

# CURRENT ELECTRICITY

## 1. OHM'S LAW AND CIRCUITS

### Points to remember

#### 1. Electric Current:

- The charge flowing any cross-section per unit time in a conductor is called electric current.
- If  $q$  is the charge flowing through any cross-section in a time  $t$  time, electric current 'I' is given by  $I = \frac{q}{t}$ .
- If the rate of flow of charge is not constant then  $I = \frac{dq}{dt}$
- Unit of current : Col/sec (or) amp DF : I (or) A
- Definition of I:** The current through a conductor is said to be 1 amp, if a charge of 1 col. flows through any cross section in one second. If  $q = 1 \text{ col.}$  and  $t = 1 \text{ sec.}$  Then  $I = 1 \text{ amp}$
- Conventional direction of current is along the direction of motion of positive charge. (ie) opposite to the flow of electron.

#### 2. Current density :

- The current flowing normally through unit area of cross-section is called current density.
- Current density is a vector and is along the motion of positive charge at that point.
- The free electrons may collide each other and these collisions are inelastic.
- The average distance traveled by an electron between two successive collisions is called mean free path.
- If  $\ell$  is the length,  $A$  is the cross sectional area of the conductor and  $v_d$  is the drift velocity, then Current density (J) =  $\frac{i}{A} = e n v_d$  amp/  $\text{m}^2$
- The mobility of free electron in a conductor is defined as the drift velocity acquired per unit strength of the electric field applied across the conductor.
- Electron Mobility =  $\mu = \frac{v_d}{E}$   $\text{m}^2 / \text{volt} - \text{sec}$

#### 3. Ohm's Law:

- At constant temperature, the potential difference across the conductor is proportional to the current passing through the conductor.
- If  $V$  is the potential difference the conductor, carrying a current  $I$ , then,
- $V \propto I$  ( $T = \text{constant}$ ) (or)  $V = IR$  (or)  $R = \frac{V}{I}$  Where  $R$  is called the electrical resistance.
- Unit of  $R$  : Ohm ( $\Omega$ ) (or) volt/amp. DF :  $[ML^2T^{-3}I^{-2}]$
- Resistance:** The ratio of potential difference across the ends of a conductor and the current passing through the conductor is said to be the resistance of the conductor.
- Ohm :** The resistance of a conductor is said to be one ohm of one ampere if current passes through a conductor of potential difference 1v across its ends. If  $v = 1 \text{ volt}$  and  $i = 1 \text{ amp}$ , then  $R = 1 \Omega$

#### 4. Resistance in series : $R = R_1 + R_2 + R_3$

The effective resistance is equal to the sum of individual resistances.

#### 5. Resistance in parallel : $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

The reciprocal of effective resistance is equal to the sum of reciprocals of individual resistances.

#### 6. Specific resistance :

- The resistance of a conductor depends on the length and area of section of the conductor.
- The resistance  $R$  of the conductor is directly proportional to the length ( $\ell$ ) of the conductor and inversely proportional to the area of cross section ( $A$ ) of the conductor.

$$R \propto \ell \quad \left| \quad R \propto \frac{\ell}{A} \right| \quad R = \frac{S\ell}{A} \quad (\text{or}) \quad s = \frac{RA}{\ell}$$

- Where the proportionality constant  $s$  or  $\rho$  is called the specific resistance (or) **Resistivity**.
- Unit of 's' : ohm - m DF of 's' :  $[ML^3T^{-3}I^{-2}]$
- Definition:** If  $\ell = 1m$  and  $A = 1m^2$  then  $s = R$  specific resistance of a conductor is numerically equal to the resistance of the conductor of unit length and unit cross-section at that temperature.

#### 7. Temperature coefficient of resistance ( $\alpha$ )

- If  $R_o$  and  $R_t$  are the resistances of the conductor at  $0^\circ C$  and  $t^\circ C$  respectively, then the temperature coefficient of resistance is given by

$$R_t = R_o (1 + \alpha t) \quad (\text{or}) \quad \alpha = \frac{R_t - R_o}{R_o t} / ^\circ C$$

- If  $R_1$  and  $R_2$  are the resistances of the conductor at  $t_1^\circ c$  and  $t_2^\circ c$  then

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} / ^\circ C$$

**8. Thermistor :** A thermistor is a heat resistive device made of a semi conductor material whose resistance always vary rapidly with temperature.

### LONG ANSWER QUESTIONS

#### 1. Define drift velocity and mobility of charge carriers in a conductor.

**Derive the relation between electric current and drift velocity of charge carriers in a conductor.**

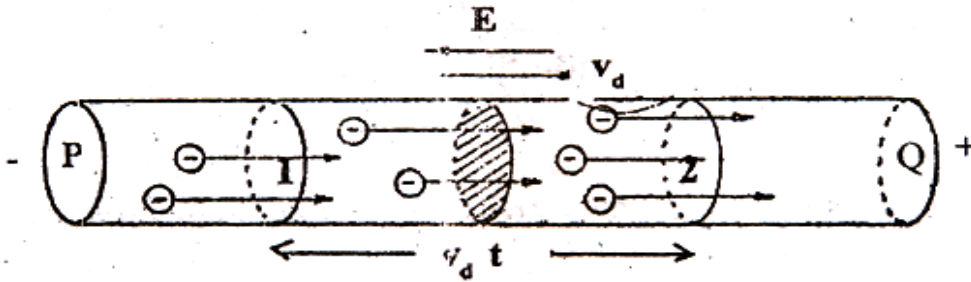
- A. **Current :** Current is defined as the rate of flow of charge through any cross-section of a conductor.

**Drift Velocity :** The speed with which an electron gets drifted in a metallic conductor under the application of an external electric field is called the drift velocity ( $v_d$ ).

