ELECTROSTATICS
4. CAPACITORS

POINTS TO REMEMBER

1. The ability of a conductor to hold the charge is called the capacity of a conductor.
2. The charge on a conductor is directly proportional to its potential. If Q is the charge present an a conductor and V is its potential then $Q \alpha V$ (or) $Q = CV$ (or) $C = \frac{Q}{V}$

   Where the constant of proportionality C is called the electric capacity (or) capacitance of a conductor.
3. The ratio of charge (q) on a conductor to its potential (V) is called the capacity of the conductor (C) (or) If $V = 1 volt$
4. Unit of capacity is Farad (or) Col /Volt
5. **Farad** : If $q = 1C$ and $V = 1 volt$ then $C = 1F$ and the capacity of a conductor is said to be one Farad if one coulomb of charge is required to raise to potential of a conductor by one volt.
6. 1 Micro Farad $(1\mu F) = 10^{-6} F$
7. 1 Pico Farad $(1PF) = 10^{-12} F$ $(1\mu\mu F)$
8. D.F: $\left[M^{-1}L^{-2}T^{4}I^{2}\right]$
9. The capacity of a conductor does not depend on the nature of the material of the conductor, but depends on the medium in which it is placed.
10. **Principle of a Condenser:** Condenser is a device used to store a large quantity of charge with out changing its potential.

11. **Effect of dielectric in a capacitor:**A dielectric is a solid insulating medium like glass, ebonite, wax paper, mica placed in between the two metal sheets in order to increase its capacity. These do not allow the electric charges to easily pass through them.
12. **Dielectrics :**Dielectric is a material in which all the electrons are tightly bound to the nuclei of the atoms. Thus there are no free electrons to carry the current. The electrical conductivity of a dielectric is zero. The molecules of dielectric may be classified as non polar and polar.
13. **A non-polar** molecule is one in which the centre of gravity of positive charges coinciding with the c.g. of negative charges. A non-polar molecule has zero electric dipole moment in the absence of the external electric field.

14. **A polar** molecule is one in which the c.g. of positive and negative charges do not coincide. They are separated by finite distances. (When such polar molecules are placed in an external electric field, the field tries to orient the positive charge centers in the direction of the field and negative charge centers in the opposite direction) The polar molecule is an electric dipole and has a permanent dipole moment.

15. Parallel plate capacitor : $C = \frac{\epsilon_o A}{d}$

16. If the space between the plates of the condenser is filled with a dielectric of constant K , then $C^1 = \frac{K\epsilon_o A}{d}$

17. **Equivalent Capacity - Series combination:** $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

The reciprocal of equivalent capacity is the sum of the reciprocals of the individual capacities

18. Parallel Combination: $C = C_1 + C_2 + C_3$

Thus the effective capacity is equal to the sum of individual capacities.

19. Let a medium of dielectric constant K and thickness t is introduced between the plates of a condenser plates which are separated by a distance d. $C_{eff} = \frac{\epsilon_o A}{d - t\left(1 - \frac{1}{k}\right)}$

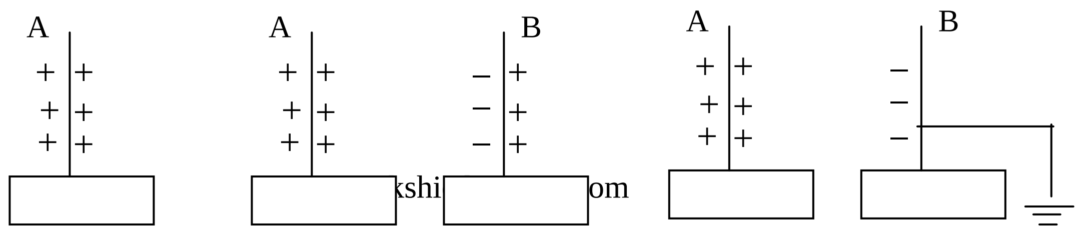
20. **Energy stored in a condenser** $U_o = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

LONG ANSWER QUESTIONS

1. Explain the principal of a capacitor. What are the uses of capacitors? What is the effect of filling the space between the plates of a capacitor with a dielectric?

A. Principle of a capacitor:

Condenser is a device used to store a large quantity of charge without changing its potential.Consider a metal plate A mounted on an insulated base and the plate A is given a positive charge till its potential becomes maximum. If Q is the charge and V is the potential of the plate then its capacity C is given by $C = \frac{Q}{V}$.



Now another metal plate B which is uncharged is brought nearer to the plate A. Then electrostatic induction takes place in B and hence negative charge is induced on the nearer face of the plate B and positive charge is induced on the farther face as shown. Let V_- be the potential at A due to induced negative charges and V_+ is the potential at A due to the induced positive charge on B. The net potential at the plate A is given by

$$V^1 = V - V_- + V_+$$

Though induced +ve and –ve charges at the plate are equal, since the negative charges are nearer to the plate A ,$V_- > V_+$.

$\therefore V^1 < V \Rightarrow C^1 = \frac{Q}{V^1}$ Since $V^1 < V$ $C^1 > C$.

To restore the initial potential of the plate A, some more charge can be given to it. There by more charge can be stored on A with out changing its potential.

