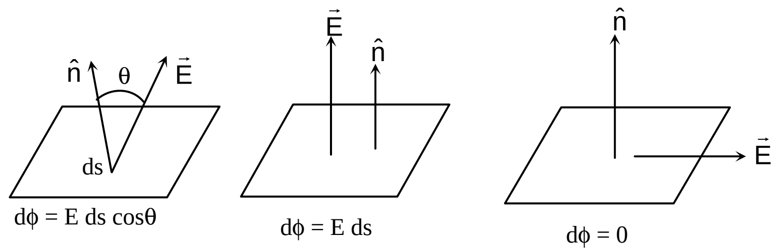
ELECTROSTATICS
3. GAUSS LAW

POINTS TO REMEMBER

1. Electric flux :

i) The number of electric lines of force crossing a surface normal to the area gives electric flux ϕ_E.



ii) Electric flux through an elementary area ds is defined as the scalar product of area and field.

$d\phi_E = \vec{E}.\vec{ds} = E ds \cos\theta$

iii) $\phi_E = \int \vec{E}.\vec{ds}$

iv) Flux will be maximum when electric field is normal to the area ($d\phi = Eds$)

v) Flux will be minimum when field is parallel to area ($d\phi$ =t 0)

vi) For a closed surface, outward flux is positive and inward flux is negative.

2. **Gauss's Law :**

i) The total flux linked with a closed surface is $(1/\epsilon_0)$ times the charge enclosed by the closed surface.

$$\oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} q$$

ii) A point charge q is placed inside a cube of edge 'a'. The flux through each face of the cube is $\frac{q}{6\epsilon_0}$.

3. Applications:

i) In the case of a charged ring of radius R on its axis at a distance x from the centre of the ring

$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(R^2 + x^2)^{3/2}}$ .At the centre x=0 & E=0

ii) In case of infinite line of charge , at a distance 'r'. $E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r} = \frac{\lambda}{2\pi\epsilon_0 r}$.

Where λ is the linear charge density.

iii) The intensity of electric field near a plane sheet of charge is $E = \frac{\sigma}{2\varepsilon_o K}$ where σ = surface charge density

iv) The intensity of electric field near a plane charged conductor $E = \frac{\sigma}{\varepsilon_o K}$ in a medium of dielectric constant K. If dielectric medium is air, then $E_{air} = \sigma / \varepsilon_o$.

v) Field between two parallel plates of a condenser is E = $\frac{\sigma}{\epsilon_0}$, where σ is the surface charge density.

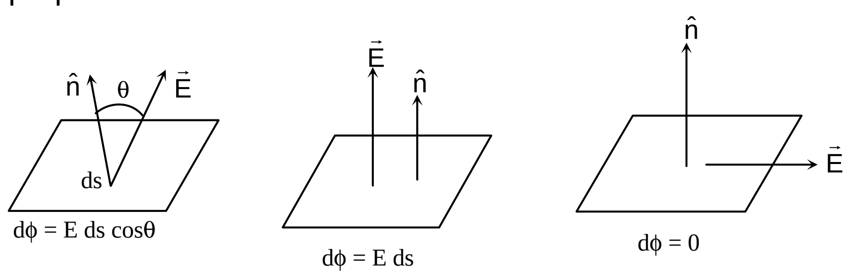
LONG ANSWER QUESTIONS

1. **Define electric flux. Applying Gauss's law derives the expression for electric intensity due to an infinite long straight charged wire. (Assume that the electric field is every where radial and depends only on the radial distance r of the point from the wire.)**

A. **Electric flux:** The total number of electric lines of force passing through a normal plane inside an electric field is called Electric flux (ϕ). It is a scalar quantity. $d\phi = \overline{E}.d\overline{s} = Eds\cos\theta$ (Or)

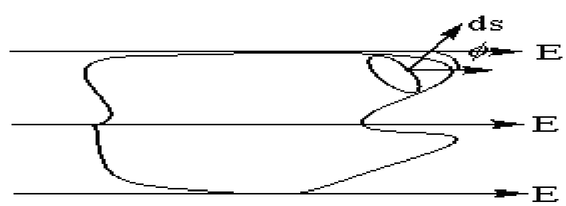
$\phi = \int E.ds = E\int ds$ Where θ is the angle between $\overline{E}$ and the normal to the area $d\overline{s}$ is along the perpendicular to the surface.