**Average drift velocity** ( $\mu$ ) : The mobility of a charge carrier is the average drift velocity due to the application of unit electric field strength.

$$\text{Mobility } (\mu) = \frac{V_d}{E} \text{ m}^2 \text{ S}^{-1} \text{ v o l t}^{-1}$$

**Relation between electric current and Drift velocity:-**



Consider a metallic conductor PQ to which an electric field E is applied from right to left as shown. Let A be the area of cross-section of the conductor and  $v_d$  be the drift velocity of the electrons. Let the free electrons move from cross-section 1 to cross-section 2 in a time interval 't'. The distance traveled by them  $l = v_d t$

The volume of region between PQ is  $V = (V_d t) A$

If the number of free electrons per unit volume in the conductor is n then the number of electrons crossing the cross section 2 in a time t will be  $n(V_d t A)$ . The total charge 'q' crossing the area of cross section 2 in a time t is given by,

$q = n(V_d t A)e$ . where e is charge of an electron

But  $i = \frac{q}{t}$  and hence  $i = (nV_d A e)$

$$\therefore i = (neA)V_d \quad \text{Or} \quad V_d = \left( \frac{i}{neA} \right)$$

### SHORT ANSWER QUESTIONS

1. **Define electric current, drift velocity and mobility of charge carriers in conductor.**

A. **i. Electric current:** Current is defined as the rate of flow of charge through any cross-section of a conductor.

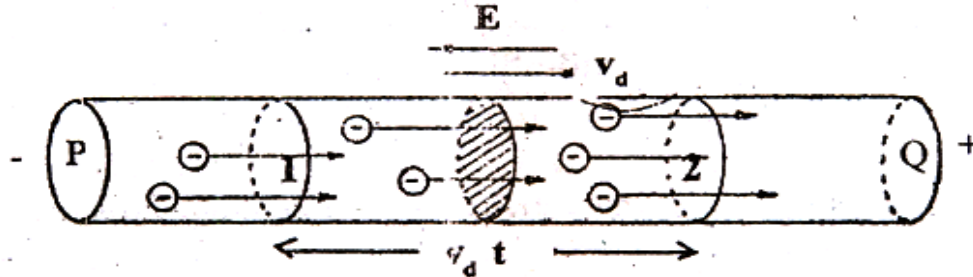
$$i = \frac{dq}{dt} \text{ (or) } i = \frac{q}{t} \text{ Its SI unit is ampere}$$

**ii. Drift velocity:** The speed with which an electron gets drifted in a metallic conductor under the application of an external electric field is called drift velocity ( $V_d$ )

**iii. Mobility of charge carriers:** The mobility of a charge carrier is the average drift velocity resulting from the application of unit electric field strength.

**2. Derive the relation between drift velocity of charge carriers and the current through a conductor.**

- A. Consider a metallic conductor PQ to which an electric field  $E$  is applied from right to left as shown. Let  $A$  be the area of cross-section of the conductor and  $v_d$  be the drift velocity of the electrons. Let the free electrons move from cross-section 1 to cross-section 2 in a time interval ' $t$ '. The distance traveled by them  $l = v_d t$



The volume of region between PQ is  $V = (v_d t) A$

If the number of free electrons per unit volume in the conductor is  $n$  then the number of electrons crossing the cross section 2 in a time  $t$  will be  $n(v_d t A)$ . The total charge ' $q$ ' crossing the area of cross section 2 in a time  $t$  is given by,

$$q = n(v_d t A)e. \text{ Where } e \text{ is charge of an electron}$$

$$\text{But } i = \frac{q}{t} \text{ and hence } i = (n v_d A e)$$

$$\therefore i = (n e A) v_d \quad \text{Or} \quad v_d = \left( \frac{i}{n e A} \right)$$

**3. State and explain ohm's law. Define ohm.**

- A. At constant temperature, the current passing through a conductor is directly proportional to the potential difference between its ends.

Let ' $V$ ' be the P.D. between the ends of a conductor and ' $I$ ' be the current through it. Then  $i \propto V \Rightarrow V = iR$  where  $R$  is electrical resistance of conductor.

Ohm's law is not a fundamental law. It is not applicable for all substances.

**Ohm:** The resistance of a conductor is said to be one ohm if one ampere of current passes through a conductor of potential difference 1v across its ends. If  $V = 1 \text{ volt}$  and  $i = 1 \text{ amp}$ , then  $R = 1\Omega$

**4. On what factors does the resistance of a conductor depend? Explain.**

- A. the resistance of a conductor depends on the following factors:

1. The resistance of a wire is proportional to its length  $R \propto l$

The resistance of a wire is inversely proportional to the area of cross section of the conductor

$$R \propto \frac{l}{A}$$

Or Resistance of a wire  $R = \rho \frac{l}{A}$  Where  $\rho$  is the specific resistance of the conductor.

2. The resistance of a wire depends on the temperature of the wire. As temperature increases, the resistance of a conductor increases.

$R_t = R_0 [1 + \alpha(\Delta T)]$  Where  $\alpha$  is called the temperature coefficient of resistance.  $\alpha = \frac{R_t - R_0}{R_0 t}$  It is positive for metals and negative for semiconductors.

**5. What is resistivity of material? Define it. What are its units? How does it change with temperature?**

- A. The resistance of a conductor depends upon its dimensions as well as the materials with which it is made. The resistance 'R' of conductor is directly proportional to its length 'l' and inversely proportional to its area of cross section 'A'

$$\text{Resistance } R \propto \frac{l}{A} \text{ or } R = \rho \frac{l}{A}$$

Where  $\rho$  is called specific resistance or resistivity of the material.

**Resistivity** : ( $\rho$ ) It is defined as the resistance of a conductor of unit length with unit area of cross section Or

The resistance between two parallel faces of a unit cube is equal to the resistivity of that substance. Resistivity is a physical constant of a substance. It depends on the nature of the material.

**Unit** : ohm – metre

**Variation of resistivity with temperature :**

The resistivity of all metallic conductors is found to increase with the increase of temperature.

