Henry (H)

3. State the factors on which self inductance depend.

Ans. Self inductance of a coil (L) depends on the dimensions of the coil, the number of turns in the coil and the relative permeability of the core material.

4. State the factors on which mutual inductance depend.

Ans. Mutual inductance between two coils (M) depends on the number of turns in each coil, the area of cross section of the coils and the permeability of core material.

5. The flux through a coil changes by 200 webers in 19 seconds. What is the induced e.m.f?

Sol: $d\phi = 200Wb$, dt = 10 seconds.

Induced e.m.f. $e = -\frac{d\phi}{dt} = \frac{200}{10} = 20V$

6. As two circular coils approach each other, what happens to the coefficient of mutual inductance between them?

Ans. When two coils approach each other, their separation decreases and coupling increases. So flux linked with one coil due to changing current in other coil increases. Hence the value of mutual inductance (M) between the coils increases.

7. If a current of 2A is reduced to zero in $\frac{1}{16}$sec, find the induced e.m.f. in an inductor of inductance 0.25H.

Ans. L = 0.25H ; di = 2 amp ; $dt = \frac{1}{16}\sec$; $e = -L\frac{di}{dt} = \frac{32}{4} = 8\,volts$

SOLVED PROBLEMS

1. Calculate the coefficient of self induction of a coil of 100 turns when a current of 5A produced a magnetic flux of 1 μ Wb

Sol: $N = 100, i = 5A, \phi = 1\mu Wb = 10^{-6}Wb, L = ?$

$$L = \frac{N\phi}{i} = \frac{100\times10^{-6}}{5} = 20\times10^{-6}H = 20\ \mu H.$$

2. The current in a coil is changed from 5A to 10A in $10^{-2}s$. Then an emf of 50 mV is induced in a coil near by it. Calculate mutual inductance of two coils. (Mrch2009)

Sol: $di = 10 - 5 = 5A; dt = 10^{-2}s$

Induced emf $(e) = 50mV = 5010^{-3}V; M = ?$

$$e = M\frac{di}{dt} \Rightarrow M = e\frac{di}{dt}$$

$$M = 50\times10^{-3}\times\frac{10^{-2}}{5} = 10^{-4}H = 100\mu H$$

3. A magnetic field in a certain region is given by $B = \left(40\hat{i} - 18\hat{k}\right)$ G. How much flux passes through a $5.0\,cm^2$ area loop in this region if the loop lies flat on xy-plane?

Sol: $A = 5.0\times10^{-4}\,\hat{k}\,m^2 \quad ; \quad B = (40\hat{i} - 18\hat{k})10^{-4}\,T$

$\phi = B.A$

$\phi = (5.0\times10^{-4})k.(40i - 18k)10^{-4}$

$\phi = -(5\times18)10^{-8}$

$\phi = -90\times10^{-8} = -900\,nwb$

4. A closed coil having 50 turns, area 300c m^2 and resistance 40 ohm is held at right angles to a uniform field of 0.02 wb/m^2. It is then turned through an angle of 30^0 about an axis at right angles to the field. Find the charge induced in the coil given Cos $30^0 = 0.8660$.

Sol:

$\phi_1 = B.A = (0.02)(3\times10^{-2}) = 6\times10^{-4}\,wb$

$\phi_2 = BA\cos\theta = (0.02)(3\times10^{-2})\cos30^0 = (0.02)(3\times10^{-2})\,0.8660$

$= (6\times10^{-4})(0.8660)\,wb$

$$e = \frac{-N(\phi_2 - \phi_1)}{t}$$

$$e = \frac{-50\left[(6\times10^{-4})(0.8660) - 6\times10^{-4}\right]}{t} = \frac{50\times6\times10^{-4}\,(0.134)}{t}V$$

But, Charge

$$q = it = \left(\frac{e}{R}\right)t = \frac{et}{R} = \frac{et}{40} = \frac{50\times6\times10^{-4}\times0.134\times t}{t\times40} = 1.005\times10^{-4}\text{ coulomb}$$

5. A conducting loop of 4 ohm is in the plane of the paper. It has a uniform induction B over its area of 0.002 m^2. The direction of B is normal to the plane of the loop. Calculate induced current in the loop, if B is decreasing at a rate of 0.1 wb/m^2

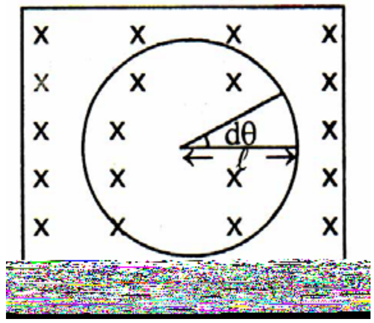
Sol: $\dfrac{d\phi}{dt} = A\dfrac{dB}{dt} = 0.002\times0.1 = 2\times10^{-4}$ Wb

