Nuclear Physics

2.RADIOACTIVITY

POINTS TO REMEMBER

1. **Radioactivity (Henry Becquerel):** The process of spontaneous disintegration of the nuclei of heavy elements with the emission of certain radiations is known as natural radioactivity.
2. The radiations emitted from a radioactive element are classified into 3 types namely
   1) α - rays 2) β - rays 3) γ - rays
3. **α-particles** are identified as helium nuclei having 2 units of +ve charge and 4 units of mass.
   The general decay of α particles is represented as $_zX^A \rightarrow {}_{z-2}Y^{A-4} + {}_2He^4$
4. **β- rays** are treated as stream of electrons having one unit of –ve charge and negligible mass.
   β decay is represented as $_zX^A \rightarrow {}_{z+1}Y^A + {}_{-1}e^0$.
5. **γ Rays** are electromagnetic waves moving with the velocity of light.
6. **Decay Law.**The rate of disintegration, i.e., the number of atoms disintegrated per second is directly proportional to the number of atoms present that moment.
   $\frac{dN}{dt} = -\lambda N$ OR $N = N_0 e^{-\lambda t}$ λ = decay or disintegration constant.
7. **Half life** of a radioactive substance is defined as the time during which half of the atoms of the substance will disintegrate $T = \frac{0.693}{\lambda}$
8. **Average Life** is the ratio of total life of all the atoms of a given sample to the total number of atoms present in the sample. $\lambda = \frac{0.693}{T}$ OR $\tau = \frac{T}{0.693}$ OR $\tau = 1.44T$.
9. After n half lifes $\left(n = \frac{t}{T}\right)$, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$
10. **Artificial transmutation :** When a nitrogen nucleus hit by an -particle, disintegrates into oxygen nucleus and a proton.
    $_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$
    This process of producing a new stable nucleus from other stable nucleus is called artificial transmutation of elements.
11. When lighter atoms are bombarded with α-particles, atoms get radio active nature.
    $_{13}Al^{27} + {}_2He^4 \rightarrow {}_{15}P^{30} + {}_0n^1$
    The half life period of the radioactive $_{15}P^{30}$ is 3.25 m and it decays in silicon and positron
    $_{15}P^{30} \rightarrow {}_{14}Si^{30} + {}_{+1}e^0$

Long Answer Questions

1. What is radioactivity? State the law of radioactive decay. Show that radioactive decay is exponential in nature. Does the activity of a radioactive element depend on external physical conditions?

Ans: **Radioactivity:** The process of spontaneous disintegration of the nuclei of heavy elements with the emission of certain radiations is known as natural radioactivity.

The nuclei of certain elements with atomic number more than 82 disintegrate by emitting alpha (α), beta (β) and gamma (γ) rays. This phenomenon is called radioactivity.

Law of radioactive Decay: This law states that the number of radioactive nuclei decreases exponentially with time.The rate of disintegration $\left(\frac{dN}{dt}\right)$, i.e., the number of atoms disintegrated per second is directly proportional to the number of atoms present (N) a that moment. This is known as Decay Law.

Explanation

Let N_0 be the number of atoms present in a radioactive sample initially i.e., at t = 0.

Let N be the no of atoms left at a time t and dN be the no of atoms disintegrating in a short interval of time t.

The rate of disintegration = $\frac{dN}{dt}$

According to decay law , $\frac{dN}{dt} \alpha N$ Or $\frac{dN}{dt} = -\lambda N$

Where, λ = decay or disintegration constant. The negative sign indicates that as time increases, the number of atoms decreases

$$\frac{dN}{dt} = -\lambda N \qquad \text{OR} \qquad \frac{dN}{N} = -\lambda dt$$

Integrating the above equation

$$\int \frac{dN}{N} = -\lambda \int dt \qquad \text{Or} \qquad \log_e N = -\lambda t + c \text{ ......(1)}$$

Where c = integration constant and it can be found in the initial condition.

i.e., If t = 0, $N = N_0$

Then, $\log_e N_0 = c$ ………… (2)

From equation (1)

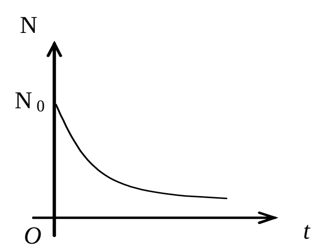
$\log_e N = -\lambda t + \log_e N_0$

$\log_e N - \log_e N_0 = -\lambda t$ (or)