Unit: Nm^2/c



Gauss theorem:

This gives the relation between electric flux through any closed surface (called Gaussian surface) and the charge enclosed by the surface. It states that "The Electric flux (ϕ) through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the net charge enclosed by the surface". $\phi = \oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} q$

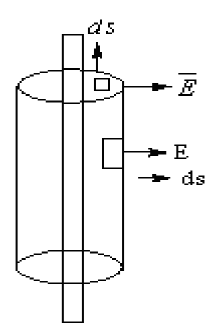


This is the integral form of Gauss law. Here ε_o is the permittivity of free space

Electric field due to a line charge :

Consider an infinitely long thin straight wire having a uniform charge density λ. . Consider a cylindrical Gaussian surface of length l and radial distance r with its axis along along the wire .

On the top and bottom surfaces $\overline{E}$ and $d\overline{s}$ are perpendicular. Hence $\phi = 0$



Along the curved surface $E\int ds = \frac{1}{\varepsilon_o} q$

$$E(2\pi r\ell) = \frac{1}{\varepsilon_o} q$$

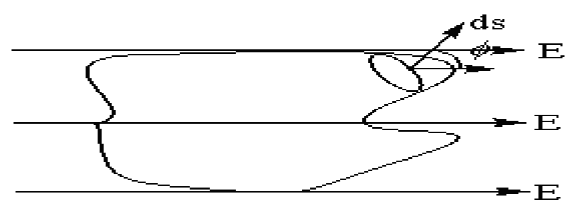
$$E = \frac{q}{2\pi\varepsilon_o r\ell} = \frac{\lambda}{2\pi\varepsilon_o r}$$

The above equation is electric intensity due to infinitely long charged wire.

2. **State Gauss's law in electrostatics. Applying Gauss's law derive the expression for electric intensity due to an infinite plane sheet of charge.**

A.

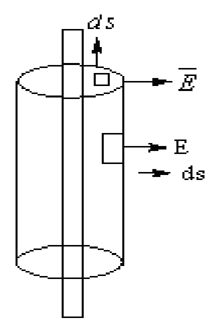
This gives the relation between electric flux through any closed surface (called Gaussian surface) and the charge enclosed by the surface. It states that "The Electric flux (ϕ) through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the net charge enclosed by the surface". $\phi = \oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} q$



This is the integral form of Gauss law. Here ε_o is the permittivity of free space

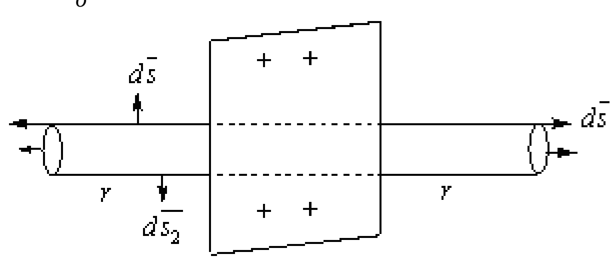
<u>Electric field due to an infinite plane sheet of charge :</u>

Consider an infinite thin plane sheet of positive charge. Consider a point P at a distance r from the sheet on either side



$$2E\int ds = \frac{q}{\varepsilon_o}$$

$$2Es = \frac{q}{\varepsilon_o} \Rightarrow E = \frac{q}{2s\varepsilon_o}$$



S is the area of each flat surface of the cylinder

Along ds_1 and $d\overline{s_2}$, $\phi = 0$

But $q = \sigma s$ where σ is the charge density on both sides

$$E = \frac{\sigma s}{2s\varepsilon_o} = \frac{\sigma}{2\varepsilon_o}$$

$$\therefore\ E = \frac{\sigma}{2\varepsilon_o}$$

If σ is positive, E will be along the outward drawn normal.
If σ is negative ,E will be along the inward drawn normal.

3. Applying Gauss's law derive the expression for Electric intensity due to charged conducting spherical shell at (i) A point outside the shell (ii) A point on the surface of the shell and (iii) A point inside the shell ?

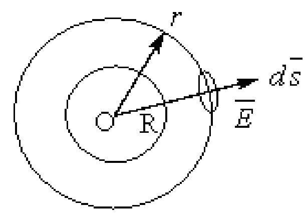
A. Electric field due to a spherical shell:

Consider a shell of radius R and a charge (+q). Let us find the electric field at a point P at a distance r from the centre 'O' of the shell.

