MAGNETISM

2.COUPLE

POINTS TO REMEMBER

1. Couple acting on a bar magnet in a uniform magnetic field. ?

i) When a magnet with magnetic moment M is suspended in a uniform field of induction B at an angle with the field direction then the couple acting on the magnet, τ = MB sin and vectorially $\bar{\tau} = \vec{M} \times \vec{B}$

ii) When = 90° , τ is maximum.

If $|\bar{\tau}_{max}|$ = MB. If = 90° and B = 1 τ_{max} = M

iii) When = 0° Torque is zero In an uniform magnetic field a bar magnet experiences only a couple but no net force. Therefore it has only rotatory motion.

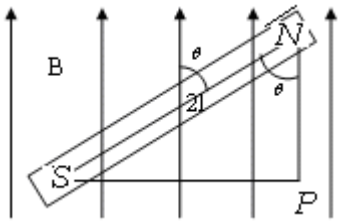
iv) In a non-uniform magnetic field a bar magnet experiences a couple and also a net force. So it undergoes both rotational and translational motion.

v) A bar magnet of moment M is initially parallel to the magnetic field of induction B. The angle through which it must be rotated so that it experiences half of the maximum couple is 30°.

Short Answer Questions:

1. Derive the equation for the couple acting on a bar magnet in a uniform magnetic field and hence deduce the definition of 'magnetic moment'?
(June 2010,March2009)

Ans: Consider a bar magnet NS of pole strength m and length 2l placed in a uniform magnetic field of indirection B. Let the axis of the bar magnet makes an angle θ with the direction of the field as shown. The north pole of the magnet experience a force mB in the direction of the field are the south pole experience a force mB in a direction opposite to the field. Hence the magnet experiences two equal and opposite forces which form a couple and there by the magnet rotates and align itself in the direction of the field.



Moment of couple (c) (or) torque (τ) = force × perpendicular distance between the forces.

From the diagram, in the triangle NSP, $\sin\theta = \frac{SP}{NS}$

(or) SP = NS $\sin\theta$ $= 2l \sin\theta$

$\tau = mB \times SP = mB \times 2l \sin\theta$

$\therefore \tau = MB \sin\theta$ *(or)* $\tau = \mu_0 MH \sin\theta$

Where M(=2lm) is called the moment of the magnet

In the vector form, $\bar{\tau} = \overline{M} \times \overline{B}$

Where the direction of the torque is perpendicular to the plane containing $\overline{M}$ and $\overline{B}$.

Definition of M : If B=1T and $\theta = 90^\circ$ then $\tau = M$

Moment of the magnet is numerically equal to the torque acting on the magnet when it is placed at right angles to a uniform magnetic field of unit induction.

Unit of M : N –m/tesla

vi)

Very Short Answer Questions:

1. When is the couple acting on a bar magnet in a uniform magnetic field (i) maximum, (ii) minimum ?

Ans :i) When the bar magnet is at right angles to the direction of uniform magnetic field $(\theta = 90^0)$, the torque on the magnet is maximum $(\tau_{max} = MB)$

(ii) When the magnet is parallel to the direction of uniform magnetic field $(\theta = 0^0)$, the torque on the magnet is minimum $(\tau_{min} = 0)$.

SOLVED PROBLEMS

1. A bar magnet of length 10cm and pole strength 2 Am is making an angle 60^0 with a uniform magnetic field of induction 50 Tesla. Find the couple acting on it.

Sol: Couple acting on the magnet, $C = MB\,Sin\theta$

Magnetic moment (M) = 2l $m = 0.1 \times 2 = 0.2\ Am^2$; B = 50T ; $\theta = 60^0$

$$C = MB\sin\theta = 0.2 \times 50 \times \sin 60^0 = 10 \times \frac{\sqrt{3}}{2} = 8.366\ Nm$$

2. A magnetic needle is acted upon by two fields of induction B_1 and B_2 respectively having an angle of 60^0 between them. The magnetic needle makes an angle of 15^0 with the direction of B_1 and comes to equilibrium. If the magnitude of B_1 is $1.2 \times 10^{-2}\ T$, find the value of B_2.

Sol: Let 'M' be the magnetic moment of the needle

Angle between the two fields = 60^0

The angle between the needle and $B_1 = \theta_1 = 15^0$

Angle between the needle and $B_2 = 45^0$

Also, $B_1 = 1.2 \times 10^{-2} T$.

At equilibrium , M $B_2 \sin\theta_2 = MB_1 \sin\theta_1$

$$\therefore B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{1.2\times10^{-2}\sin 15^0}{\sin 45^0} = 4.392\times10^{-3}T$$

Unsolved Problem

1. **If the maximum couple acting on a magnet in a field of induction 0.2 Tesla is 10Nm, what is its magnetic moment?**

Sol : B = 0.2 Tesla, $\tau = 10Nm$

Maximum torque $(\tau) = MB \qquad (\because \theta = 90^0)$

10 = M (0.2)

$\therefore$ Magnetic moment (M) = $\frac{10}{0.2} = 50Am^2$

2. **The maximum torque acting on the magnet of length 2×10^{-1} m in a uniform magnetic field having induction 2×10^{-1} Tesla is 10 Nm. Calculate its pole strength.?**

Sol: Maximum torque = MB =10 N –m

Length of the magnet $2l = 2\times10^{-1}m$.

Magnetic induction field strength $B = 2\times10^{-1}Tesla$

For maximum torque, $\theta = 90^0$

Pole strength $m = \frac{\tau}{2lB\sin\theta} = \frac{10}{2\times10^{-1}\times2\times10^{-1}} = 250Am$

3. **Calculate the moment of couple required to keep a bar magnet of magnetic moment $2\times10^2 Am^2$ in a uniform field of induction 0.36T at an angle 30^0 with the direction of uniform field?**

Sol: $M = 2\times10^2 A-m^2$; B = 0.36 T; $\theta = 30^0$

Moment of couple $(\tau) = MB\sin\theta$

$\tau = 2\times10^2\times0.36\times\sin 30^0$ = 36 Nm.

4. **When a bar magnetic is placed at 90^0 to uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to be half of the maximum value, at what angle should the magnet be inclined to the magnetic field (B)?**

Sol: <u>case 1:</u> From $\theta_1 = 90^0$

$\tau = MB\,Sin\theta$

$\tau = \tau_{max}\,Sin\theta_1 = 1 \qquad (\because \theta_1 = 90^0)$

$\Rightarrow \tau_{max} = MB$

Case (2) : $\tau = \frac{1}{2}\tau_{max}$

$\Rightarrow MB\,Sin\theta_2 = \frac{1}{2}\tau_{max} = \frac{1}{2}(MB)$ $\Rightarrow Sin\,\theta_2 = \frac{1}{2} = Sin\,30^0$ $\Rightarrow \theta_2 = 30^0$

ASSESS YOURSELF:

1. **Two magnets of magnetic moments M_1 and M_2 are joined are joined at their centers to form a cross (perpendicular to each other). The combination is suspended in a uniform magnetic field directed vertically upwards. It stands in equilibrium when either of them makes an angle of 45^0 with the vertical field. What is the ratio of their magnetic moments?**

Ans: $M_1 : M_2 = 1:1$