$\log_e \left(\frac{N}{N_0}\right) = -\lambda t$

$\frac{N}{N_0} = e^{-\lambda t}$

OR $N = N_0 e^{-\lambda t}$

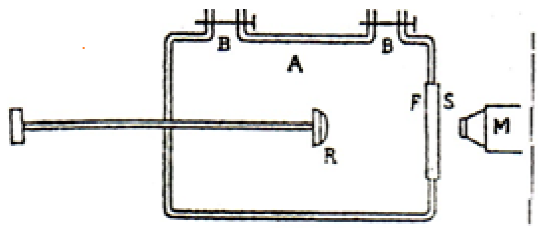


a) The number of radioactive nuclei decreases exponentially with time and reduced to zero after infinite time

b) The above equation represent radioactive decay is exponential in nature. The activity of a radioactivity element does not depend on external physical conditions like temperature, pressure etc.

2. What is artificial transmutation? Describe the experiment that led to the discovery of artificial transmutation? Why only high energy α-Particles were used in the discovery of artificial transmutation and neutron?

Ans: **<u>Artificial Transmutation:</u>** The process of converting a stable nucleus into other stable nucleus by bombarding with high energetic particles like α-particles is called artificial transmutation of elements.



Experiment: Rutherford allowed α-particles to pass through different gases and studied the different processes that may happen.

The apparatus consists of a thick glass chamber A provided with an adjustable rod, carrying a radioactive substance R (radium C). The side of the glass tube facing R is covered by metal plate with a central hole which is closed by a thin silver foil . A screen S coated with a fluorescent material like zinc sulphide is arranged in front of the silver foil and the scintillations produced on it can be observed through the microscope M.

The radioactive substance emits α-particles whose range in air is about 8cm. when the glass tube is filled with nitrogen gas scintillations are observed even when R is at a distance of 40cm from the foil. These particles producing scintillations cannot be α-particles as they cannot have such a long range.

Further analysis proved that each of these particles had a mass nearly equal to that of hydrogen atom and carried a positive charge equal to that of an electron. The new particles were named as protons.

When nitrogen nucleus hit by an $\alpha({}_2He^4)$-particle, it disintegrates into oxygen nucle4us and a proton ${}_1H^1$. The nuclear reaction can be represented as

$${}_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$$

The light elements from boron to potassium, with the exception of carbon and oxygen will be transmuted by bombarding with α-particles $({}_2He^4)$. One such study led to the discovery of neutron.

3. Explain the discovery of neutron. Mention its properties.

Ans: **Discovery of Neutron:** The existence of neutron was first predicted by Rutherford in the year 1920. But it was discovered by Chadwick in 1932. In 1930, Bothe and Becker, two German scientists observed that a highly penetrating radiation was emitted when Boron or Beryllium were bombarded with particles of energy about 5 MeV emitted from polonium (P_0) .These were thought to be high energetic γ-rays because these are not affected by electric or magnetic fields.

$$_4Be^9 +_2 He^4 \rightarrow \left[_6C^{13} \right] \rightarrow _6C^{13} + \gamma$$

In this process, the total energy available is 14MeV which must be shared by the $^{13}_{6}C$ and γ-photon. Hence their energy should be slightly less than 14MeV. Absorption measurements estimated that the γ-photon energy should be about 7MeV. Curie and Joliot observed that when this radiation is passed through hydrogenated materials like paraffin, water, etc., high energy protons were ejected with a maximum energy of about 7.5MeV. From the calculation, it has shown that ejection of 7.5MeV protons needed γ-photons as estimated by the measurement of the energy of protons expelled from paraffin is far below the value of 64MeV. Besides this, the energy of the radiation coming out from beryllium was predicted to be different, when the same radiation was incident on other hydrogenous substances. In other words, the energy of the same radiation was found to be having different values, when incident of different hydrogenous substances. Thus this led to controversies about the energy of the γ-photons.

Late in the year 1932, J.Chadwick observed concluded that these are a group of neutral particles of mass equal to that of protons. These neutral particles are called neutrons.

$$_4Be^9 +_2 He^4 \rightarrow \left[_6C^{13} \right] \rightarrow _6C^{12} + _0n^1 + Q \text{ (Q = energy)}$$

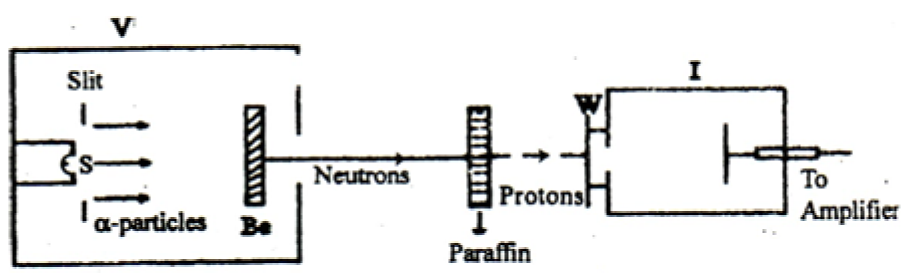


Figure shows the schematic diagram of Chadwick's experiment; α-particles were emitted from the sources S (polonium). These α-rays were allowed to bombard the Beryllium layer. The particles (Neutrons) coming out from the Beryllium were allowed to be incident on a paraffin block (w). It emits high energy protons which were then allowed to pass through an ionization chamber. Thus the neutron was discovered by Chadwick.

