ELECTROMAGNETICS

5. D.C CIRCUITS

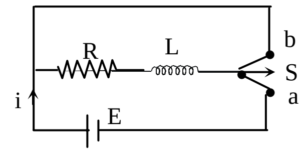
POINTS TO REMEMBER

1. Electric currents whose magnitudes vary for a small time while growing to maximum or decaying to zero are called transient currents.

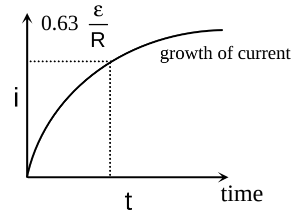
GROWTH OF CURRENT IN LR CIRCUIT

2. When switch "S" is closed at t=0; $\varepsilon - L\dfrac{di}{dt} = Ri$

3. At time t, current $i = \dfrac{\varepsilon}{R}\left(1 - e^{-t\frac{R}{L}}\right)$



1. The constant L/R has dimensions of time and is called the **inductive time constant** (τ) of the LR circuit.
2. $t = \tau;\ i = 0.63 i_0$, in one time constant, the current reaches 63% of the maximum value. The time constant tells us how fast will the current grow.



3. $i = i_0$, when $t = \infty$, where $i = \dfrac{\varepsilon}{R}$.

   Theoretically current grows to maximum value after infinite time. But practically it grows to maximum after 5τ.

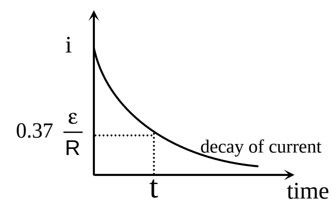
Decay of current:

4. When switch "S" is open at t=0; $-L\dfrac{di}{dt} = Ri$

   at t=0, $i = i_0$

   at time t, $i = i_0 e^{-\frac{t}{\tau}}$

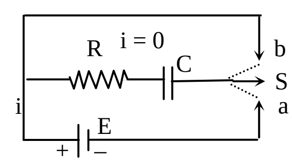
   The current reduces to 37% of the initial value in one time constant i.e., 63% of the decay is complete.

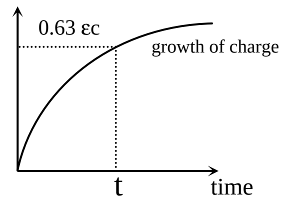


5. Energy stored in inductor $E = \dfrac{1}{2}Li^2$.

Charging of a capacitor :

When a capacitor is connected to a battery, positive charge appears on one plate and negative charge on the other. The potential difference between the plates ultimately becomes equal to e.m.f of the battery. The whole process takes some time and during this time there is an electric current





6. through connecting wires and the battery.

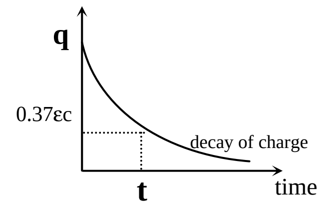
7. Using Kirchoff's loop law $\dfrac{q}{C} + Ri - \varepsilon = 0$.

8. At any time t, $q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{CR}}\right)$

   $V = E\left(1 - e^{-\frac{t}{CR}}\right)$; $i = i_0\, e^{-\frac{t}{CR}}$.

9. The constant RC has dimensions of time and is called **capacitive time constant** (τ).

10. In one time constant (τ=RC), the charge accumulated on the capacitor is q=0.63εC.



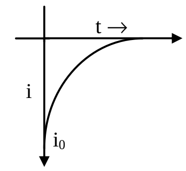
Discharging of a capacitor :

11. When the plates of a charged capacitor are connected through a conducting wire, the capacitor gets discharged, again there is a flow of charge through the wires and hence there is a current

12. $\frac{q}{C} - Ri = 0$

13. $q = Qe^{-\frac{t}{RC}}$, where $Q = \varepsilon C$

$V = E\,e^{-\frac{t}{CR}}$; ; $i = -\, i_0\, e^{-\frac{t}{CR}}$.



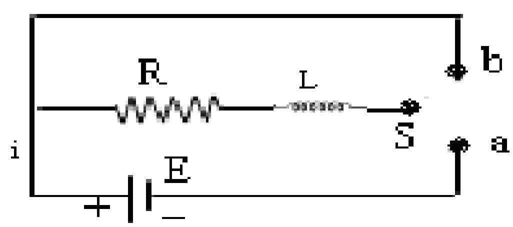
14. At t=RC, q=0.37Q, i.e., 63% of the discharging is complete in one time constant.