If  $\rho_1$  and  $\rho_2$  be the resistivities of the same material at temperature  $t_1$  and  $t_2$ , then

$$\rho_2 = \rho_1 [1 + \alpha(t_2 - t_1)]$$

Where  $\alpha$  is the temperature coefficient of resistivity of the material.

- i. For all metallic conductors over a small range of temperature is found to increase linearly with temperature.
- ii. For certain metals at very low temperature, the temperature dependence is non-linear.
- iii. Resistivity of manganin and constantan are nearly independent of temperature
- iv. The resistivity of carbon and semiconductors decrease with increase of temperature and hence they have a negative temperature coefficient of resistivity.

**6. Define conductance. What are its units? How does it change with temperature?**

- A. The reciprocal of resistance (R) of a conductor is known as conductance of a conductor.

Conductance of a conductor is also defined as the ratio of current ( $i$ ) flowing through a conductor to the potential drop ( $V$ ) across the terminals of that conductor.

$$\text{Conductance } C = \frac{i}{V} = \frac{1}{V/i} = \frac{1}{R}$$

The unit of resistance is ohm. The unit of conductance  $G = \frac{1}{\text{Ohm}}$  called mho. The

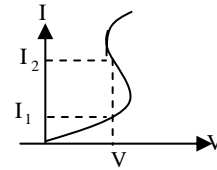
**Unit:** Siemens or mho.

For metals and alloys conductance is inversely proportional to temperature. For semi conductors it is directly proportional to temperature.

## 7. Write short notes on thermistor.?

A. A thermistor is a heat resistive device made of a semi conductor material whose resistance always varies rapidly with temperature. The characters of a thermistor are

1. The resistance changes very rapidly with the change of temperature.
2. The temperature coefficient of resistance is very high.
3. The temperature coefficient of resistance may be positive or negative.



1. Thermistors are made from semi conductor oxides of iron cobalt and nickel. These are in the form of beads (or) discs enclosed in a small glass bulb. A pair of platinum leads is attached at its ends for electric connection.
2. There are two types of thermistors one with positive and other with negative temperature co-efficient of resistance. In general a thermistor has high negative temperature. Coefficient of resistance. Such thermistors are used as resistance thermo metres to measure a temperature of order of 10k. These have high resistivity at low temperature and can be used to measure low temperature accurately.

## 8. Explain the temperature dependence of resistivity.?

A. Resistivity of a material varies with temperature. For pure metals, within limited range of temperature, resistivity increases linearly with the increase of temperature let  $\rho_0$  and  $\rho$  be the resistivities of a material at  $0^\circ C$  and  $t^\circ C$  respectively. Then  $\rho = \rho_0(1 + \alpha t)$  Here,  $\alpha$  is a constant called the temperature coefficient of resistivity of the material.

$$\alpha = \frac{\rho - \rho_0}{\rho_0 t}$$

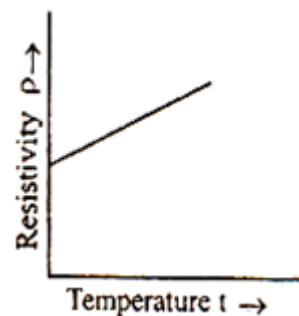
$$\text{Unit : } ^\circ C^{-1} \text{ or } K^{-1}$$

**Definition of  $\alpha$**  : It is the increase in the resistivity of a material for  $1^\circ C$  rise of temperature per unit original resistivity at  $0^\circ C$

Let  $\rho_1$  and  $\rho_2$  be the resistivities of a material at  $t_1^\circ C$  and  $t_2^\circ C$  respectively. Then

$$\rho_1 = \rho_0(1 + \alpha t_1) \text{ and } \rho_2 = \rho_0(1 + \alpha t_2)$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2} \Rightarrow \alpha = \frac{\rho_2 - \rho_1}{\rho_1 t_2 - \rho_2 t_1}$$



- a) for metallic conductors,  $\alpha$  is positive.
- b) For electrolytes, semiconductors and insulators,  $\alpha$  is negative.
- c) For thermistor,  $\alpha$  is very high and negative. For pure metals,  $\rho-t$  graph is a straight line with positive slope  
For semiconductors,  $\rho-t$  graph is a curve with a negative slope

### **VERY SHORT ANSWER QUESTIONS**

**1. Define electric current is it a scalar quantity or a vector quantity?**

- A. It is defined as the rate of flow of electric charge through any cross-section of a conductor. It is a scalar quantity because it does not obey the laws of vector addition.

$$\text{Average current } (i) = \frac{\text{Charge}}{\text{Time}} = \frac{q}{t}$$

**2. Define drift velocity of charge carriers in a conductor. Mention the units of drift velocity.**

- A. When an external electric field  $E$  is applied on a conductor, the random motion of electrons get modified and they drift slowly in a direction opposite to the direction of electric field  $E$ . The velocity with which the charge carriers like electrons are drifted is called drift velocity ( $V_d$ ). Its SI units are  $ms^{-1}$ .

**3. Define mobility of charge carriers in a conductor. Mention the units of mobility.**

- A. Mobility of a charge carrier like electron is defined as the average drift velocity resulting from the application of unit electric field strength  $\mu = \frac{V_d}{E}$ . Its SI units are  $m^2s^{-1}volt^{-1}$

**4. A conductor has got different areas of cross section. Are the currents same at different cross sections? What conservation law helps you to decide the answer?**

- A. Even though a conductor has different areas of cross-section at different points along its length, the current will be same for all cross sections of the conductor. This is a consequence of the law of conservation of charge.

**5. State the units and dimensions of resistance.**

- A. SI units : ohm ( $\Omega$ ) (or)  $VA^{-1}$

Dimensional formula :  $ML^2T^{-3}A^{-2}$

**6. Explain Ohm's law**

- A. At constant temperature, the current passing through a conductor is directly proportional to the potential difference between its ends.