Properties of Neutron:

a) Neutron is an uncharged particle and hence it is not deflected by electric and magnetic field.

b) It was very high penetrating power and has ver low ionization power.

c) A free neutrons is unstable and spontaneously decays into a proton, electron and an antineutrino $\left(\bar{\upsilon}\right)$ $\quad {}_0n^1 \rightarrow {}_1^1H + {}_{-1}e^0 + \gamma$

d) If fast neutrons pass through substances like heavy water, paraffin wax, graphite etc., they are slowed down.

e) Neutrons are diffracted by crystals.

SHORT ANSWER QUESTIONS

1. State the properties α-rays.

Ans: **Properties of α- Rays :**

1. α- rays are identified as helium nuclei.
2. Velocity is of order of 10^6m/s. These posses energy of about 6MeV
3. These (among α, β, γ rays) have less penetrating power since these are massive
4. These have intense ionizing power
5. These range is 3cm - 8cm (distance up to which ionizing power exists is called range)
6. These are deflected by both electric & magnetic fields
7. These affect the photographic plates.
8. These produce fluorescence on zinc sulphide screens or barium platino-cyanide coated screens.

2. State the properties of β-rays.

Ans: **Properties of β- rays**

1. β - rays are stream of electrons.
2. The velocity of β particles ranges from 33% - 99.8% of velocity of light.
3. The energy ranges from 2-3 MeV.
4. These can pass through few millimeter thick aluminum sheets.
5. Their ionizing power is less than that of α rays.
6. Their range is several meters.
7. They produce fluorescence on zinc sulphide screens.
8. These affect photographic plates.
9. These are deflected by both electric and magnetic fields

3. State the properties of γ-rays.

Ans: **Properties of γ Rays**

1. These are electromagnetic waves moving with the velocity of light.
2. These are not deflected by both electric and magnetic fields
3. These have high penetrating power.
4. These have less ionizing power
5. These produce fluorescence on zinc sulphide screens
6. These affect photographic plates.

4. Define the term 'decay constant' for a radioactive substance. Deduce the relation between decay constant and half life period.

Ans: **Decay constant:** The ratio of number of nuclei decaying per unit time $\left(\frac{dN}{dt}\right)$ to the number of nuclei (N) present at the instant is known as decay constant.

The half life of a radioactive substance is defined as the time during which half of the atoms of the substance will disintegrate.

When t = T or $T_{1/2}$, then $N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T} \text{ (OR) } 2 = e^{\lambda T} \text{ (OR) } \lambda T = \log_e 2$$

$$T = \frac{\log_e 2}{\lambda} = \frac{2.303 \quad \log_{10} 2}{\lambda} \qquad \Rightarrow \; T = \frac{0.693}{\lambda}$$

5. Define average life on a radioactive substance. Obtain the relation between decay constant and average life.

Ans: **Average life:** It is the ratio of total life of all the atoms of a given sample to the total number of atoms present in the sample.

Let N_0 be the number of atoms present at t = 0 in the substance. Let N be the number of atoms present in a time t.Let dN be the number of atoms disintegrated in a time interval of t and t + dt, i.e., each of dN atoms lived for a time t.

Total life of dN atoms = tdN

$$\text{Average life } (\tau) = \frac{\text{Total life of all atoms}}{\text{Number of atoms}} = \frac{\int_0^\infty t\,dN}{N_0}$$

$$\text{But } \frac{dN}{dt} = -\lambda N \qquad \text{OR} \qquad dN = -\lambda N dt$$

$$\tau = \int_0^\infty \frac{-t\lambda N dt}{N_0} \qquad \text{OR} \qquad \tau = \int_0^\infty \frac{-t\lambda N_0 e^{-\lambda t} dt}{N_0}$$

$$\therefore \text{Average life } \tau = \frac{1}{\lambda}$$

But T or $T_{1/2} = \frac{0.693}{\lambda}$

$$\lambda = \frac{0.693}{T} \qquad \text{OR} \qquad \tau = \frac{T}{0.693} \qquad \text{OR} \qquad \tau = 1.44T$$

7. What do you meant by half-life and mean-life of a radioactive substance? (May2009)
Deduce the relation between them.

Ans: **Half-life:** The half life of a radioactive substance is defined as the time during which half of the atoms of the substance will disintegrate.