Further, if the metal plate B is earthed as shown, the induced the charges flow to the earth. Hence the net potential at the plate A is given by

$V^{11} = V - V_-$ And hence $V^{11} << V$

$\therefore C^{11} = \frac{Q}{V^{11}}$ Where $C^{11} >> C$

Again to raise the potential of A to the initial value a large amount of charge can be given to it.

Thus the capacity of a conductor can be increased without changing its potential.

<u>Uses of capacitors</u>:

1. Capacitors are used to store electric charge.
2. They can establish uniform electric field within a small region.
3. A capacitor will block D.C. and allows A.C through it. So capacitors are used in filter circuits.
4. Capacitors are widely used in tuning circuits of radio, T.V. and wireless sets.
5. Capacitors are used to produce and to detect oscillating electric field.
6. Capacitors are used to reduce voltage fluctuations in power supply systems and in time delay circuits, etc.

<u>Effect of dielectric on the capacity of a capacitor</u>:

Whenever a dielectric is placed between the plates of a capacitor its capacity will increase**.** Capacity of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

Case1: When the space between the plates is totally filled with a dielectric of constant ‘K’, then new capacity $C_1 \frac{\varepsilon A}{d} = K\frac{\varepsilon_0 A}{d} = KC\left[\because \varepsilon = K\varepsilon_0\right]$

So capacity increases by K times.

Case 2: When a dielectric of thickness't' is introduced between the plates, then new capacity

$$C_1 \frac{\varepsilon_0 A}{d-t+\frac{1}{k}} = \frac{\varepsilon_0 A}{d-1\left(1-\frac{1}{k}\right)}$$

In this case increase in capacity depends on thickness of dielectric 't' and on its dielectric constant.

2. Derive an expression for the energy stored in a capacitor. What is the change in the energy stored when the space between the plates is filled with a dielectric?

a)_ With charging battery disconnected

b) With charging battery connected in the circuit

A. it.**<u>Energy stored in a condenser :</u>**

Consider a condenser of capacity C which is charged to a potential V by giving a charge q on to it . $\therefore V = \frac{q}{C}$

The work done in increasing the charge on the condenser by a small amount dq is given by $dW = V\ dq\ = \frac{q}{c}$ dq.

The total work done in increasing the charge on the condenser from 0 to q is given by

$$W = \frac{1}{C}\int_0^q q\ dq = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^q = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electric potential energy of the condenser.

$$\therefore U_o = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}C\,V^2 = \frac{1}{2}QV$$

<u>Effect of dielectric on energy :</u>

Consider a capacitor of capacity C_o with air as dielectric and charged to a potential V_o with the help of a battery. Let the final charge on the capacitor is q_o .

<u>When the charging battery is disconnected:</u>

Let the battery is disconnected from the capacitor after charging it and the space between the plates is filled completely with a dielectric of constant k. Hence the potential of the condenser becomes $V = \frac{V_o}{k}$.

Since the battery is disconnected the charge on it is constant. (i.e.) $q = q_o$. Hence energy stored is

$$U = \frac{1}{2}qV = \frac{1}{2}q_o\frac{V_o}{k} = \frac{1}{2}\frac{q_oV_o}{k}$$

$$\therefore U = \frac{U_o}{k} \qquad \left(\because \; U_o = \frac{1}{2} q_o V_o\right)$$

Energy reduces by a factor k.

When the charging battery connected.

Let the dielectric of constant k is introduced between the plates of the condenser keeping the battery connected to the capacitor. Hence the charge on the plates increases to $q = kq_o$

Since the battery is not disconnected, the potential difference remaining the same.

The energy stored is given by $U = \frac{1}{2} qV = \frac{1}{2}(kq_o)V_o = kU_o \Rightarrow \; U = kU_o$

Hence the energy increases by a factor k.

3. Describe three different types of capacitors, their construction and uses.

A. 1 .Variable air capacitor :

The variable capacitor consists of two sets of plates. One set is fixed and it is called stator. The other set, called rotor can be rotated over stator by rotating a piston. During rotation the effective area of plates and hence the capacity of the condenser can be varied. If n is the number of plates in the capacitor, then $(n-1)$ capacitors are in parallel

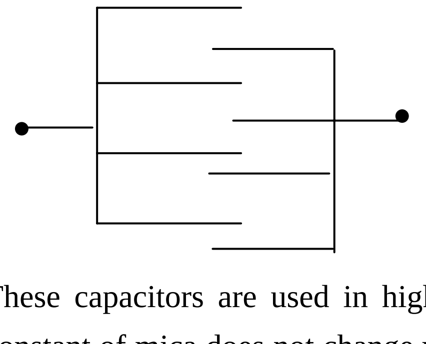
$$\therefore C = \frac{(n-1)\in_o A}{d}$$

Variable capacitors are widely used in tuning circuits in radio, T.V receivers.

2.Multiple capacitor :

This is a parallel combination of several parallel plate capacitors and has a fixed capacity. In this mica is used as dielectric between a number of tin foils arranged in parallel as shown. If n is the total number of plates then

$$C = (n-1)\frac{\in_o A}{d}$$



These capacitors are used in high frequency oscillating circuits. The dielectric constant of mica does not change with temperature and hence these capacitors are used as standard capacitors in laboratory.

3. Paper condenser :

In paper condenser a paper soaked in oil (or) wax is used as dielectric in between tin foils that serve as capacitor plates. To increase the capacitance to large extent several strips of metal foils and waxed paper are arranged alternately. The entire capacitor can be rolled and sealed in a cylinder.