Let 'V' be the potential difference, between the ends of a conductor and 'I' be the current through it. Then  $i \propto V; V = iR$

'R' is electrical resistance of the conductor

**7. What are ohmic and non ohmic devices?**

- A. **Ohmic devices:** The devices which obey ohm's law are known as ohmic devices  
e.g. Metals

**Non-ohmic devices :** The devices which do not obey ohm's law are known as non-ohmic devices

e.g. Diodes, Valves, Thermistor, etc.

**8. Two copper wires of radii in the ratio 1 : 2 carry the same current. What is the ratio of the drift velocities in them?**

- A. Drift velocity  $(V_d) = \frac{i}{neA} = \frac{i}{ne(\pi r^2)} \Rightarrow \frac{V_{d_1}}{V_{d_2}} = \frac{r_2^2}{r_1^2} = \frac{4}{1}$

**9. What is conductance? Give its unit,**

- A. Reciprocal of resistance is called conductance. Its unit is siemens

**10. On what factors, does the resistance of a conductor depend?(March2011)**

- A. The resistance of a conductor  $R = \rho \frac{l}{A}$

The resistance depends on

- 1) resistivity of material  $\rho$
- 2) On length of the conductor and
- 3) On the area of cross section of the conductor
- 4) Resistance of a conductor depends on the temperature. It follows the equation  $R_t = R_0 (1 + \alpha t)$

**11. The manganin wire is used in the preparation of standard resistances. Why?**

- A. Manganin is an alloy of Cu, Ni and Manganese. Since it is an alloy, it has high resistivity and has negligible temperature co-efficient of resistance. Its resistance does not vary with temperature and hence it is used as standard resistance.

**12. What is temperature coefficient of resistivity? What is its unit? (June2010)**

- A. Temperature coefficient of resistance is defined as the ratio of change in resistance of a conductor to its original resistance for  $1^\circ C$  rise in its temperature.



Temperature coefficient of resistance  $\alpha = \frac{R_1 - R_0}{R_0 \cdot t} / ^\circ C$

Unit of  $\alpha$  :  $^\circ C^{-1}$  or  $K^{-1}$

**13. How many electrons flow through a wire when 1A current passes for 1 millisecond? (June 2010)**

A.  $i = 1A$  ;  $t = 1 \text{ millisecond} = 10^{-3} \text{ second}$  ;  $q = ?$

$$i = \frac{q}{t} = \frac{ne}{t} \Rightarrow q = it = 1 \times 10^{-3} C$$

Charge  $q = ne$

$$\therefore n = \frac{q}{e}$$

$$n = \frac{1 \times 10^{-3}}{1.60 \times 10^{-19}} ; n = 6.25 \times 10^{15} \text{ electrons}$$

**14. What is a thermistor? What is its use?**

A. A thermistor is a heat resistive device made of a semi conductor material whose resistance always varies rapidly with temperature.

Uses :

1. These are used in measuring the rate of energy flow in microwave beams
2. These are also used as thermostats and to prevent a sudden surge in electric circuits

**15. "Electrons flows along are the current carriers in conductors". Explain whether this statement is correct or not?**

A. Yes, this statement is correct.

The atoms which loose electrons are positively charged and the substances which gains electrons are negatively charged. In an electric field, negative charges will move towards positive terminal and positive charges will move towards negative terminal. Hence current flows.

## **SOLVED PROBLEMS**

**1. An aluminium conductor is carrying a current of 1A. How many electrons per second are passing across any point in the conductor?**

Sol.  $e = 1.6 \times 10^{-19} C$  ;  $i = 1A$

$$\text{Current, } i = \frac{q}{t} = \frac{ne}{t}$$

$$1 = n \times 1.6 \times 10^{-19}$$

$$\therefore n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

**2. A wire carries a current of 1A. (a) How much charge flows through the wire in 5.0 minutes? (b) How many electrons will cross over a particular point in the conductor during this period ?**

Sol.  $i = 1A$  ;  $t = 5 \text{ minutes} = 5 \times 60s$

a) Charge  $q = it = 1 \times (5 \times 60) = 300\text{C}$

b) Let the number of electrons crossing over a particular point in 5 min be 'n'.

$q = ne$  where  $e$  is the charge of the electron

$$\therefore 300 = n \times 1.6 \times 10^{-19} \text{C}$$

$$n = \frac{300}{1.6 \times 10^{-19}} = 1.875 \times 10^{21}$$

3. **Current of 5A flows through a copper wire. The area of cross section of the wire is  $3 \text{ mm}^2$ . Assuming that there are  $8.5 \times 10^{28}$  atoms/ $\text{m}^3$  in copper and there is one free electron per atom in copper, find the drift velocity of electrons in copper.**

Sol. Drift velocity  $v_d$ , 
$$v_d = \frac{1}{neA}$$

Number of conduction electrons per unit volume  $n = 8.5 \times 10^{28}/\text{m}^3$  as each atom gives one free electron.

$$e = 1.6 \times 10^{-19} \text{C} \quad ; \quad i = 5\text{A} \quad ; \quad A = 3 \text{ mm}^2 = 3 \times 10^{-6} \text{ m}^2$$

$$\therefore \text{Drift velocity } v_d = \frac{5}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(3 \times 10^{-6})} = 1.2 \times 10^{-4} \text{ ms}^{-1}$$

4. **The number of conduction electrons per unit volume of copper is  $8.5 \times 10^{28}/\text{m}^3$ . The length of a copper wire is 3.0 m and area of cross section is  $3 \text{ mm}^2$ . A copper wire is carrying a current of 3A. Find the time taken by an electron to drift from one end of the wire to the other end.**

Sol. The drift velocity  $v_d$  
$$v_d = \frac{i}{neA} \quad ; \quad i = 3\text{A}$$