The relation between half-life (T) and disintegration constant (λ) of radioactive substance is

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} ....(1)$$

Mean life time (or) average life time (T_{avg}) : It is define as the ratio of the total life time of all the N_0 nuclei to the total number of original nuclei N_0.

$$T_{avg} = \left(\frac{1}{\lambda}\right) ....(2)$$

From (1) and (2) we have relation between average life and half-life and half-life of a radioactive substance as

$$T_{\frac{1}{2}} = 0.693 \Rightarrow T_{avg} = \frac{T_{\frac{1}{2}}}{0.693}$$

8. Write a short note on the discovery of neutron (March2011,June2010)

Ans: **Discovery of Neutron:** The existence of neutron was first predicted by Rutherford in the year 1920. But it was discovered by Chadwick in 1932. In 1930, Bothe and Becker, two German scientists observed that a highly penetrating radiation was emitted when Boron or Beryllium were bombarded with particles of energy about 5 MeV emitted from polonium (P_0) .These were thought to be high energetic γ-rays because these are not affected by electric or magnetic fields.

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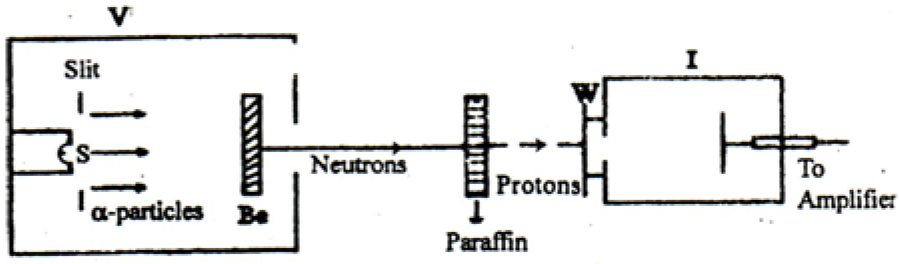

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9. Mention the uses of radioisotopes.

Ans: **<u>Uses of Radioisotopes</u>**

Name — **Use**

Radio carbon – Carbon dating.

Radio tritium – a suitable substitute for radium

Radio cobalt – Radiography or radiotherapy

Radio sulphur – chemical and industrial applications.

Radio phosphorus – treatment of blood cancer.

Radio iodine – thyroid diseases

Radio sodium – circulatory disorders in blood vessels can be studied.

Radio iron – deficiency in red blood cells.

Radio tritium age of underground water or deuterium

Radio oxygen – photosynthesis.

Radio antimony – defects in pipes used for underground fluid flow (tracing

VERY SHORT ANSWER QUESTIONS

1. What are the characteristic of radioactivity?

Ans: 1. The process of spontaneous disintegration of the nuclei of heavy elements with the emission of certain radiations is known as natural radioactivity.

2. Natural radioactivity is associated with the emission of radiations and is exhibited by the heavy elements (above lead Pb = 82)
3. These nuclei are unstable and they disintegrate to acquire move stable state.
2. Radio-activity is a nuclear property.
3. Radio-activity is practically unaffected by the conditions like temperature pressure etc.

2. Natural radioactive nuclei are mostly nuclei of high mass number. Why?

Ans: In general, natural radioactivity is displayed by heavy nuclei beyond lead in the periodic table because of the instability of the nuclei.

3. Does the ratio of neutrons to protons in a nucleus increase, decrease or remain the same after the emission of an α-particle?

Ans: During α-decay atomic number decreases by two units and its mass number A decreases by four units. Both electric charge and nucleon number are conserved in this process. Therefore the ratio of neutrons to proton in a nucleus remains the same after emission of an α-particle.

4. A nucleus contains no electrons but can eject them. How?

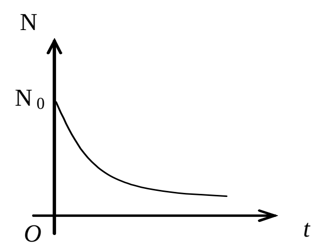
Ans: When a nucleus disintegrates and radiates β-rays, it undergoes β –decay. β-particle is nothing but fast moving electros. In the conversion of a neutron into a proton a β -particle is emitted. $_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \overline{\gamma}$ (Anti-neutrino)

5. State the law of radioactive decay. Sketch a graph to illustrate radioactive decay.

Ans: **<u>Law of Radioactive decay :</u>** The rate of radioactive decay $\left(\frac{dN}{dt}\right)$ i.e., the number of nuclei decaying per unit time at any instant is directly proportional to the number of nuclei (N) present at that instant and is independent of the external physical conditions like temperature, pressure etc.,

$$\frac{dN}{dt} \propto N \Rightarrow dN = -\lambda N\, dt$$

Where λ is decay constant or disintegration constant.