As these capacitors occupies a small space and as they are cheap, these are widely used in radio circuits and in laboratories

4) Electrolytic condenser :

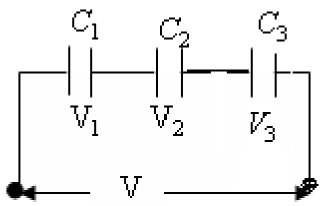
This is made of two Aluminum electrodes dipped in a solution of ammonium borate. When D.C. is passed into the solution Aluminum oxide is obtained on the positive electrode (Anode). This oxide film of thickness of order of 10^{-6} cm acts like a dielectric.Due to this small thickness, capacity will be very high. Anode is one plate, solution is other plate and Aluminum oxide is dielectric.

This capacitor does not work for AC. Using DC the anode Aluminum plate is always positive otherwise the oxide film breaks down.

These are widely used when high capacitances are required. Capacity of order of $10^3 \mu F$ can be easily obtained with electrolytic capacitors of small volumes.

4. **Explain series and parallel combination of capacitors. Derive the formula for equivalent capacitance in each combination.**

A. **Series combination**: Consider three capacitors of capacities C_1, C_2 and C_3 are connected in series across a battery of potential difference V. Let V_1, V_2 and V_3 be the potential difference across the capacitors respectively. As the capacitors are in series the charge an all capacitors is same and let it be q.



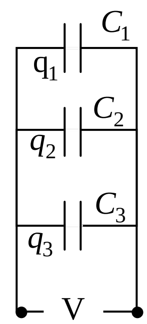
$$V_1 = \frac{q}{C_1} \; ; \; V_2 = \frac{q}{C_2} \text{ and } V_3 = \frac{q}{C_3}$$

But $V = V_1 + V_2 + V_3$

$$\therefore \; \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \text{ (or) } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus the reciprocal of equivalent capacity is the sum of the reciprocals of the individual capacities.

Parallel Combination:



Consider three capacitors of capacity C_1 C_2 and C_3 connected in parallel across a ball of potential difference on the capacitors respectively. As the capacitors are connected parallel the potential difference across each capacitor is same and let it be V.

$q_1 = C_1V$; $q_2 = C_2V$ and $q_3 = C_3V$

But $q = q_1 + q_2 + q_3$

$CV = C_1V + C_2V + C_3V$ (or)

$\therefore C = C_1 + C_2 + C_3$

Thus the effective capacity is equal to the sum of individual capacities.

SHORT ANSWER QUESTIONS

1. Define capacitance or capacity of conductor and explain the principle of capacitor. (June2010)

A. **Capacity:** The capacity of a conductor is defined as the ratio of charge to the potential on the conductor (or). It is a charge required to raise the potential of the conductor by one unit.

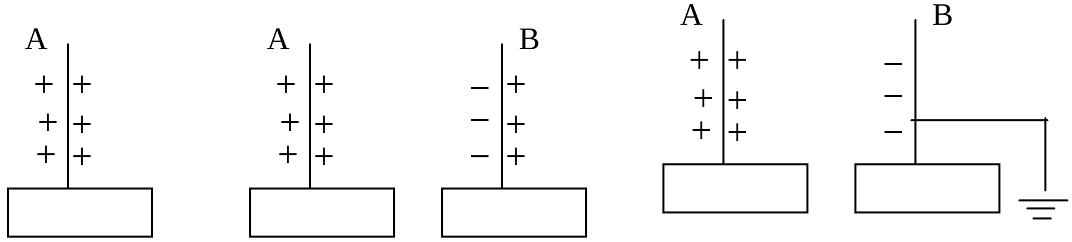
$$\textit{Capacity } C = \frac{Charge}{Potential} = \frac{Q}{V}$$

S.I. unit of capacity is farad (F)

Principle of a capacitor:

Condenser is a device used to store a large quantity of charge with out changing its potential.

Consider a metal plate A mounted on an insulated base and the plate A is given a positive charge till its potential becomes maximum. If Q is the charge and V is the potential of the plate then its capacity C is given by $C = \frac{Q}{V}$



Now another metal plate B which is uncharged is brought nearer to the plate A. Then electrostatic induction takes place in B and hence negative charge is induced on the nearer face of the plate B and positive charge is induced on the farther face as shown. Let V_- be the potential at A due to induced negative charges and V_+ is the potential at A due to the induced positive charge on B. The net potential at the plate A is given by

$$V^1 = V - V_- + V_+$$

Though induced +ve and –ve charges at the plate are equal, since the negative charges are nearer to the plate A , $V_- > V_+$.

$$\therefore V^1 < V \quad \Rightarrow \quad C^1 = \frac{Q}{V^1} \text{ Since } V^1 < V \quad C^1 > C .$$

To restore the initial potential of the plate A, some more charge can be given to it. There by more charge can be stored on A with out changing its potential.

Further, if the metal plate B is earthed as shown, the induced the charges flow to the earth. Hence the net potential at the plate A is given by

$$V^{11} = V - V_- \text{ And hence } V^{11} << V$$

$$\therefore \; C^{11} = \frac{Q}{V^{11}} \text{ Where } C^{11} >> C$$

Again to raise the potential of A to the initial value a large amount of charge can be given to it.

Thus the capacity of a conductor can be increased without changing its potential.