Number of conduction electron per unit volume  $n = 8.5 \times 10^{28}/\text{m}^3$

$$e = 1.6 \times 10^{-19} \text{C}$$

$$\text{Area of cross section } A = 3 \text{ mm}^2 = 3 \times 10^{-6} \text{ m}^2$$

$$\therefore v_d = \frac{3}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(3 \times 10^{-6})} = 1.10 \times 10^{-4} \text{ ms}^{-1}$$

The time taken by an electron to travel a distance  $l$  is  $t = \frac{l}{v_d}$

Here,  $l = 3\text{m}$

$$\therefore t = \frac{3}{1.10 \times 10^{-4}} = 27.2 \times 10^3 \text{ s}$$

5. **Drift velocity of electrons ( $v_d$ ) in metals is very small and is of the order of  $10^{-4} \text{ ms}^{-1}$ . Explain how the electric bulb glows almost immediately after the switch is on.**

Sol. The drift velocity  $v_d$  is  $10^{-4} \text{ ms}^{-1}$  and from the previous example we find that to drift through a distance of 3 m, the electron takes  $27.2 \times 10^3 \text{ s}$  or about 7 hours 30 minutes.

But, changes in electric field configuration (electrical energy) travel at nearly the speed of light ( $3 \times 10^8 \text{ m/s}$ ) along the wire and reach the filament immediately on switching on.. Thus the electrons in the filament get immediately accelerated and start colliding with atoms and ions in the filament. These collisions result in heat and light generated by the filament.

6. **A copper wire of cross sectional area  $0.01 \text{ mm}^2$  is used to prepare a resistance of  $1\text{k}\Omega$ . The resistivity of copper is  $1.7 \times 10^{-18} \Omega\text{m}$ . Calculate the length of the wire.**

**Sol.** Let the length of the wire be  $l$  m.

$$A = 0.01 \text{ mm}^2 = 0.01 \times 10^{-6} \text{ m}^2$$

$$R = 1 \text{ k}\Omega = 10^3 \Omega \quad ; \quad \rho = 1.7 \times 10^{-8} \Omega \text{m}$$

$$\text{But, } R = \rho \frac{l}{A}$$

$$\therefore \text{ The length of the wire required, } l = \frac{RA}{\rho} = \frac{10^3 \times 0.01 \times 10^{-6}}{1.7 \times 10^{-8}} = 588.2 \text{ m}$$

**7. A wire of length 1m and diameter 0.2 mm has a resistance of 100Ω. Calculate the specific resistance of the material of the wire.**

**Sol.**  $l = 1 \text{ m}$  ;  $d = 0.2 \text{ mm}$  ;  $r = \frac{d}{2} = \frac{0.2}{2} = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$ .

$$A = \pi r^2 = 3.14 \times (0.1 \times 10^{-3})^2 \text{ m}^2 \quad ; \quad R = 100 \Omega .$$

$$\text{Specific Resistance} = \frac{100 \times 3.14 \times (0.1 \times 10^{-3})^2}{1} = 3.14 \times 10^{-6} \Omega \text{m}$$

**8. Two wires of the same material and having lengths in the ratio of 2 : 3 are connected in series. The p.d.s across the wires are 4.2V and 3.6V respectively. Compare their radii.**

**Sol.**  $\frac{l_1}{l_2} = \frac{2}{3}$  and  $\frac{V_1}{V_2} = \frac{4.2}{3.6} = \frac{7}{6}$

But  $V_1 = i_1 R_1$  and  $V_2 = i_2 R_2$ . As the two wires are in series, current will be the same i.e.  $i_1 = i_2$

$$\therefore \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$\text{But } R_1 = \frac{\rho_1 l_1}{A_1} = \frac{\rho_1 l_1}{\pi r_1^2} \quad \text{and} \quad R_2 = \frac{\rho_2 l_2}{A_2} = \frac{\rho_2 l_2}{\pi r_2^2}$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_1 l_1}{\rho_2 l_2} \times \frac{r_2^2}{r_1^2} = \frac{V_1}{V_2}$$

As the two wires are made of the same material,

$$\therefore \frac{l_1 r_2^2}{l_2 r_1^2} = \frac{V_1}{V_2} = \frac{7}{6}$$

$$\therefore \frac{r_1^2}{r_2^2} = \frac{6}{7} \times \frac{l_1}{l_2} = \frac{6}{7} \times \frac{2}{3} = \frac{4}{7}$$

$$r_1 : r_2 = 2 : \sqrt{7}$$

**9. A copper conductor has a resistance of 1.72Ω at a temperature of 20°C and 2.26Ω at 100°C. Calculate the temperature coefficient of resistivity of copper and also the resistance of the conductor at 0°C.**

**Sol.**  $R_1 = 1.72 \Omega$  at a temperature  $t_1 = 20^\circ \text{C}$

$$R_2 = 2.26 \Omega \text{ at a temperature } t_2 = 100^\circ \text{C}$$

A) If  $\alpha$  is the temperature coefficient of resistivity of copper,

$$R_2 = R_1 [1 + \alpha(t_2 - t_1)]$$

$$\text{Or } 2.26 = 1.72 [1 + \alpha(100 - 20)] = 1.72 [1 + 80\alpha]$$

$$\therefore \alpha = 3.93 \times 10^{-3} \text{ K}^{-1} \text{ or } (^\circ \text{C})^{-1}$$

B) Let the resistance of the conductor be  $R_2$  at temperature  $t_2 = 0^\circ\text{C}$

$$R_2 = R_1 [1 + \alpha(t_2 - t_1)]$$

$$R_2 = 1.72 [1 + 3.93 \times 10^{-3} (0 - 20)] = 1.72 [1 - 3.93 \times 20 \times 10^{-3}] = 1.58 \Omega$$

- 10. If a copper wire is stretched to make it 0.1% longer, find the percentage of increase in its resistance.**