6. Which of the following radiations, α-rays, β, -rays, γ-rays (i) are similar to X-rays (ii) are easily absorbed by matter (iii) travel with greatest speed (iv) are similar in nature to cathode rays ?

Ans: i) γ-rays are electromagnetic radiation which similar to X-rays

ii) α-rays can be easily absorbed by matter

iii) γ-rays travel greatest speed i.e. with the speed of light.

iv) β-rays are similar in nature to cathode rays.

7. What are the units and dimensions of disintegration constant?

Ans: SI unit : $(\text{second})^{-1}$

Dimensional formula: $\left[M^0L^0T^{-1}\right]$

8. What is meant by the activity of a radioactive material? How is it related to decay constant?

Ans: **T**he number of radio-active nuclei that decay per unit time is called activity of the sample.

Activity of a radioactive sample = λN

9. A radioactive substance having N nuclei has activity A. Write down an expression for its half-life in terms of A and N ?

Ans: **A**ctivity of a radioactive sample $A=\lambda N \Rightarrow \lambda=\frac{A}{N}$

$\therefore$ Half life period $T=\frac{0.693}{\lambda}=\frac{0.693}{\left(\frac{A}{N}\right)}=\frac{N}{A}0.693$

10. Write a general equation that represents α-emission?

Ans: The general equation of α-decay can be written as ${}_{z}X^{A} \rightarrow {}_{z-2}Y^{A-4}+{}_{2}He^{4}$ where X refers to parent nucleus and Y refers to daughter nucleus.

11. Why do all the electrons emitted during β-decays not have the same energy?

Ans: The velocities of β-rays from 1% to 99% of the velocity of light. Hence their mass increases as the velocity increases. According to Einstein's theory of relativity

$$m=\frac{m_0}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$$

Where m_0 rest mass and c is is velocity of light.

Hence as $'\upsilon'$ increases, mass 'm' also increases and thereby they do not have same energy.

12. Neutron is the best projectiles to produce nuclear reactions. Why?

Ans: Neutrons are electrically neutral and they are not deflected by both magnetic and electric field. They have high penetrating power. Therefore neutrons enter the nucleus easily and are not deflected by the positive charge of the nucleus. Thus neutrons are the best projectiles to produce nuclear reactions.

13. Neutrons cannot produce ionization. Why?

Ans: As they are electrically neutral and highly penetrating, they cannot produced ionization.

14. State any two properties of neutron.

Ans: 1 Neutrons are electrically neutral and they are not deflected by both magnetic and electric field. 2) It has high penetrating power and has very low ionizing power.

15. What are radio isotopes? Give some examples of radio isotopes.

Ans: The isotopes of elements which are radioactive are called radio isotopes.

Ex: Radio phosphorus P^{32}, radio carbon C^{14}, radio iodine I^{131}, Radio Sodium Na^{24} .

16. The half-life of ofa substance is 69.3 days. Calculate its average life.(May2009)

Ans. Average life $\tau = \frac{T}{0.693} = \frac{69.3}{0.693} = 100\ days$.

SOLVED PROBLEMS

1. The half –life of radius is 1600 years. How much time does 1g of radius take to reduce to 0.125g?

Sol: $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{0.125}{1} = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$

$\therefore$ Time taken = 3 half lives = 3 x 1600 = 4800 years

2. The half-life of ^{58}Co is 72days. How much time does it take for $\frac{3}{4}th$ of the initial mass to disintegrate?

Sol: Mass of the substance remaining $= 1 - \frac{3}{4} = \frac{1}{4}$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)^n \Rightarrow n = 2$

$\therefore$ Time taken = 2half lives =2 x 72 = 144 days.

3. The half life of radium is 1600 years. Find its average life

Sol: Half-life T = 1600 years

Average life $\tau = \frac{T}{0.693} = \frac{1600}{0.693} = 2308\, days$.