LONG ANSWER QUESTIONS

1. Describe the growth and decay of current in an inductance, resistance circuit. How the growth and decay are affected with difference values of inductance discuss?

Ans. **Growth of Current :** L.R. circuit is as shown in the figure. When the key 'S' is thrown over to 'a', the current begins to flow and a magnetic flux is linked with the coil.The value of current increases from zero to maximum but not suddenly because the induced emf across the inductor opposes the growth of current.

Let i be the instantaneous current and $\frac{di}{dt}$ be the rate of rise of current.



$$E - L.\frac{di}{dt} = iR$$

But $E = i_0 R$

$$i_0 R - L.\frac{di}{dt} = iR \qquad \Rightarrow i_0 R - iR = L.\frac{di}{dt}$$

$$\Rightarrow (i_0 - i)R = L.\frac{di}{dt} \qquad \Rightarrow \frac{di}{(i_0 - i)} = \frac{R}{L}dt$$

On integrating, $\int \frac{di}{(i_0 - i)} = \frac{R}{L}\int dt$

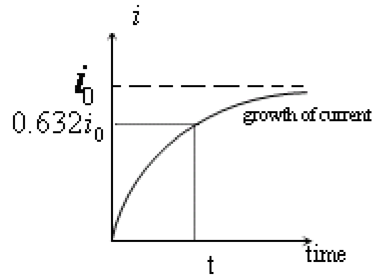
$$\Rightarrow \log_e (i_0 - i) = -\frac{R}{L}t + A$$

Where A is the integration constant.

If t = 0, i = 0 , then $A = \log_e i_0$

$$\therefore \log_e (i_0 - i) = -\frac{R}{L}t + \log_e i_0$$

$$\Rightarrow \log_e (i_0 - i) - \log_e i_0 = -\frac{R}{L}t$$



$$\Rightarrow \log_e \left(\frac{i_0 - i}{i_0} \right) = -\frac{R}{L}t$$

$$\Rightarrow \frac{i_0 - i}{i_0} = e^{-\frac{R}{L}t} \qquad \Rightarrow 1 - \frac{i}{i_0} = e^{-\frac{R}{L}t}$$

$$\Rightarrow \frac{i}{i_0} = \left(1 - e^{-\frac{R}{L}t} \right) \qquad \Rightarrow i = i_0 \left(1 - e^{-\frac{R}{L}t} \right)$$

The factor $\frac{L}{R}$ has the dimensions of time and it is called inductive time constant.

If $t = \frac{L}{R} \Rightarrow i = i_0 \left(1 - e^{-1} \right) \Rightarrow i = 0.632\, i_0$

The time taken for the L– R circuit to grow its current value from zero to 63% of its maximum value is called capacitive time constant.The instantaneous value of current rises exponentially.

Also, $i = i_0 \left(1 - e^{-\frac{R}{L}t} \right) \Rightarrow i = i_0 - e^{-\frac{R}{L}t} \Rightarrow i_0 e^{-\frac{R}{L}t} = i_0 - i$

The rate of growth of current is given by,

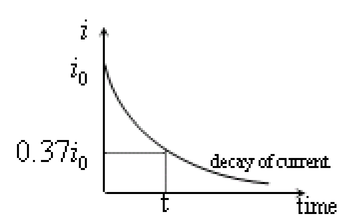
$$\frac{di}{dt} = \frac{d}{dt}\left[i_0 \left(1 - e^{-\frac{R}{L}t} \right) \right] \qquad \Rightarrow \frac{di}{dt} = i_0 \left[-\left(-\frac{R}{L} \right) e^{-\frac{R}{L}t} \right]$$

$$\Rightarrow \frac{di}{dt} = \frac{R}{L}.i_0 e^{-\frac{R}{L}t} \qquad \Rightarrow \frac{di}{dt} = \frac{R}{L}(i_0 - i)$$

As $\frac{R}{L}$ is greater or $\frac{L}{R}$ is smaller, rate of growth of current is greater. Hence for a smaller inductive time constant, the current attains its maximum steady value more rapidly.

<u>Decay of current</u>: When the key ‘S’ is thrown over to ‘a’, the current in the circuit suddenly does not become zero, but it gradually decreases.