**Sol.**  $R = \frac{s\ell}{A} \times \frac{\ell}{\ell} = \frac{s\ell^2}{A\ell} = \frac{s\ell^2}{V} \Rightarrow R\alpha\ell^2$  (volume = constant)

$$\frac{\Delta R}{R} = 2 \frac{\Delta \ell}{\ell} = 2 \times 0.1 = 0.2 \%$$

$\therefore$  Percentage increase in resistance is 0.2%

- 11. The resistance of a platinum wire of a platinum resistance thermometer at the ice point is  $5\Omega$  and at steam point is  $5.4\Omega$ . When the thermometer is inserted in a hot bath, the resistance of the platinum wire is  $6.2\Omega$ . Find the temperature of the hot bath.**

**Sol.**  $t_1 = 0^\circ\text{C}$  ;  $R_1 = 5\Omega$

$$t_2 = 100^\circ\text{C} ; R_2 = 5.4\Omega$$

Temperature coefficient of resistance  $\alpha$  of platinum is

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{5.4 - 5}{5 \times 100} = \frac{0.4}{500} = 8 \times 10^{-4} \text{ K}^{-1} \text{ or } (^\circ\text{C})^{-1}.$$

Now,  $t_1 = 0^\circ\text{C}$ ,  $R_1 = 5\Omega$ ,  $R_2 = 6.2\Omega$ ,  $t_2 = ?$

$$R_2 = R_1 [1 + \alpha(t_2 - t_1)]$$

$$6.2 = 5 [1 + 8 \times 10^{-4} t_2] = 5 + 40 \times 10^{-4} \times t_2$$

$$t_2 = \frac{6.2 - 5}{40 \times 10^{-4}} = 300^\circ\text{C}$$

## UNSOLVED PROBLEMS

- 1. The number of electrons striking the screen of CRT is  $7.5 \times 10^{15}$  in 10. Calculate the electric current.?**

A.  $n = 7.5 \times 10^{15}$  ;  $e = 1.6 \times 10^{-19} \text{ C}$  ;  $t = 10\text{s}$

$$i = \frac{Q}{t} = \frac{ne}{t} = \frac{7.5 \times 10^{15} \times 1.6 \times 10^{-19}}{10}$$

$$\therefore i = 120 \mu\text{A}$$

- 2. A wire carries 0.16 A steady current. Calculate the time required to pass  $36 \times 10^{19}$  electrons through it.?**

A.  $I = 0.16 \text{ A}$

$$n = 36 \times 10^{19}; t = ?$$

$$i = \frac{ne}{t} \Rightarrow t = \frac{ne}{i}$$

$$t = \frac{30 \times 10^{19} \times 1.6 \times 10^{-19}}{0.16} = 360s = 6\text{min.}$$

3. **A rectangular block has dimensions  $5\text{cm} \times 5\text{cm} \times 10\text{cm}$ . Calculate the resistance measured between a) two square ends and b) the opposite rectangular ends? Specific resistance of the material is  $3.5 \times 10^{-5} \Omega\text{m}$ .**

A. a) Two square ends:

$$l = 10\text{cm} = 10^{-1}\text{m}; A = 5 \times 5 = 25\text{cm}^2 = 25 \times 10^{-4}\text{m}^2$$

$$S = 3.5 \times 10^{-5} \Omega\text{m}; R = ?$$

$$R = \frac{Sl}{A} \Rightarrow R = \frac{3.5 \times 10^{-5} \times 10^{-1}}{25 \times 10^{-4}} \Rightarrow R = 1.4 \times 10^{-3} \Omega$$

b) Opposite rectangular ends:

$$l = 5\text{cm} = 5 \times 10^{-2}\text{m}; A = 10 \times 5 = 50\text{cm}^2 = 50 \times 10^{-4}\text{m}^2$$

$$R = \frac{Sl}{A} = \frac{3.5 \times 10^{-5} \times 10^{-2} \times 5}{50 \times 10^{-4}} \Rightarrow R = 3.5 \times 10^{-4} \Omega$$

4. **Two wires of same material have their lengths in the ratio of 2:3 and radii 8 : 9 and equal value of p.d is applied between their ends (separately). Calculate the ratio of currents through them.?**

A.  $S = \text{same}$

$$l_1 : l_2 = 2 : 3; r_1 : r_2 = 8 : 9; i_1 : i_2 = ?; V = \text{same}$$

$$i = \frac{V}{R} \text{ and } R = \frac{Sl}{A} \Rightarrow i = \frac{V \times A}{Sl} \Rightarrow i = \frac{V(\pi r^2)}{Sl}$$

$$i \propto \frac{r^2}{l} \Rightarrow \frac{i_1}{i_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{l_2}{l_1}$$

$$\frac{i_1}{i_2} = \left(\frac{8}{9}\right)^2 \left(\frac{3}{2}\right) = \frac{32}{27} \Rightarrow i_1 : i_2 = 32 : 27$$

5. **The temperature coefficient of resistance of platinum  $\alpha = 3.92 \times 10^{-3} K^{-1}$  at  $0^\circ C$ . Find the temperature at which the increase in the resistance of platinum wire is 10% of its value of  $0^\circ C$ .?**

A.  $\alpha = 3.92 \times 10^{-3} K^{-1}, t_1 = 20^\circ, t_2 = ?$

$$R_2 = \frac{110}{100} R_1 = \frac{11}{10} R_1$$

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \Rightarrow t_2 - t_1 = \frac{R_2 - R_1}{R_1 \alpha}$$

$$t_2 - t_1 = \frac{\frac{11}{10} R_1 - R_1}{R_1 \times 3.92 \times 10^{-3}} = \frac{1}{10} \times \frac{10^3}{3.92} \Rightarrow t_2 = \frac{100 \times 100}{392}$$

$$\Rightarrow t_2 - 20 = \frac{100 \times 100}{392} = 25.50^\circ C \Rightarrow t_2 = 25.5^\circ C$$

6. A current of 5A is passing through a metallic wire of cross sectional area  $4 \times 10^{-6} m^2$ . If the density of the charge carries in the wire  $5 \times 10^{23} / m^3$ , find the drift speed of the electrons (charge carriers).?