4. The half-life of ^{58}Co is 72 days. Calculate its average life.

Sol: half – life T = 72 days

Average life $\tau = \frac{T}{0.693} = \frac{72}{0.693} = 103.9 days$

5. **Find the kinetic energy of the α-particle emitted in the decay $^{238}_{94}Pu \rightarrow ^{234}_{92}U + ^{4}_{2}He$ .The atomic masses of ^{238}Pu ¸ ^{234}U and α-particle are 238.04955u, 234.04095u and 4.002603u respectively. Neglect any recoil of the nucleus.**

Sol: Q = [Total mass of the reactants – Total mass of the products] c^2

$$= [Mass\,of\ ^{238}Pu - (mass\,of\ ^{234}U + Mass\,of\ ^{4}_{2}He]c^2$$

= [238.04955 – (234.04095 + 4.002603)] $u \times c^2$

= (0.0060030) 931.5MeV = 5.592MeV.

This is converted as kinetic energy of the α-particle.

6. **After a certain lapse of time, the fraction of radioactive polonium un-decayed is found to be 12.5% of initial quantity. What is the duration of this time lapse, if the half-life of polonium is 138 days.**

Sol: $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{12.5}{100} = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$

∴ Time taken = 3 half lives =3 x 138 = 414 days.

7. **A radioactive sample has 2.0×10^{20} active nuclei at a certain instant of time. How many of them will still be in the same active state after three half-lives?**

Sol: $n = 3$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{2 \times 10^{20}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \Rightarrow N == \frac{2 \times 10^{20}}{8} = 2.5 \times 10^{19}$$

8. **The half-life of ^{238}U for α-decay is 4.5×10^9 years. How many disintegrations per second occur in 1g of ^{238}U ? (Avogadro's number = $6.023 \times 10^{23}\ mol^{-1}$)**

Sol: Half life $T = 4.5 \times 10^9 \times 365 \times 86400\,s = 1.419 \times 10^{17}\ s$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{1.419 \times 10^{17}} = 4.882 \times 10^{-19} s^{-1}.$$

Number of ^{238}U atoms in 1g, $N = \frac{\text{Avogadro's number}}{\text{mass number}} = \frac{6.023 \times 10^{23}}{238} = 2.530 \times 10^{21}$

Number of disintegrations per second,

$$\frac{dN}{dt} = \lambda N = 4.882\times10^{-18}\times2.530\times10^{21} = 1.235\times10^{4}\,s^{-1}$$

9. **A radioactive substance has 6.0×10^{18} active nuclei initially. What time is required for the active nuclei of the same substance to become 1.0×10^{18} if its half-life is 40s?**

Sol: The number of active nuclei at any instant of time t, $N = N_0 e^{-\lambda t}$ (or) $\frac{N_0}{N} = e^{+\lambda t} \Rightarrow \log_e \frac{N_0}{N} = \lambda t$

$$\therefore t = \frac{\log_e\left(\frac{N_0}{N}\right)}{\lambda} = \frac{2.303\ \log_{10}\left(\frac{N_0}{N}\right)}{\lambda}$$

But $N_0 = 6.0\times10^{18}$

$N = 1.0\times10^{18}$, T = 40s, $\lambda = \frac{0.693}{T} = \frac{0.693}{40} = 1.733\times10^{-2}\ s^{-1}$

$$t = \frac{2.303\log_{10}\left(\frac{6.0\times10^{18}}{1.0\times10^{18}}\right)}{1.733\times10^{-2}} = \frac{2.303\times0.7782}{1.733\times10^{-2}} = 103.4\,s$$

10. **Two radioactive substances X and Y initially contain an equal number of atoms. Their half-lives are 1 hour and 2hr respectively. Calculate the ratio of their rates of disintegration after four hours.**

Sol: Rate of disintegration $(R) = \frac{dN}{dt} = \lambda N = N\times\frac{0.693}{T}$

$\therefore$ The ratio of rates of disintegration $\frac{R_1}{R_2} = \left(\frac{N_1}{N_2}\right)\frac{T_2}{T_1}$

$T_1 = 1\text{hour}$, $T_2 = 2\text{hour}$

N_0 is same for both the radio active substance. In four hours, X completes 4 half-lives and Y completes 2 half-lives in four hours. $N_1 = \frac{N_0}{2^n} = \frac{N_0}{2^4} = \frac{N_0}{16}$ *and* $N_2 = \frac{N_0}{2^2} = \frac{N_0}{4}$

$$\therefore \frac{R_1}{R_2} = \frac{N_0}{16}\times\frac{4}{N_0}\left(\frac{2}{1}\right) = \frac{1}{2}$$

11. **A radioactive sample can decay by two different processes. The half-life for the first process is T_1 and that for the second process is T_2. Find the effective half-life T of the radioactive sample?**