(a) When P is outside the shell

$$\phi = E\int ds = E\left(4\pi r^2\right)$$

$$E\left(4\pi r^2\right) = \frac{1}{\varepsilon_o}q \quad \Rightarrow E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2}$$



$$\text{But, } \sigma = \frac{q}{4\pi R^2} \Rightarrow q = 4\pi R^2\sigma$$

$$E = \frac{4\pi R^2\sigma}{4\pi\varepsilon_o r^2} \quad \Rightarrow E = \frac{\sigma}{\varepsilon_o}\frac{R^2}{r^2}$$

$$\text{Also, } E = \frac{-dV}{dr} \Rightarrow V = -Edr = \int\frac{\sigma}{\varepsilon_o}\frac{R^2}{r^2}dr$$

$$\therefore V = \frac{\sigma}{\varepsilon_o}\frac{R^2}{r^2}$$

(b) When the point is on the surface

$$E\left(4\pi R^2\right) = \frac{q}{\varepsilon_o}$$

$$E = \frac{q}{4\pi\varepsilon_o}R^2 \Rightarrow E = \frac{\sigma}{\varepsilon_o}$$

$$E = -\frac{dV}{dr} \Rightarrow V = -Edr$$

$$V = \int\frac{\sigma}{\varepsilon_o}dr = \frac{\sigma}{\varepsilon_o}r \quad \Rightarrow V = \frac{\sigma r}{\varepsilon_o}$$

(c)When P is inside the shell :

In this case Gaussian surface does not enclose any charge and hence according to Gauss law.

$$E\left(4\pi r^2\right) = \frac{0}{\varepsilon_o}$$

$$\therefore E = 0$$

$$E = -\frac{dV}{dr} = 0$$

$\therefore$ V is constant

$\therefore$ On the surface r = R

$$V = \frac{\sigma r}{\varepsilon_o} = \frac{q}{4\pi R^2}\frac{R}{\varepsilon_o} \quad \Rightarrow V = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

SHORT ANSWER QUESTIONS

1. **Using Gauss's law derive Coulomb's inverse square law in electrostatics.**

A. **Coulomb's law from Gauss law :**

Consider a charge 'q' and draw a Gaussian spherical surface of radius 'r' with q as centre. By symmetry the electric field $\overline{E}$ at any point on the surface of the sphere surface is along the outward normal at that point and has same magnitude at every point on the surface. Both electric field vector $\overline{E}$ and the area vector $d\overline{s}$ are along the same direction (radially outward) (i.e) $\theta = 0$.

$\therefore\ E.ds = E\,ds\cos 0 = Eds$

$\therefore\ \phi = \int \overline{E}.d\overline{s} = \int Eds$

Since E is constant at all patches $\phi = E\int ds = E.4\pi r^2$

From Gauss law, $E\,4\pi r^2 = \dfrac{q}{\varepsilon_o}$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

This is the magnitude of electric field at a distance r from the point charge. If a test charge q_o is placed at that point. The force experienced by it is

$$F = Eq_o = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

This is Coulomb's law

Thus Gauss law is equivalent to Coulomb's law. Gauss law and Coulomb's law are not two independent laws, but the same law expressed in different forms.

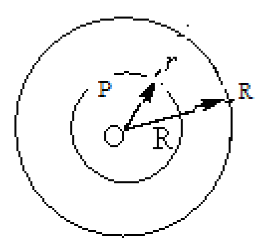
2. **State Gauss law in Electro – Statics and its importance?**

A. Gauss's Law: "The total Electric flux through any closed surface is equal to $\mathbf{\dfrac{1}{\varepsilon_0}}$ times the net charge enclosed by the surface."

Importance: In case of complex configuration of charge, the electric field strength can be more easily calculated by using Gauss law.

3. **Applying Gauss law show that the intensity of Electric field E is zero everywhere inside a charged conducting spherical shell.**

A. Consider a point P in a spherical shell. Its surface is Gaussian surface of concentric sphere with centre at 'o' and radius OP = r. The Electric field (E) and Elementary surface 'ds' are parallel to each other.



Gaussian surface passes through the point P. Now E and ds on the Gaussian surface will be every where parallel to each other and E.ds = Eds and $\int_8 \boldsymbol{E.ds}\ \int_8 \boldsymbol{ds} = \boldsymbol{E}.\left(\boldsymbol{4\pi r^2}\right)$

Since the point P is inside the sphere, there is no charge inside the spherical shell. Hence the charge $\boldsymbol{q^1}$ inside the Gaussian surface is zero and we have

$\int E.ds = E\left(4\pi r^2\right) = 0$

$\therefore$ E = 0 inside the charged shell.

The electric field intensity due to a uniformly charged spherical shell is zero at all points inside the shell.

4. Write down the expressions for electrostatic potential due to
i) An infinite long straight charged wire.
ii) An infinite plane charged sheet and
iii) A charged conducting spherical shell at paints outside and inside the shell.