Let i be the instantaneous current and $\frac{di}{dt}$ be the rate of decay of current.



$$\Rightarrow -L.\frac{di}{dt} = iR \qquad \Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

On integrating, $\int \frac{di}{L} = -\frac{R}{L}\int dt$

$$\Rightarrow \log_e i = -\frac{R}{L}t + B$$

Where B is the integration constant

If t = 0, i = i_0 ,then $\log_e i_0 = B$

$$\therefore \log_e i = -\frac{R}{L}.t + \log_e i_0 \qquad \Rightarrow \log_e i - \log_e i_0 = -\frac{R}{L}t$$

$$\Rightarrow \log_e\left(\frac{i}{i_0}\right) = -\frac{R}{L}.t \qquad \Rightarrow \frac{i}{i_0} = e^{-\frac{R}{L}t} \qquad \Rightarrow i = i_0 e^{-\frac{R}{L}t}$$

The factor $\frac{L}{R}$ has the dimensions of time and it is called inductive time constant.

If $t = \frac{L}{R}$, Then $i = i_0 e^{-1} \Rightarrow i = 0.37 i_0$

The time taken for the current value to decay from maximum value to 37% of its maximum value is called inductive time constant.

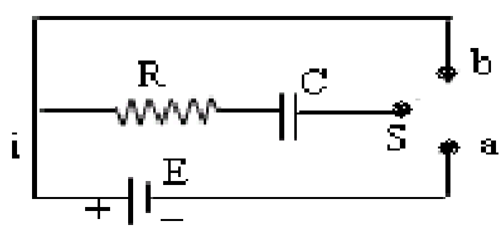
The rate of decay of current is, $\frac{di}{dt} = \left(-\frac{R}{L}\right) i_0 e^{-\frac{R}{L}t} \qquad \Rightarrow \frac{di}{dt} = -\frac{R}{L}i$

Hence for a smaller inductive time constant, the current decays more rapidly.

2. Describe the growth and decay of current in resistance, capacitance circuit? Define time constant.

Ans. **<u>C – R circuit :</u>**Consider a circuit of a capacitance ‘C’ and a resistance ‘R’ are connected in series to a cell of emf ‘E’ and a two way plug key ‘S’ .

<u>Growth of charge :</u> When the key ‘S’ is thrown over to ‘a’ , the capacitor begins to collects the charge. After some instant of time, the charge accumulated on the capacitor opposes the further growth of charge i.e. an emf is induced in the capacitor called induced emf (or) back emf. Then the net emf of the circuit is equivalent to p.d. across ‘R’.



Let q be the charge on the capacitor and i be the current flowing in the circuit after a time t. If q_0 is the maximum charge of the capacitor, then

q_0 = CE

$$E - \frac{q}{C} = iR \quad \text{and} \qquad i = \frac{dq}{dt}$$

$$\Rightarrow \frac{q_0}{C} - \frac{q}{C} = iR$$

$$\Rightarrow \frac{1}{C}[q_0 - q] = \frac{dq}{dt}.R.$$

$$\Rightarrow \frac{dt}{CR} = \frac{dq}{q_0 - q}$$

$$\Rightarrow \int \frac{dt}{CR} = \int \frac{dq}{q_0 - q}$$

$$\Rightarrow -\frac{t}{CR} = \log_e (q_0 - q)$$

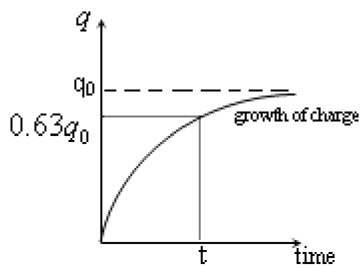
$$\Rightarrow -\frac{t}{CR} + A = \log_e (q_0 - q)$$

Where A is the integration constant. When t = 0, q = 0 , then $A = \log_e q_0$

$$\Rightarrow -\frac{t}{CR} + \log_e q_0 = \log_e (q_0 - q)$$

$$\Rightarrow -\frac{t}{CR} = \log_e \left(\frac{q_0 - q}{q_0} \right)$$

$$e^{-\frac{t}{CR}} = \left(\frac{q_0 - q}{q_0} \right) \Rightarrow e^{-\frac{t}{CR}} = 1 - \frac{q}{q_0}$$



$$\Rightarrow \frac{q}{q_0} = 1 - e^{-\frac{t}{CR}} \qquad \Rightarrow q = q_0 \left(1 - e^{-\frac{t}{CR}} \right)$$

The factor RC has the dimensions of time and it is called capacitive time constant.