Sol: Let N be the total number of atoms of the radioactive sample initially. Let $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ be the initial rates of disintegrations of the radioactive sample by the two processes respectively. Then $\frac{dN_1}{dt}=\lambda_1 N$ and $\frac{dN_2}{dt}=\lambda_2 N$ Where λ_1 and λ_2 are the decay constants for the first and second processes respectively. The initial rate of disintegrations of the radioactive sample by both the process $=\frac{dN_1}{dt}+\frac{dN_2}{dt}=\lambda_1 N+\lambda_2 N=(\lambda_1+\lambda_2)N$

If λ is the effective decay constant of the radioactive sample, its initial rate of disintegration,

$$\frac{dN}{dt}=\lambda N$$

But , $\frac{dN}{dt}=\frac{dN_1}{dt}+\frac{dN_2}{dt}$

$$\lambda N=(\lambda_1+\lambda_2)N \qquad \Rightarrow \quad \lambda=\lambda_1+\lambda_2$$

$$\frac{0.693}{T_1}+\frac{0.693}{T_2}=\frac{0.693}{T}$$

$$\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2};T=\frac{T_1T_2}{T_1+T_2}$$

12. How many α and β-particles are emitted when uranium nucleus $\left({}^{238}_{92}U\right)$ decay to ${}^{214}_{82}Pb$?

Sol: Let n be the number of α -particles and m be the number of β-particles emitted

$${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb+n\,{}^{4}_{2}He+m\,{}^{0}_{-1}\beta$$

As mass is conserved, 238 = 214 + 4n + m (0) = 214 + 4n $\Rightarrow$ 4n = 24 $\Rightarrow$ n = 6

As charge is conserved, 92 = 82 + 2n + m(-1) = 90 + 2(6) –m $\Rightarrow$ m = 2

$\therefore Six\,\alpha$ Particles and two β-particles are emitted.

UNSOLVED PROBLEMS

1. Plutonium decays with a half – life of 24,000 years. If plutonium is stored for 72,000 years, what fraction of it remains?

Sol: Half life of plutonium = T = 24,000 yrs ; Number of years stored = t = 72,000 yrs

Number of half lives $n=\frac{t}{T}=\frac{72000}{24000}=3$

Fraction that remains = $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^8=\frac{1}{8}$

2. **A certain substance decays to $\frac{1}{32}$ of its initial activity in 25days. Calculate its half – life.**

Sol: Fraction of substance un-decayed = $\frac{N}{N_0}=\frac{1}{32}$

Number of days taken = t = 25 days

Half life = T

$$\left(\frac{N}{N_0}\right)=\left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{32}=\left(\frac{1}{2}\right)^n \quad \Rightarrow \frac{1}{2^n}=\frac{1}{2^5}\Rightarrow n=5$$

But, $t=nT \Rightarrow T \Rightarrow \frac{t}{n}=\frac{25}{5}=5\,days$

3. **The half-life period of a radioactive substance is 20 days. What is the time taken for $\frac{7}{8}th$ of its original mass to disintegrate?(June2010)**

Sol: Half life period T = 20 days

Fraction of original mass disintegrated $=\frac{7}{8}$

Fraction of original mass left $\left(\frac{N}{N_0}\right)=1-\frac{7}{8}=\frac{1}{8}$

Now $\left(\frac{N}{N_0}\right)=\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^n=\frac{1}{8} \;\Rightarrow 2^n=2^3 \Rightarrow n=3$

Time taken t = nT = 3 x 20 = 60 days

4. **A radioactive nucleus undergoes a series of decays according to the sequence $A\xrightarrow{\beta}A_1\xrightarrow{\alpha}A_2\xrightarrow{\alpha}A_3$. If the mass number and atomic number of A_3 are 172 and 69 respectively, what is the mass number and atomic number of A?**

Sol: $A\xrightarrow{\beta}A_1\xrightarrow{\alpha}A_2\xrightarrow{\alpha}A_3$

Since $A_3=172$

$A_2=A_3+{}_2He^4=172+4=176$, $Z_2=69+2=71$

$A_1 = A_2 + 2^{He^4} = 176 + 4 = 180, Z_1 = 71 + 2 = 73$

$A = A_1 + 0 = 180$; Z = 73-1 = 72

5. The isotope $^{238}_{92}U$ decays successively to form $^{234}_{90}Th$, $^{234}_{91}Pa$, $^{234}_{92}U$, $^{230}_{90}Th$ and $^{226}_{88}Ra$. What are the radiations emitted in these five steps?