2. Derive an expression for the energy stored in a capacitor. If a dielectric is introduced between the plates how will the energy change ?

A. **Energy stored in a condenser :**

Consider a condenser of capacity C which is charged to a potential V by giving a charge q. $\therefore V = \frac{q}{C}$

The work done in increasing the charge on the condenser by a small amount dq is given by $dW = V\,dq = \frac{q}{c}$ dq.

The total work done in increasing the charge on the condenser from 0 to q is given by

$$W = \frac{1}{C}\int_0^q q\,dq = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^q = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electric potential energy of the condenser.

$$\therefore U_o = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Effect of dielectric on energy :

Consider a capacitor of capacity C_o with air as dielectric and charged to a potential V_o with the help of a battery. Let the final charge on the capacitor is q_o.

When the charging battery is disconnected:

Let the battery is disconnected from the capacitor after charging it and the space between the plates is filled completely with a dielectric of constant k. Hence the potential of the condenser becomes $V = \frac{V_o}{k}$.

Since the battery is disconnected the charge on it is constant. (i.e.) $q = q_o$.

Hence energy stored is

$$U = \frac{1}{2}qV = \frac{1}{2}q_o\frac{V_o}{k} = \frac{1}{2}\frac{q_oV_o}{k}$$

$$\therefore U = \frac{U_o}{k} \qquad \left(\because\ U_o = \frac{1}{2}q_oV_o\right)$$

Energy reduces by a factor k.

When the charging battery connected.

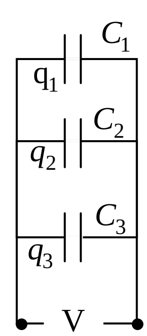
Let the dielectric of constant k is introduced between the plates of the condenser keeping the battery connected to the capacitor. Hence the charge on the plates increases to $q = kq_o$

Since the battery is not disconnected, the potential difference remaining the same.

The energy stored is given by $U = \frac{1}{2}qV = \frac{1}{2}(kq_o)V_o = kU_o \Rightarrow\ U = kU_o$

Hence the energy increases by a factor k.

3. **Derive an equation for equivalent capacitance when a number of capacitors are connected in parallel. (May2009)**



A. Consider three capacitors of capacity C_1 C_2 and C_3 connected in parallel across a ball of potential difference on the capacitors respectively. As the capacitors are connected parallel the potential difference across each capacitor is same and let it be V.

$q_1 = C_1V$; $q_2 = C_2V$ and $q_3 = C_3V$

But $q = q_1 + q_2 + q_3$

$CV = C_1V + C_2V + C_3V$ (Or)

$\therefore C = C_1 + C_2 + C_3$

Thus the effective capacity is equal to the sum of individual capacities.

4. Derive an equation for the equivalent capacitance when capacitors are connected in series.

A. Let three capacitors say C_1, C_2, and C_3 are connected in series as shown in figure. In series combination, current in the circuit is constant, So same amount of charge (Q) flows through each capacitor, Potential drop across AC is the sum of potential drops on each capacitor $\therefore V = V_1 + V_2 + V_3$

But Capacity , $C = \frac{Charge}{Potential}\frac{Q}{V}$ $\quad \therefore V = Q/C$

$$\therefore V = Q/C \text{ or } V_1 = \frac{Q}{C_1}; V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

Since Q is constant

Total potential $V = V_1 + V_2 + V_3$

$$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \text{ or } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

5. Write a note on different types of capacitors.

A. Capacitors are of different types. They are

1) Variable capacitors (2) Multiple capacitors (3) Paper capacitors (4) Electrolyte capacitors

1. Variable capacitors: Variable capacitor consists of two sets of parallel plates in the form of semicircle. Generally they are made of aluminium. One set of parallel plates are fixed called stator. The other set can be rotated in the stator. This is called rotor.

When rotor is slowly rotated the effective area between the plates will change so the capacity of this capacitor will also change.

Uses: These variable capacitors are widely used in tuning circuits of radio, and T.V. receivers.

2. Multiple capacitors: This type of capacitors consists of two sets of parallel plates. Alternate plates are grouped as a set. A dielectric slab made of mica is placed between each set of plates. Let capacity between two plates with mica dielectric be 'C' and number of dielectric slabs are n then total capacity of the capacitor $C_1 = nC$

3. Paper capacitors: This type of capacitors consists of two parallel tin foils. Between these tin foils a paper soaked in oil or wax is used as a dielectric. This combination is rolled and sealed in a cylinder with the terminals given out of cylinder for connection.

Uses : These capacitors are small in size and very cheap. They are used in laboratories and in electronic circuits of radios or T.V. receivers.

4. Electrolyte capacitors: In these capacitors an electrolyte like aluminium borate is coated as a thin film of thickness $10^{-6}\ cm$ between two aluminium foils. When D.C is passed with given polarity the electrolyte will form an ultra thin layer of aluminium oxide on anode (+ ve terminal) .

This is bad conductor of electricity and act as a dielectric. Due to extremely thin separation capacity of this capacitor is very high. In electrolyte capacitors polarity of terminals must be followed. Otherwise the capacitor is damaged.

Uses: These electrolyte capacitors are widely used as high capacitances with small size.