A.  $I = 5A$  ;  $A = 4 \times 10^{-6} m^2$  ;  $n = 5 \times 10^{26} m^3$  ; Drift speed =  $v_d$

$$I = nAev_d \Rightarrow v_d = \frac{I}{nAe} = \frac{5}{5 \times 10^{26} \times 4 \times 10^{-6} \times 1.6 \times 10^{-19}} = \frac{1}{64} = 1.562 \times 10^{-2} ms^{-1}$$

7. The temperature coefficient of resistivity of a material is  $0.0004 K^{-1}$ . When the temperature is increased by  $50^\circ C$ , the resistivity increases by  $2 \times 10^{-8} ohm$  metre. Find the initial resistivity of the material. ?

A. Initial resistivity =  $S_1$

Final resistivity =  $S_2$

Increase in resistivity =  $S_2 - S_1 = 2 \times 10^{-8} \Omega m$

Temperature coefficient of resistivity =  $\alpha = 4 \times 10^{-4} K^{-1}$

Rise in temperature =  $t_2 - t_1 = 50^\circ C$

$$\alpha = \frac{S_2 - S_1}{S_1(t_2 - t_1)} \Rightarrow S_1 = \frac{S_2 - S_1}{\alpha(t_2 - t_1)} = \frac{2 \times 10^{-8}}{4 \times 10^{-4} \times 50} = 100 \times 10^{-8} \Omega - m$$

8. Two wires of equal diameters of resistivities  $\rho_1$  and  $\rho_2$  and lengths  $X_1$  and  $X_2$  respectively are joined in series. Find the equivalent resistivity of the combination.?

A. Total resistance =  $R = R_1 + R_2$

$$\frac{\rho X}{A} = \frac{\rho_1 X_1}{A_1} + \frac{\rho_2 X_2}{A_2}$$

$$\rho(X_1 + X_2) = \rho_1 X_1 + \rho_2 X_2$$

[Since  $A_1 = A_2 = A$ ]

$$\rho = \frac{\rho_1 X_1 + \rho_2 X_2}{X_1 + X_2}$$

9. The temperature coefficient of resistance of a wire is  $0.00125^{\circ}C^{-1}$ . At 300 K the resistance of the wire is one ohm. Find the temperature at which the resistance of the wire will be 2 ohm.?

A.  $\alpha = 0.00125^{\circ}C = 125 \times 10^{-5}^{\circ}C^{-1}$

$$t_1 = 300K \quad ; \quad R_1 = 1\Omega \quad ; \quad R_2 = 2\Omega$$

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \Rightarrow 125 \times 10^{-5} = \frac{2 - 1}{1(t_2 - 300)}$$

$$\Rightarrow t_2 - 300 = \frac{1}{125 \times 10^{-5}} = \frac{100000}{125} = 800 \Rightarrow t_2 = 800 + 300 = 1100K$$

10. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. The lengths of the wires are in the ratio of 4:3 and radii of the wires are in the ratio of 2 : 3. Find the ratio of the currents passing through the wire.?

A. When wires are in parallel,  $I \propto \frac{1}{R}$

$$\frac{l_1}{l_2} = \frac{4}{3} \quad ; \quad \frac{r_1}{r_2} = \frac{2}{3} \Rightarrow \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{2^2}{3^2} = \frac{4}{9} \quad ; \quad \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\text{Since } R = \frac{Sl}{A} \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{3}{4} \times \frac{4}{9} = \frac{1}{3} \Rightarrow I_1 : I_2 = 1 : 3$$

11. A wire of resistance 10 ohm is elongated by 10%. Find the resistance of the elongated wire.?

(Hint : when the same wire is elongated, the volume of the wire remains constant)

A. From  $R = \frac{Sl}{A} \Rightarrow R = \frac{Sl \times l}{A \times l} = \frac{Sl^2}{V}$  (S,V are constants.)

$$\Rightarrow R \propto l^2 \Rightarrow \frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \left( \frac{110}{100} \frac{l_1}{l_1} \right)^2$$

$$\Rightarrow \frac{R_2}{10} = (1.1)^2 = 1.21 \Rightarrow R_2 = 1.21 \times 10 = 12.1\Omega$$

12. A copper wire is elongated by 0.1%. i) Find the percentage increase in the resistance. ii) If the same wire is now elongated such that the radius becomes 0.1% shorter, find the percentage increase in resistance.?

A. i) From  $R = \frac{Sl}{A} \Rightarrow R = \frac{Sl \times l}{A \times l} = \frac{Sl^2}{V}$  (S, V are constant)

$$\Rightarrow R \propto l^2 \Rightarrow \frac{\Delta R}{R} \times 100 = 2 \left( \frac{\Delta l}{l} \times 100 \right) = 2(0.1) = 0.2\%$$

$$\text{ii) From } R = \frac{Sl}{A} = \frac{Sl \times A}{A \times A} = \frac{SV}{A^2} = \frac{SV}{(\pi r^2)^2} = \frac{SV}{\pi^2 r^4}$$

$$\Rightarrow R \propto \frac{1}{r^4} \text{ (or) } R \propto r^{-4}$$

$$\frac{\Delta R}{R} \times 100 = -4 \left( \frac{\Delta r}{r} \times 100 \right) = -(4)(-0.1) = 0.4\%$$

**13. A wire has a resistance of  $2.5\Omega$  at  $100^\circ\text{C}$ . Temperature coefficient of resistance of the material of the wire is  $\alpha = 3.6 \times 10^{-3} \text{K}^{-1}$ . Find its resistance at  $25^\circ\text{C}$  . ?**