Sol: ${}_{92}U^{238} \xrightarrow{\alpha} 2^{He^4} + {}_{90}Th^{234}$ α-particle is emitted

${}_{90}Th^{234} \xrightarrow{\beta} {}_{-1}e^0 + {}_{91}Pa^{234}$ β-particle is emitted

${}_{91}Pa^{234} \xrightarrow{\beta} {}_{-1}e^0 + {}_{92}U^{234}$ β-particle is emitted

${}_{92}U^{234} \xrightarrow{\alpha} {}_2He^4 + {}^{230}_{90}Th$ α-particle is emitted

${}_{90}Th^{230} \xrightarrow{\alpha} {}_{88}Ra^{226} + 2He^4$ α-particle is emitted

${}_{92}U^{238} \xrightarrow{\alpha} {}_{90}Th^{234} \xrightarrow{\beta} {}_{90}Pa^{234} \xrightarrow{\beta} {}_{92}U^{234} \xrightarrow{\alpha} {}_{90}Th^{230} \xrightarrow{\alpha} {}_{88}Ra_{226}$

The radiations emitted are $\alpha, \beta, \beta, \alpha$ *and* α

6. How many disintegrations per second will occur in one gram of $^{238}_{92}U$, If its half-life against α-decay is $1.42 \times 10^{17} s$?

Sol: half life of substance T $= 1.42 \times 10^{17}$sec

$$\lambda = \frac{0.693}{T} = \frac{0.693}{1.42 \times 10^{17}} = 4.882 \times 10^{-19}\ \text{sec}^{-1}$$

Number of U^{238} atoms is 1gm $= n = \frac{N}{A}$ $\Rightarrow n = \frac{6.023 \times 10^{23}}{238} = 2.530 \times 10^{21}$

Number of disintegrations per second $= \frac{dN}{dt} = \Delta n = 4.8802 \times 10^{-18} \times 2.530 \times 10^{21}$

$$= 1.235 \times 10^4\ \text{sec}^{-1}$$

7. The half-life of a radioactive substance is 100 years. Calculate in how many years the activity will decay to $\frac{1}{10}th$ of its initial value.

Sol: Half life = T = 100 years

Substance remained $= \frac{N}{N_0} = \frac{1}{10}$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{10} = \left(\frac{1}{2}\right)^n \quad \Rightarrow 2^n = 10 \Rightarrow n\log 2 = \log(10)$$

$$\Rightarrow n = \frac{1}{\log 2} = \frac{1}{0.3010} = 3.323$$

Time taken = t = nT= 3.323 x 100 = 332.3 yrs.

8. **One gram of radium is reduced by 2milligram in 5 years by** α-decay**. Calculate the half-life of radium.**

Sol:

Amount left = 1- 0.002 = 0.998 gm

Time taken t = 5 years

Fraction remained $= \frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{0.998}{1} = \frac{1}{2^n}$

$$\Rightarrow 2^n = \frac{1000}{998} \Rightarrow n\log 2 = \log 1000 - \log 998 \quad \Rightarrow n = \frac{3 - 2.9991}{0.3010} = \frac{0.0009}{0.3010} = 0.00299$$

Half life of radius = $T = \frac{t}{n} = \frac{5}{0.00299} = 1672.2\,yrs$

9. **The half-life of a radioactive substance is 500years. In how many years, its activity will decay to 0.2 times of its initial value.** $\left(\textbf{Given}\log_{10} 5 = 0.6990\right)$

Sol: Half- life of substance T = 5000 years

Fraction of substance left = $\frac{N}{N_0} = 0.2 = \frac{1}{5}$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{5} = \frac{1}{2^n} \Rightarrow 2^n = 5 \Rightarrow n\log_{10}^2 = \log 5$$

$$n = \frac{0.6990}{\log 2} = \frac{0.690}{0.3010} = 2.3222$$

Time taken = t = nT= 2.3222 x 5000 $= 1.16\times 10^4\ yrs$

<u>ASSESS YOUR SELF</u>

1. **Why is a nucleus compared with a liquid drop?**

A. Because nucleus has properties like short range and saturation nature of nuclear force which are similar to that of the forces in a liquid drop.

2. Even if some of the γ - rays and X-rays have same wavelength, how can they be distinguished?

A. X-ray emission is an atomic phenomenon γ-ray emission is a nuclear phenomenon

3. When a nucleus undergoes β-decay an electron is emitted. Does this mean that a nucleus consists of electron also?

A. No. When a neutron inside a nucleus converts into a proton an electron is emitted.

4. Which types of decay does not change the parent element and why?

A. γ-decay, because γ-rays have no charge and no mass.

5. The rate of radioactive decay is independent of the external physical conditions like temperature, pressure. Why?

A. Because the radioactive decay is a nuclear phenomenon.