If $t = CR$, then $q = q_0 \left(1 - e^{-1} \right)$ or $q = 0.63 q_0$

The growth of charge across the capacitance is also exponential and is as shown in the figure.

The time taken for the C – R circuit to grow its charge value from zero to 63% of its maximum value is called **capacitive time constant**.

Rate of growth of charge across the capacitor.

$$q = q_0 \left(1 - e^{-\frac{t}{CR}} \right) \qquad \Rightarrow q = q_0 - q_0^{-\frac{t}{CR}}$$

$$q_0 e^{-\frac{t}{CR}} = q_0 - q$$

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[q_0 \left(1 - e^{-\frac{t}{CR}} \right) \right]$$

$$\frac{dq}{dt} = \frac{1}{CR} q_0 e^{-\frac{t}{CR}} = \frac{CE}{CR} e^{-\frac{t}{CR}}$$

Or i = i_0 $e^{-\frac{t}{CR}}$

And $\quad \frac{dq}{dt} = \frac{1}{CR}(q_0 - q)$

Hence the charge on the capacitor increases and current in the circuit decreases with time exponentially.

Decay of Charge :

Let the capacitor is charged to a value 'q_0' and the key 'S' is thrown over to 'b', to allow the capacitor to discharge through the resistance R.

Let q be the charge on the capacitor and i be the current flowing in the circuit after a time t during the discharge.

$$E - \frac{q}{C} = iR \qquad \Rightarrow -\frac{q}{C} = \frac{dq}{dt}.R$$

$$\frac{dq}{q} = -\frac{dt}{RC} \qquad \Rightarrow \int \frac{dq}{q} = -\frac{1}{RC}\int dt$$

$$\log_e q = -\frac{t}{RC} + A$$

Where A is the integration constant. When

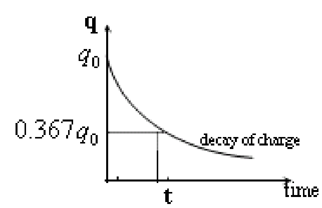
$$t = 0, q = q_0$$

$$A = \log_e q_0$$

$$\log_e q = -\frac{t}{RC} + \log_e q_0 \qquad \Rightarrow \log_e\left(-\frac{q}{q_0}\right) = -\frac{t}{RC}$$

$$\text{Or } \frac{q}{q_0} = e^{-\frac{t}{RC}} \qquad \Rightarrow q = q_0\left(e^{-\frac{t}{RC}}\right)$$

Hence the charge on the capacitor decreases with time exponentially.



t = RC

$$q = q_0\left(e^{-1}\right) \qquad \Rightarrow q = 0.367q_0$$

The time taken for the C – R circuit to decay its charge value from zero to 37% of its maximum value is called capacitive time constant

$$\text{Also,} \quad q = q_0\left(e^{-\frac{t}{RC}}\right)$$

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[q_0\left(e^{-\frac{t}{RC}}\right)\right] \qquad = -\frac{1}{RC}e^{-\frac{t}{RC}}q_0$$

$$i = \frac{dq}{dt} = -\frac{1}{RC}q$$

$$\text{Also,} \quad i = -\frac{q_0}{RC}e^{-\frac{t}{RC}} = -\frac{CE}{RC}e^{-\frac{t}{RC}} \quad \Rightarrow i = -i_0\, e^{-\frac{t}{RC}}$$

SOLVED PROBLEMS

1. A coil has a time constant of 1s and an inductance of 8 H. If the coil is connected to a 100 V d.c. source, determine (i) the resistance of the inductor, (ii) steady value of the current and (iii) the time taken by the current to reach 60% of the steady value of the current.

Sol: i) $\tau = \frac{L}{R}; R = \frac{L}{\tau} = \frac{8}{1} = 8\Omega$

ii) $I_0 = e/R = 100/8 = 12.5A$

iii) $i = \frac{60}{100} I_0 = 0.6 I_0, \tau = 1s$.

$i = I_0\left(1 - e^{-t/\lambda}\right) \Rightarrow \; 0.6 I_0 = I_0\left(1 - e^{-t/1}\right) \quad \Rightarrow \; e^t = \frac{1}{0.4} = 2.5$

$t = \log_e 2.5 = 2.303 \log_{10} 2.5 = 2.303 \times .3979 = 0.9164 s$.

2. A constant voltage is applied to a series L-R circuit at t = 0 by closing a switch. The voltage across L is 25 V at t = 0 and drops to 5 V at t = 0.025 s. If L=2 H, what must be the value of R?