A. Resistance at  $100^\circ\text{C} = R_2 = 2.5\Omega$  ; Resistance at  $25^\circ\text{C} = R_1$

$$\text{Temperature coefficient of resistance} = \alpha = 36 \times 10^{-4} (^\circ\text{C})^{-1}$$

$$\text{From, } \alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \Rightarrow 36 \times 10^{-4} = \frac{2.5 - R_1}{R_1(100 - 25)}$$

$$\Rightarrow 2.5 - R_1 = 75R_1(36 \times 10^{-4}) = 0.27R_1$$

$$\Rightarrow 2.5 - R_1 + 0.27R_1 = 1.27R_1 \Rightarrow R_1 = \frac{2.5}{1.27} = 2\Omega \text{ (nearly)}$$

**14. The resistance of a wire of length 2m and area of cross section  $0.5\text{mm}^2$  is  $2.2\Omega$ . Find the length of the wire of the same material that will have a resistance of  $22\Omega$  .?**

A. From  $R = \frac{Sl}{A}$

$$R \propto l \Rightarrow \frac{l_2}{l_1} = \frac{R_2}{R_1}$$

$$\frac{l_2}{l_1} = \frac{22}{2.2} \Rightarrow l_2 = 2 \times 10 = 20\text{m}$$

**15. A wire of silver has a resistance of 1 ohm. Specific resistance of constantan is 30 times the specific resistance of silver. Find the resistance of a constantan wire whose length is one third the length of the silver is one third the length of the silver wire and radius one half the radius of the silver wire.?**

A.  $R_1 = 1\Omega$ ,  $S_2 = 30S_1$  ;  $l_2 = \frac{1}{3}l_1$  ;  $r_2 = \frac{1}{2}r_1$

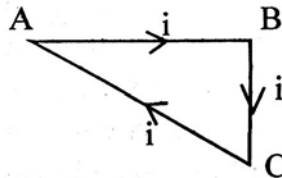
$$\text{But, } R = \frac{Sl}{A} = \frac{Sl}{\pi r^2}$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{S_2}{S_1} \times \frac{l_2}{l_1} \times \frac{r_1^2}{r_2^2} \Rightarrow \frac{R_2}{1} = 30 \times \frac{1}{3} \times 4 \Rightarrow R_2 = 40\Omega$$



## ASSESS YOURSELF

1. **An uncharged conductor is electrically neutral. It is only a charged conductor that produces an electric field outside. Now think of a current carrying conductor (for simplicity let us assume a long straight wire). Will it give rise to an electric field ? (It gives rise to a magnetic field as discussed in the chapter of electromagnetism).**
  - A. No
2. **Does the random motion of free electrons in a conductor contribute to the drift of the electrons?**
  - A. No.
3. **How can you ascertain that the drift is steady and not accelerated?**
  - A. An accelerated charge gives out radiation. Here there is no radiation.
4. **We put an arrow mark ( $\rightarrow$ ) to show the direction of current flow. Why is the current not a vector quantity?**
  - A. The arrow of current only denotes the sense of current in which direction the charge flows. But current does not obey the laws of vector addition and hence current is not a vector.

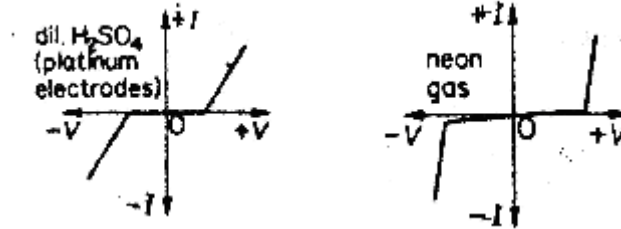


- It is a scalar only. Consider a current flowing along ABCA. If current is a vector, the resultant should be zero. However, the current remains the same  $i$ . Thus, current is **not a vector**.
5. **(a) Is the drift speed a characteristic of the material of the conductor ? (b) A given conductor has got different areas of cross sections. From eqn. (6.10) can we say the current  $i$  will be different at different cross sections?**
    - A. (a) No.  $v_d = \frac{i}{neA}$  For a given material  $n$  and  $e$  are constants. But  $v_d$  depends on the amount of current  $i$  flowing through the conductor and also on the area of cross section  $A$ . That is why we find  $v_d = 10^{-3} \text{ ms}^{-1}$  and  $10^{-4} \text{ ms}^{-1}$  etc. with different orders of magnitude for copper, (b) No. current will be the same at all cross sections.
  6. **The electron gets drift speed under the influence of external force (due to applied electric field). Why the force does not give rise to acceleration of the electron ?**
    - A. Due to collisions with positive ions inside the conductor the electron acquires only a drift speed.
  7. **When no external field is applied (that is when no current is passing through the conductor) the average velocity of a conduction electron over a large time will be zero and at any given time the average velocity of all the free electrons will also be zero.**

**Will the situation be same when current is passing (electric field is applied) ?**

    - A. No
  8. **When a conductor is heated due to the passage of current through it, the resistivity (or resistance) of the conductor increases. What happens to the drift speed  $v_d$  of conduction electrons in this case ?**
    - A. The drift speed decreases. This is due to increase in energy of free electrons, which results in more number of collisions with lattice ions over a given time and consequent loss of more energy.

9. **V- i characteristics of dilute sulphuric acid with tungsten electrodes and neon gas are shown in figure 6.5 (e) and (f) respectively.**



**What is your inference about the nature of these conductors?**

- A. They are non - ohmic.
10. **A non-ohmic resistor does not obey Ohm's law. Can such non - ohmic resistors be of any use?**
- A. Actually many useful components in the electrical industry and in electronics are non - ohmic in nature. Diode, transistor, light emitting diodes (LED), thermistor are all non-ohmic resistors (conductors), without the non - ohmic resistors (conductors) there would have been no progress in electronics
11. **What are good conductors? Give examples.?**
- A. The substances which allow electricity to pass through them are called good conductors.

Ex. silver, gold, aluminium etc. are good conductors of electricity