6. Explain the behavior of a dielectric in an external field.

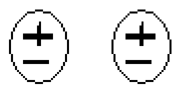
A. **<u>Effect of dielectric in a capacitor:</u>**

A dielectric is a solid insulating medium like glass, ebonite, wax paper, mica placed in between the two metal sheets in order to increase its capacity. These do not allow the electric charges to easily pass through them.

<u>Polar and non-polar dielectrics :</u>

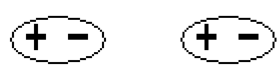
Dielectric is a material in which all the electrons are tightly bound to the nuclei of the atoms. Thus there are no free electrons to carry the current. The electrical conductivity of a dielectric is zero. The molecules of dielectric may be classified as non polar and polar.

A non-polar molecule is one in which the centre of gravity of positive charges coinciding with the c.g. of negative charges. A non-polar molecule has zero electric dipole moment in the absence of the external electric field.



Ex: $H_2, N_2, O_2, CO_2 CH_4, CCl_4$ etc.

A polar molecule is one in which the c.g. of positive and negative charges do not coincide. They are separated by finite distances. (When such polar molecules are placed in an external electric field, the field tries to orient the positive charge centers in the direction of the field and negative charge centers in the opposite direction) The polar molecule is an electric dipole and has a permanent dipole moment.

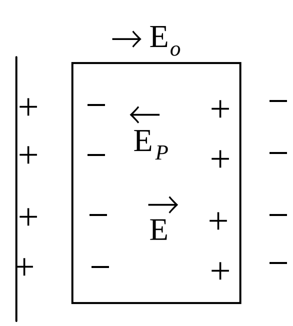


Ex HCL, CO, H_2O, NH_3 etc.

<u>Electric polarization of dielectric :</u>

When a dielectric (polar or non-polar) is placed in an electric field , then the material gets polarized and the net dipole moment is induced in the direction of the external field.

If a dielectric slab of dielectric material is placed in a uniform electric field E_o between the parallel plates of a capacitor. Then the slab becomes electrically polarized. These induced charges produce an electric field (E_P) inside the dielectric the opposite direction to that of external field. Hence the magnitude of the resultant field intensity with in the dielectric is given by



$|\bar{E}| = |\bar{E}_o| - |\bar{E}_P|$ (or) $E = E_o - E_P$

When a dielectric material is placed between the plates of a capacitor (Keeping the charge constant) the electric field and hence the potential decreases by a factor K. Hence capacity increases by a factor K

$$\therefore\ E = \frac{E_o}{K} \quad \text{And} \quad V = \frac{V_o}{K}$$

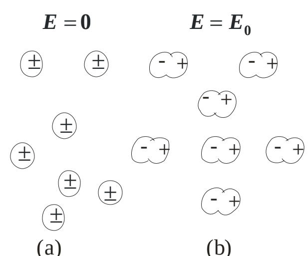
$$\therefore\ \frac{E_o}{K} = E_o - E_P \ \text{(or)} \quad E_P = E_o\left(1 - \frac{1}{K}\right)$$

$$\frac{\sigma_P}{\epsilon_o} = \frac{\sigma_o}{\epsilon_o}\left(1 - \frac{1}{K}\right) \ \text{(or)}\ \sigma_p = \sigma_o\left(1 - \frac{1}{K}\right) \ \text{and}\ q_p = q_o\left(1 - \frac{1}{K}\right)$$

7. **Explain the behavior of a material made of non polar molecules when it is placed is an external electric field.**

A. Substances which do not allow the electric charges to flow through them are called insulators or dielectrics. All substances are made with molecules and these molecules are electrically neutral. So these molecules contains equal amount of positive and negative charges.

In non polar molecules, the molecules will have symmetric distribution of positive and negative charges. So the centre of positive charge coincides with centre of negative charge when there is no external electric field.



Dielectric materials (Non-polar)

When these non – polar dielectric substances are placed in an electric field, charge polarization takes place. i.e., Negative charges will move nearer to the positive side of external field and positive charges will move nearer to negative side of electric field due to attraction. So the centre of the charges will move in opposite direction as shown in figure. So each molecule is said to be polarized.

Let the magnitude of external electric field be $\boldsymbol{E_0}$. As the molecules are polarized in an electric field then surface charge will develop on each molecule and these induced charges are in opposite direction to the applied field. These induced charges will develop electric field in opposite direction. Let their magnitude be $\boldsymbol{E_1}$. So due to charge polarization intensity of electric field in a dielectric reduces. The value of actual intensity $\boldsymbol{E = \frac{E_0}{K}}$ similarly actual potential across dielectric $\boldsymbol{V = \frac{V_0}{K}}$ where $\boldsymbol{V_0}$ is applied potential When a dielectric is placed in an electric field the intensity of electric field and potential across the dielectric are less than the intensity and potential in that region without dielectric.