A. i) The expression for Electrostatic potential due to An infinite long straight charged wire is at a distance r is given by,

$V = -\left[\frac{\lambda}{2\pi\epsilon_0}\log_e r\right] + k$ Where K is the integration constant, λ is linear charge density of wire.

ii) For a infinite plane charged sheet is at a distance r is given by,

$V = -\left[\frac{\sigma}{2\epsilon_0} r\right] + K$, where σ is surface charge density. K is the integration constant .

iii) A charged conducting spherical shell at points outside and inside the shell is For outside the conducting spherical shell at a distance r is given by,

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{r}\right)$$

Inside the shell, the potential is given by

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q}{R}\right)$$

VERY SHORT ANSWER QUESTIONS

1. Write the expression for electric intensity due to an infinite long charged wire.

A. The electric intensity due to an infinitely long charged wire perpendicular to the wire $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ Where λ linear charge density and r is is radial distance.

2. Write the expression for electrostatic potential due to an infinite plane sheet of charge.

A. The electrostatic potential due to an infinite plane sheet of charge at a perpendicular distance ‘r’ from the sheet is $V = -\left[\frac{\sigma}{2\epsilon_0 r}\right] + K$

Where σ is surface charge density and K is constant of integration .

3. Write the expression for electric intensity due to a charged conducting spherical shell at points outside and inside the shell.

A. The electric intensity at a point outside a uniformly charged spherical shell at a distance r from the centre of the shell.

$R = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$ Where q is the charge on the shell.

The electric intensity at any point inside a uniformly charged spherical shell is zero .

4. **Write the expression for electrostatic potential due to an infinite a charged conducting spherical shell at points (a) outside the shell (d) on the surface of the shell and (c) inside the shell.**

A. The electrostatic potential due to an infinite charged conducting spherical shell of radius R at a point

(i) Outside the shell at a distance r , $V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q}{r}\right)$

(ii) on the surface of the shell , $V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q}{R}\right)$

(iii) Inside the spherical shell, $V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q}{R}\right)$

5. **State Gauss's law in electrostatics.**

A. **<u>Gauss's theorem:</u>** It states that the total electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface. Here ϵ_0 is the permittivity of free space. Mathematically gauss's law can be given as $\int \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

6. **When will be the electric flux negative when is it positive?**

A. Electric flux through an elementary are ds is defined as the scalar product of area and field. $d\phi_E = E.ds = E\,ds\,\cos\theta$

Ie., $\phi_E = \int \vec{E}.\vec{ds}$

It represents the total lines of force passing through the given area. For a closed body, outward flux is taken to be positive while inward flux is taken to be negative.

<u>ASSESS YOURSELF</u>

1. **Even though electric flux is a scalar quantity, we consider the flux flowing out of a surface as positive and flux entering into the surface as negative. Keeping this fact in mind answer the following question.**
The total electric flux through a Gaussian surface is zero when there is a charge outside the surface. Why is it so when the charge produces electric field?

Sol: The electric field will always be along the direction away from the charge (assumed positive). But the flux will be both positive and negative when charge is outside the closed surface and equal in magnitude. Hence total flux will be zero for charge outside the closed surface.

2. **In conductors, the outer electrons of each atom or molecule are weakly bound to the atom or molecule. So, these electrons are almost free to move**

throughout the conductor. Hence, these are called free electrons or conduction electrons. When such a conductor is in an electric field, the free electrons inside redistribute themselves on the surface of the conductor in such a way that the electric field at

3. **We have derived considering only a finite length l of the wire. How are we justified in taking the equation valid for the entire of infinite length?**

Sol: This is because, the total charge enclosed will be λL and total surface area will be $(2\pi r)$L and L cancels out in Eqn. $E(2\pi r)L = \frac{\lambda L}{\varepsilon_0},$ even when L is infinity.

4. **The magnitude of the electric field of an infinite plane sheet of charge (as given by $E = \frac{\sigma}{2\varepsilon_0}$) is independent of the distance r from the sheet. $E \propto \frac{1}{r^2}$ is not followed here. Can you guess why?**

Sol: This is because the entire charge is on the infinite plane and does not depend on the distance r from the plane.

5. **Let us consider an infinite plane charged conducting plate. Now the charge distributes on both sides of the conducting plate. The field of such charged plate arises from the superposition of the fields of two sheets of charge. Can you guess the value of E for points outside the plate? What will be the field inside the plate, and why?**

Sol: $E = \frac{\sigma}{\varepsilon_0}$ and inside the plate E = 0.