Induced emf $e = -\dfrac{d\phi}{dt} = \dfrac{-2\times10^{-4}}{1} = -2\times10^{-4}$ V

Induced current $i = \dfrac{e}{R} = \dfrac{2}{4}\times10^{-4} = 0.5\times10^{-4}\,A$

6. A metal rod of length 1m is rotated about one of its ends in a plane at right angles to a uniform magnetic field of induction $2.5\times10^{-3} Wb/m^2$. If it makes 1800 rev/min, calculate the induced emf between its ends.

Sol: When a metal rod of length ℓ is rotated at right angles to a uniform magnetic field of induction B, the area swept by it as it rotates through an angle dθ in a time interval dt is



dA = Area of the circular arc $=\frac{1}{2}\ell^2 d\theta$

The magnitude of induced emf $e=\frac{d\phi}{dt}=\frac{d}{dt}(BA) \quad = B\frac{dA}{dt}$

$(\because B \text{ is constant})$

$$=B\times\frac{1}{2}\ell^2\frac{d\theta}{dt}=\frac{1}{2}B\ell^2\omega$$

Where ω is the angular velocity of rotation of the metal rod.

Now, $B=2.5\times10^{-3}Wb/m^2; \ell=1m; \ \omega=\frac{2\pi n}{t}=\frac{2\pi\times1800}{60}=60\pi \text{ rad/s}$.

The induced emf, $e=\frac{1}{2}B\ell^2\omega==\frac{1}{2}\times2.5\times10^{-3}\times(1)^2\times60\pi=75\pi\times10^{-3}=0.2357\text{ V}$

<u>UNSOLVED PROBLEMS</u>

1. A coil has inductance 0.05H and 100 turns. Calculate the flux linked with it when 0.02A current is passed through it.

Sol: n = 100 ; L = 0.05H; i = 0.02A;

Now, $N.\phi=L.i$ or $\phi=\frac{Li}{N}$

$\therefore \phi=\frac{0.05\times0.02}{100}=10^{-5}Wb$

2. A current of 2A is passed through a coil of 1000 turns, to produce a flux of 0.5μ .wb. **Calculate self inductance of the coil?**

Sol: i = 2A; n = 1000

$\phi=0.5\mu Wb=0.5\times10^{-6}Wb$; L = ?

But, $n\phi=Li$

$\therefore L=\frac{n\phi}{i}=\frac{1000\times0.5\times10^{-6}}{2}=2.5\times10^{-4}H$

3. A coil has self inductance of 0.01H. The current through it is allowed to change at the rate of 1A in $10^{-2}s$ **,calculate the e.m.f. induced.**

Sol: L = 0.01H

$\frac{di}{dt}=\frac{1}{10^{-2}} \ amp/\text{sec}$

$$e = L.\frac{di}{dt} = 0.01 \times \frac{1}{10^{-2}} = 1V$$

4. **The current decays from 5A to 2A in 0.01sec in a coil. The e.m.f. induced in a coil near by it is 30V. Calculate the mutual inductance of the coil.**

Sol: $e = M.\frac{di}{dt} \Rightarrow 30 = M.\frac{5-2}{0.01} \Rightarrow M = 0.1H$

5. **The e.m.f induced in a secondary coil is 20000V when the current breaks in the primary. The mutual inductance is 5H and the current reaches to zero in 10^{-4} s in primary. Calculate the maximum current in the primary before the break.**

Sol: $e = M.\frac{di}{dt} \Rightarrow 20,000 = \frac{5 \times (I_{\max} - 0)}{10^{-4}}$

$$\therefore I_{\max} = \frac{20,000 \times 10^{-4}}{5} = 0.4\,amp$$

<u>**ASSESS YOURSELF**</u>

1. **A copper ring is held horizontally and a bar magnet is dropped through the ring with a its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than or less than that due to gravity?**

Ans. When the magnet approaches the ring, the magnetic flux linked with the ring increases. Now induced emf is developed in the ring. As the ring is a loop current is induced in it such that it opposes the approach of the magnet. The acceleration of the magnet becomes less than g.

2. **A short magnet is allowed to fall along the axis of a horizontal metallic ring. Straight from rest, the distance fallen by the magnet in one second may be**
a) 4.0m b) 5.0m c) 6.0m d) 7.0m

Ans. (a)

3. **A conductor of length l moves with a velocity v in a uniform magnetic field of induction B. If the magnetic field is acting normally what is the induced emf?**

Ans.. **B *l* V**