<u>VERY SHORT ANSWER QUESTIONS</u>

1. **Three capacitors of capacitances $4\mu F, 6\mu F$ *and* $8\mu F$ are connected in parallel**

a) What is the ratio of charges b) What is the ratio of potential differences? (June2001)

A. **a)** In parallel combination potential is constant and charge changes

$Q \alpha C \ (\because V \ is \ constant)$

$\therefore Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

$\Rightarrow \ Q_1 : Q_2 : Q_3 = 4:6:8 \ \Rightarrow \Rightarrow Q_1 : Q_2 : Q_3 = 2:3:4$

b) As potential is constant ,the ratio is $V_1 : V_2 : V_3 = 1:1:1$

2. If the above capacitors are connected in series then what is the ratio of a) charges b) potential differences?

A. **a)** In series, same charge is stored on all capacitors.

$\therefore$ Ratio of charges $Q_1 : Q_2 : Q_3 = 1:1:1$

b) $V \alpha \frac{1}{C}$

Ratio of potentials $V_1 : V_2 : V_3 = \frac{1}{4} : \frac{1}{6} : \frac{1}{8}$ ($\because Q$ is constant)

Or $V_1 : V_2 : V_3 = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6:4:3$

3. What happens to the capacity of a parallel plate capacitor if its radius of plates is doubled?

A. Capacity of parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$ Where A = Area of the plate

Area of circular plate, A = πr^2

$C = \frac{\varepsilon_0 \pi r^2}{d} \Rightarrow C \ \alpha \ r^2 \ \Rightarrow \ \frac{C_2}{C_1} = \frac{r_2^2}{r_1^2}$

$r_2 = 2r_1 \Rightarrow \frac{C_2}{C_1} = \left(\frac{2r_1}{r_1}\right)^2 \Rightarrow C_2 = 4C_1$

Hence capacity increases by four times.

4. Write down the uses of capacitors. (Or) condensers. ?

A. a) In tuning circuit of radio.

b) As filters to stop D.C. and allow A.C.

c) To produce strong electric fields within a small space.

d) To reduce sparks in induction coil.

e) To store energy in electronic flash units.

UNSOLVED PROBLEMS

1. Calculate the capacity of a parallel plate capacitor of each plate area of 10 cm ×10 cm and separated by a distance 1 mm.?

A. A = 10×10 cm^2 = 10^{-2}m^2 ; d = 1 mm = 10^{-3} m ;

$\varepsilon_0 = 8.842 \times 10^{-12} \ farad/meter$

$C = \frac{\varepsilon_0 A}{d}$

$\therefore C = \frac{8.84 \times 10^{-12} \times 10^{-2}}{10^{-3}} = 8.842 \times 10^{-11} \ farad = 88.42 \, PF$

2. **Three capacitors of capacitance $2\mu F, 4\mu F$ and $6\mu F$ are connected in parallel and a pd of 12 V applied, Calculate charge on each capacitor.?**

A. V = 12 V; $C_1 = 2\mu F$; $C_2 = 4\mu F$; $C_3 = 6\mu F$

q = CV

$q_1 = C_1V = 24\mu C$; $q_2 = C_2V = 48\mu C$; $q_3 = C_3V = 72\mu C$

3. **Two capacitors have a capacity of 25 μF when connected in parallel and $6\mu F$ when connected in series. Find their individual capacities.**

A. $C_1 + C_2 = C_p = 25$ …………(1)

$$\frac{C_1C_2}{C_1+C_2} = C_8 = 6 \Rightarrow C_1C_2 = 6\times 25 = 150$$

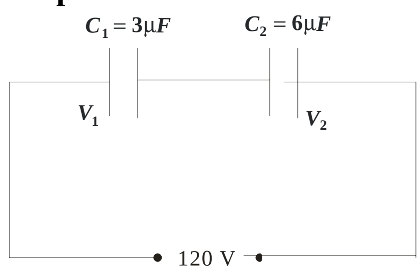
$$(C_1 - C_2)^2 = (C_1 + C_2)^2 = 4C_1C_2 = 25^2 - 4\times 150$$

$C_1 - C_2 = 5$ ……….(2)

From equation (1) and (2)

$C_1 = 15\mu F$ And $C_2 = 10\ \mu F$

4. **Two capacitors of capacities $C_2 = 3\mu F$ and $C_2 = 6\mu F$ are connected in series and a source of emf 120 V is connected across the combination. Find the potential difference across each of them. ?**



A. $V_1 = V\left(\frac{C_2}{C_1+C_2}\right)$ and $V_2 = V\left(\frac{C_1}{C_1+C_2}\right)$

$V_1 = 80 Volt$ and $V_2 = 40 Volt$.

5. **A parallel plate capacitor of capacitance (capacity) 5 μF has got a plate separation of 6 cm. It is connected to a battery of emf 1V and is fully charged. Now, with the battery still present in the circuit, a dielectric of dielectric constant 4 and of thickness 4 cm is introduced between the plates of the capacitor. How much additional charge flows on the plates of the capacitor from the battery?**

(Hint: As the battery is still connected in the circuit the potential remains unaltered even when the dielectric is introduced.)

A. $C_a = \frac{\varepsilon_0 A}{d} = 5\mu F$

Distance between plates d = 6cm

Dielectric constant of slab K = 4

Thickness of slab t = 4 cm

New capacity of capacitor.

$$C_d = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{6 - 4 + \frac{4}{4}} = \frac{\varepsilon_0 A}{3} \quad \Rightarrow \frac{C_d}{C_a} = \frac{\frac{\varepsilon_0 A}{3}}{\frac{\varepsilon_0 A}{d}} = \frac{d}{3} = 2$$

$$\Rightarrow C_d = 2C_a = 2(5) = 10\ \mu F$$

Since battery is not disconnected

$V_a = V_d = IV$

$\therefore$ Additional charge flow $n = Q_d - Q_a = C_d V_d - C_a V_a = 1(10-5) = 5\mu F$

6. If the charge on a body is increased by an amount of 2 C, the energy stored in it increases by 21 %. Find the original charge on the body.?