Sol: At t = 0, I = 0 Hence the voltage across L at t = 0 is $e = 25V$.

At t = 0.25 s, voltage across L is 5 V.

Hence voltage across R = applied emf –voltage across L = 25 – 5 = 20 V

$I_0 = \frac{E}{R} = \frac{25}{R}; i = \frac{\text{Voltage across R}}{R} = \frac{20}{R}$.

But, $i = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \quad \Rightarrow \frac{20}{R} = \frac{25}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \quad \Rightarrow \frac{t}{\tau} = \log_e 5 = 2.303 \times \log_{10} 5$

Or $\frac{tR}{L} = 2.303 \times 0.6990$

$R = \frac{2.303 \times 0.6990}{0.025} = 128.6\Omega$

3. A capacitor of $100\mu F$ capacitance is connected in series with a resistance of $8000\,\Omega$. (i) Estimate the time constant of the circuit (ii) If the combination is suddenly connected to a 100 V direct current supply, find the initial rate of rise of potential across the capacitor.

Sol: (i) $\tau = CR = 100 \times 10^{-6} \times 8000 = 0.8\text{s}$

(ii) At t = 0, potential difference across capacitor is zero i.e., putting V = 0 in $e = V = iR$

$e = iR = R\frac{dq}{dt} = RC\frac{dV}{dt}$.

$\left(\frac{dV}{dt}\right)_{t=0} = \frac{e}{RC} = \frac{100}{0.8} = 125\ V/S$.

4. Capacitor of 0.1 μF is charged from a 100 V battery through a series esistance of $1000\,\Omega$. Find i) the time for the capacitor to receive 63.2% of its final charge, (ii) the charge received in this time, (iii) the initial rate of charging and (iv) the rate of charging when the charge is 63.2% of the final charge.

Sol: i) 63.2% of charge is received in a time equal to the time constant of the circuit.

$\tau = CR = 0.1 \times 10^{-6} \times 1000 = 1 \times 10^{-4} s$

ii) $q_0 = CV = 0.1 \times 1000 = 10\mu C$

Charge received during one time constant = $0.632 \times q_0 = 0.632 \times 10 = 6.32\mu C$

iii) The rate of charging at any time t is $\frac{dV}{dt} = \frac{e - V}{CR}$

$e - V = iR\left(\frac{dq}{dt}\right)R = R\frac{d}{dt}(CV) = RC\frac{dV}{dt}$.

Initially, V = 0

Hence, $\frac{dV}{dt} = \frac{e}{CR} = \frac{100}{0.1 \times 10^{-6} \times 10^3} = 10^6 V/s$

iv) $V = q/c = 0.632q_0$; $V = 0.632\times100 = 63.2V$

$$\therefore \frac{dV}{dt} = \frac{e-V}{CR} = \frac{100-63.2}{10^{-4}} = 368KV/S.$$

UNSOLVED PROBLEMS

1. A coil with self inductance of 2.4H and resistance 12Ω is suddenly switched across a 120V direct current supply of negligible internal resistance. Determine (i) the time constant of the coil (ii) the instantaneous value of the current after 0.1s (iii) the final steady current and (iv) the time taken for the current to reach 5A.

Sol: L = 2.4H ; $R = 12\Omega$

emf of D.C. supply = 120V

i) Time constant $= \tau = \frac{L}{R} = \frac{2.4}{12} = 0.2\sec$

ii) $I = I_0\left[1-e^{-t\left(\frac{R}{L}\right)}\right] = \frac{E}{R}\left[1-e^{-(0.1)/(0.2)}\right] = 3.94A$

iii) Final steady current = $I_0 = \frac{E}{R} = \frac{120}{12} = 10A$

iv) If t_1 is the time taken for current to reach 5A, then I = 5A

$$I_0 = 10A \Rightarrow I = \frac{I_0}{2} \Rightarrow I = I_0\left[1-e^{-t\left(\frac{R}{L}\right)}\right]$$

$$\frac{I_0}{2} = I_0\left[1-e^{-t_{1l}/0.2}\right] \quad \Rightarrow \frac{1}{2} = \left[1-e^{-t_1/0.2}\right]$$

$$\Rightarrow e^{-5t_1} = 1-\frac{1}{2} = \frac{1}{2} \Rightarrow e^{5t_1} = 2$$

$$\Rightarrow \log_e^2 = 5t_1 \Rightarrow t_1 = \frac{\log_e^2}{5} = \frac{0.693}{5} = 0.139 \text{ sec (nearly)}$$

2. A 200v direct current supply is suddenly switched to a coil which has a time constant of 1 milli second. If the current in the coil reaches 0.2A after 3 millisecond, determine the final steady value of the current, the resistance and the inductance of the coil.