A. $U = \frac{1}{2}\frac{Q^2}{C} \Rightarrow Q^2 \alpha U \Rightarrow Q \alpha \sqrt{U} \Rightarrow \frac{Q_2}{Q_1} = \sqrt{\frac{U_2}{U_1}}$

$$\Rightarrow \frac{Q_1 + 2}{Q_1} = \sqrt{\frac{121U_1}{100U_1}} = \frac{11}{10} \quad \Rightarrow Q_1 = 20C$$

7. Two metal spheres A and B have their capacities in the ratio 3: 4. They are put in contact with each other and an amount of charge $7x10^{-6}C$ is given to the combination. Next, the two spheres are separated wide a part so that one has no electrical influence on the other. Find the potential due to the smaller sphere at a distance of 50 m from it.
(Hint: Charges are shared according to the capacities at the same potential.)

A. $\frac{C_1}{C_2} = \frac{3}{4} \Rightarrow C_2 = \frac{4}{3}C_1$

Total charge = $q_1 + q_2 = q_1^1 + q_2^1 = 7\times10^{-6}C$

After Separation new charge on 1st sphere,

$$q_1^1 = \left(\frac{C_1}{C_1 + C_2}\right)(q_1 + q_2) = \frac{C_1}{C_1 + \frac{4}{3}C_1}\left(7\times10^{-6}\right) = 3\times10^{-6}C$$

Potential due to 1st sphere = $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1^1}{d} = \frac{9\times10^9 53\times10^{-6}}{50} = \frac{27000}{50} = 540V$.

8. A parallel plate capacitor (condenser) has a certain capacitance (capacity). When 2/3 rd of the distance between the plates is filled with a dielectric, the (capacity) capacitance is found to be 2.25 times the initial capacitance. Find the dielectric constant of the dielectric.?

A. Capacity with air medium = $C_a = \frac{\varepsilon_0 A}{d}$

Capacity with dielectric = $C_d = 2.25\ C_a$

$$\Rightarrow \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = (2.25)\left(\frac{\varepsilon_0 A}{d}\right) \Rightarrow d - t + \frac{t}{k} = \frac{d}{2.25}$$

Here $t = \frac{2}{3}d \Rightarrow$ K = 6

9. A parallel plate capacitor is filled with a dielectric of dielectric constant (relative permittivity) 5 between its plates and is charged to acquire an energy E. Then it is isolated (disconnected from the battery) and the

dielectric is replaced by another dielectric of dielectric constant (relative permittivity) 2. Find the new energy stored in the capacitor.?

A. $\boldsymbol{E = K_1\left(\frac{1}{2}CV^2\right) = k_1U}$........(1)

Where $\boldsymbol{K_1 = 5}$,

$\boldsymbol{U = \frac{1}{2}CV^2\left(or\right)\frac{1}{2}QV}$

After battery is disconnected depolarization of charges of second dielectric takes place and the energy stored becomes $\boldsymbol{E^1 = \frac{1}{2}QV^1}$

$$\boldsymbol{E^1 = \frac{1}{2}Q\left(\frac{V}{K_2}\right)}$$

$$\boldsymbol{E^1 = \frac{U}{K_2}} \quad \text{..........(2)}$$

$$\boldsymbol{\frac{E^1}{E} = \frac{\frac{U}{K_2}}{K_1U} \quad \Rightarrow E^1 = \frac{E}{K_1K_2}}$$

$$\boldsymbol{E^1 = \frac{E}{2\times 5} = \frac{E}{10}}$$

10. The plates of a parallel plate capacitor are charged to 200 V and then the charging battery is disconnected. Now, a dielectric slab of dielectric constant 5 and thickness 4 mm is inserted between the capacitor plates. To maintain the original capacitance (capacity) of the capacitor, how much is the plate separation to be increased?

A. $\boldsymbol{C = \frac{\varepsilon_0 A}{d}}$ and $\boldsymbol{C^1 = \frac{\varepsilon_0 A}{d + x - t + \frac{t}{k}}}$

Since $\boldsymbol{C^1 = C}$

$$\boldsymbol{\Rightarrow \frac{\varepsilon_0 A}{d + x - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d} \quad \Rightarrow x = \frac{16}{5} = 3.2\,mm}.$$

11. A dielectric of thick ness 5 cm and dielectric constant 10 is introduced between the plates of a parallel plate capacitor having plate area 500 sq.cm and separation between the plates 10 cm. Calculate the capacitance of the capacitor with dielectric slab . $\left[\text{Take}\,\varepsilon_0 = 8.8\times10^{-12}C^2/N-m^2\right]$

Sol: A = 500 sq.cm = $500\times10^{-4}m^2$; d = 10 cm = 0.1 m ; t = 5 c, = 0.05m, ; k = 10 ; $\varepsilon_0 = 8.8\times10^{-12}C^2/N-m^2$.

$$C = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)} = \frac{\left(8.8\times10^{-12}\right)\left(500\times10^{-4}\right)}{\left[0.1 - 0.05 + \frac{0.05}{10}\right]} = 8\times10^{-12}F = 8pF$$

12. **The plates of a parallel plate capacitor are separated by 0.1 mm. What should be the plate area to have a capacitance of 2 pF?**
$\left[\text{Take}\,\varepsilon_0 = 8.85\times10^{-12}C^2N^{-1}m^{-2}\right]$

Sol: $\epsilon_0 = 8.85\times10^{-12}C^2N^{-1}m^{-2}$; d = 0.1 mm = $0.1\times10^{-3}m$; C = 2pF = $2\times10^{-12}F$.

Let the area of each plate be A m^2

$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad A = \frac{dC}{\varepsilon_0} = \frac{(0.1\times10^{-3})(2\times10^{-12})}{8.85\times10^{-12}} = 22.6\times10^{-6}m^2$$

13. **A capacitor of 10 μF capacitance is charged by a 12 V battery. What is the magnitude of charge on each plate of the capacitor? Next, the battery is continued in the circuit while the space between the plates of capacitor is filled with a dielectric of dielectric constant K = 3. Now what is the magnitude of the charge?**

Sol: $C_0 = 10\mu F = 10\times10^{-6}F$; V = 12 V

a) $Q_0 = C_0V_0 = (10\times10^{-6})\times12 = 120\times10^{-6}C = 120\mu C$

b) Dielectric constant K = 3
Capacitance with dielectric $C = KC_0 = 3\times10\times10^{-6}F$
As the battery is still present in the circuit, the potential will not change and hence $V = V_0 = 12V$

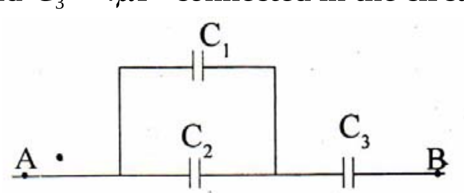
$\therefore$ Charge on each plate $Q = CV = (3\times10\times10^{-6})12 = 360\ \mu C$.

14. **Three capacitors of capacitances $5\mu F$, 10 μF, and 20 μF are connected in series. Calculate the effective capacitance'?**

Sol: $C_1 = 5\mu F$, $C_2 = 10\mu F$ and $C_3 = 20\mu F$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{7}{20} \text{ and } C = \frac{20}{7} = 2.86\mu F.$$

15. **Calculate the equivalent capacitance between A and B with $C_1 = 10\mu F$, $C_2 = 5\mu F$ and $C_3 = 4\mu F$ connected in the circuit as shown in the figure.** ?



Sol: Equivalent capacitance of $C_1 = 10\mu F$ and $C_2 = 5\mu F$ is given by
$C_4 = 10 + 5 = 15\mu F$.
Now, this 15 μF capacitor is in series with $C_3 = 4\mu F$ capacitor.
The equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_4} + \frac{1}{C_3} = \frac{1}{15} + \frac{1}{4} = \frac{(4+15)}{60} = \frac{19}{60} \text{ or } C = \frac{60}{19} = 3.16\mu F$$

16. **A 10 μF parallel plate capacitor is charged by a 12 V battery. What is the energy stored in it?**

Sol: $C = 10\mu F = 10\times10^{-6}F.$; V = 12 V

Energy stored in the capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}(10\times10^{-6})(12)^2 = 720\times10^{-6}J$

17. **A capacitor of capacitance 10 μF is charged by a 12 V battery. While the battery is still present in the circuit, the space between the capacitor plates is**

filled with a dielectric of dielectric constant K = 3. Calculate the energy stored in the capacitor filled with dielectric?

Sol: $C_0 = 10\mu F$; l V = 12V ; K = 3

As the battery is still present in the circuit, the potential V will not change.

Capacitance of capacitor with dielectric $C = KC_0 = 3\times10\mu F = 3\times10\times10^{-6}F$ and hence, energy stored in capacitor with dielectric

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(3\times10\times10^{-6}\right)\left(12^2\right) = 15\times10^{-6}\times144 = 2160\times10^{-6}J$$

18. A capacitor of 10 μF capacitance is charged with a 12 V battery. Now, the battery is disconnected from the circuit and the space between the plates is filled with a dielectric of dielectric constant K = 3. Calculate the energy stored in the capacitor with dielectric.?

Sol: $C_0 = 10\mu F$; $V_0 = 12V$; K = 3

Capacitance with dielectric $C = KC_0 = 3\times10\mu F = 3\times10\times10^{-6}F$

Now, as the battery is disconnected from the circuit, the potential will drop to

$V = \frac{V_0}{K} = \frac{12}{3} = 4V$.

$\therefore$ Energy stored in the capacitor with dielectric

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(3\times10\times10^{-6}\right)\left(4^2\right) = 240\times10^{-6}J$$

<u>ASSESS YOURSELF</u>

1. The dielectric strength of air is $3\times10^6\,Vm^{-1}$ at a certain pressure. A parallel plate capacitor with air in between the plates has a plate separation of 1cm. Can you charge the capacitor to 3×10^6 V?

Sol: No. There will be electrical break down

2 When a dielectric is placed in external electric field of strength E_0, inside the dielectric, due to polarization, and electric field of strength E_i is developed in a direction opposite to E_0 and the resultant electric field inside the dielectric is $E = E_0 - E_1 = \frac{E_0}{K}$ Where K is the dielectric constant of the dielectric.

What will be the value of E_i in case of a perfect conductor?

From this result, find the value of K for a perfect conductor.

Sol: For a perfect conductor $E_i = E_0$ and hence E = 0 inside. K will be infinity for a perfect conductor.