Sol: $E_0 = 200V$; $\tau = 1 milli\sec = 3\times10^{-3}\sec$

1) $i_0 = ?$

But $i = i_0\left[1-e^{\frac{-t}{\lambda}}\right] \Rightarrow 0.2 = i_0\left[1-e^{\frac{-3\times10^{-3}}{1\times10^{-3}}}\right] \Rightarrow i_0 = \frac{0.2}{1-e^{-3}}$

$$i_0 = \frac{0.2}{1-0.05}$$

$$i_0 = 0.21 amp$$

But $R = \frac{E}{i_0} = \frac{200}{0.21} = 952.38\Omega$

$$L = R\tau = 952.38\times1\times10^{-3} = 0.952H$$

2. An $8\mu F$ Capacitor is being charged by a 400V supply through 0.1 mega ohm resistor. How long will it take the capacitor to develop a potential difference of 300V?

Sol: $C = 8\mu F = 8\times10^{-6}F$; R = $0.1M\Omega = 0.1\times10^6\,\Omega = 10^5\,\Omega$

$\tau = RC = 10^5 \times 8 \times 10^{-6} = 0.8 \sec$

$q = q_0\left[1-e^{-t/RC}\right] \quad \Rightarrow CV = CV_0\left[1-e^{-t/0.8}\right] \qquad \Rightarrow 300 = 400\left[1-e^{-t/0.8}\right]$

$\Rightarrow 0.75 = 1-e^{-t/0.8}$

$t = 1.11 \sec$

3. A resistance R and a $4\mu F$ capacitor are connected in series across a 220v direct current supply. Across the capacitor a neon lamp that glows at 120V is connected. Calculated the value of R to make the lamp glow 5 seconds after the switch has been closed.

Sol: $q = q_0\left(1-e^{-\frac{t}{RC}}\right)$

$q = CV_{applied} = \left(4\times10^{-6}\right)(120)$

$q_{0(\max)} = CV_{\max} = \left(4\times10^{-6}\right)(200)$

t = 5 sec

$\Rightarrow \left(4\times10^{-6}\right)(120) = \left(4\times10^{-6}\right)(200)\left[1-e^{\frac{5}{RC}}\right]$

$\Rightarrow \frac{120}{200} = 1-e^{\frac{5}{RC}} \Rightarrow e^{\frac{5}{RC}} = 1-\frac{3}{5} = 0.4$

$\Rightarrow R = \frac{5}{C\times0.915} = \frac{5}{4\times10^{-6}\times0.915} = 1.366\times10^6\,\Omega = 1.366\, M\Omega$

<u>ASSESS YOURSELF</u>

1. What are the dimensions of R/L?

Ans. $[T^{-1}]$

2. What are the dimensions of RC?

Ans. [T]

3. The current which varies for a small finite time, while growing to maximum or decaying to zero value is called transient current. Where do you find such currents?

Ans. In the charging and discharging of a capacitor through a resistor and in the growth and decay of current through an inductor and resistor in series.

4. A constant voltage of 25V is applied to a series L-R circuit at t = 0 by closing a switch. What is the potential difference across (i) the resistor and (ii) the inductor at t = 0?

Ans. 0; 25V

5. A constant voltage of 50V is applied to a series L-R circuit at t = 0 by closing a switch. What is the potential difference across (i) the resistor and ii) the inductor at $t = \infty$?

Ans. 50V, 0

6. When a capacitor is charged through a resistor, the charge on the capacitor at any time t is $q = q_0(1-e^{-t/\lambda})$ where $\lambda = CR$. What is the expression for the potential difference across the capacitor at that instant of tiem?

Sol: $q = q_0(1-e^{-t/\lambda})$

$\frac{q}{c} = \frac{q_0}{c}(1-e^{-t/\lambda})$

$V = V_0(1-e^{-t/\lambda})$

7. **A constant voltage of 100V is applied a series C-R circuit at $t = 0$ by closing a key. What is the potential difference across (i) the resistor and (ii) the capacitor at $t = 0$, ∞.**

Ans. At $t = 0$; 100V; 0 ; at $t = \infty$; 9, 100V