PHYSICAL OPTICS
1. INTERFERENCE

POINTS TO REMEMBER

1. The condition which allows us to use the principles of geometry is $b^2 >> l\lambda$ Where
   b = size of the object interacting with light
   l = distance between the object and the screen
   λ = wavelength of light

2. Coherent Sources

A. Two light sources are said to be coherent if they emit waves of same frequency which are in phase or which maintain a constant phase difference.

B. Two independent sources cannot be coherent.

C. A real source and its virtual image (Lloyd's mirror), two virtual images of a real source (Fresnel's biprism) are the examples of coherent sources.

D. A LASER (light amplification by stimulated emission of radiation) is a source in which the atoms are made to emit radiations of same wavelength and at the same instant of time. Two independent lasers can be coherent.

3. Conditions for Interference

A. The two sources should produce waves of same wavelength continuously and waves must be coherent.

B. For complete destructive interference the amplitudes of the two waves must be the same.

C. The distance between the two sources must be as small as possible and the slits must be very narrow and should have same width.

D. In interference there are two methods.
   a. Method of involving the division of wave front. E.g.: Young's experiment, Lloyd's mirror, Fresnel's biprism.
   b. Method involving the division of amplitude. E.g.: Colours in the films, Newton's rings.

4. Principle of Superposition

A. When two or more waves reach a point in space simultaneously, the resultant displacement at that point at any instant of time is the algebraic sum of the displacements produced by the individual waves. This is known as the principle of superposition.

B. If y_1, y_2, y_3….. are the displacements at a given point due to a number of waves, then the net displacement is given by $y = y_1 + y_2 + y_3 + \ldots$

5. Young's Double – Slit Experiment

A. The energy distribution on the screen is done according to the relation
   $I = 4a^2 \cos^2 \frac{\phi}{2}$ or $I = 4I_0 \cos^2 \frac{\phi}{2}$

B. Condition for constructive Interference: $\cos\frac{\phi}{2} = \pm 1$ (or) $\phi = 2n\pi$

i. Path difference (δ) = $n\lambda$ Where n = 0, 1, 2, 3, 4……

C. Condition for destructive Interference : $\cos\frac{\phi}{2} = 0$ (or) $\phi = (2n-1)\pi$

Where n = 1, 2, 3…

6. The distance between two consecutive bright or consecutive dark fringes is known as fringe **width** β.

$\beta = \frac{D\lambda}{d}$. Where the separation between the two slits is d and the distance of the screen from the plane of the two slits be D.

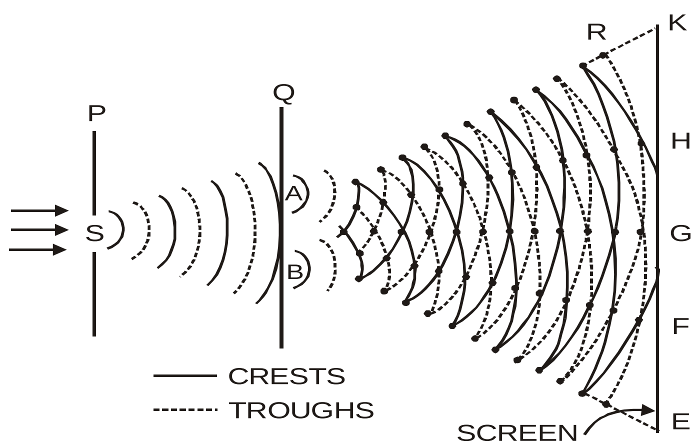
7. Angular fringe width (ω) : The ratio of fringe width to source screen distance is defined as angular fringe width (ω) i.e., $\omega = \frac{\beta}{D} = \frac{\lambda}{d}$.

LONG ANSWER QUESTIONS

1. Describe Young's double slit experiment and give the necessary theory to explain formation of dark and bright interference pattern.

(March 2006)

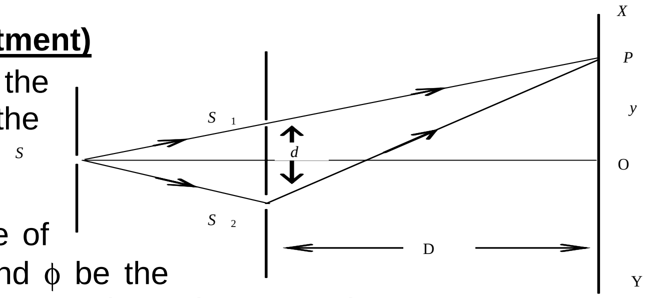
A. Young's double slit experiment: Consider a source of monochromatic light placed in front of a narrow slit S. The crests of the waves are represented by complete arcs and the troughs are represented by dotted arcs. A and B are identical narrow slits placed very close to the source slit S which behaves like coherent sources. At any particular instant, the displacement at any point is the resultant of the displacements produced by the two waves.



Constructive interference takes place at the points when the crest of one wave falls on the crest of the other or when the trough of one wave falls on the trough of the other. Destructive interference takes place when the crest of one wave falls on the trough of the other. As the total energy is redistributed, energy (intensity) at certain points is maximum and minimum at the other points. Thus alternately bright and dark bands are formed at equal distances.

Experiment: The experimental arrangement of Young's experiment to study the interference on the screen is shown. Let S be a narrow slit illuminated with a source of monochromatic light. S_1 and S_2 are the two narrows slits that are very close to each other and equidistant from S. The light waves from S arrive at S_1 & S_2 in the same phase and produce interference on the screen XY.

Explanation (Analytical Treatment)



Let the separation between the two slits S_1 and S_2 be d and the distance of the screen from the plane of the two slits be D.

Let ‘a’ be the amplitude of each wave from S_1 and S_2 and ϕ be the phase difference between them at the point P on the screen. The intensity of the individual wave is given by $I_0 = Ka^2$ Where k = constant of proportionality

$y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$

$$y = y_1 + y_2 = a \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= \sin \omega t [a_1 + a_2 \cos \phi] + a_2 \cos \omega t \sin \phi$$

Let $R \cos\theta = (a_1 + a_2 \cos \phi)$ & $R \sin \theta = a_2 \sin \phi$

$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$

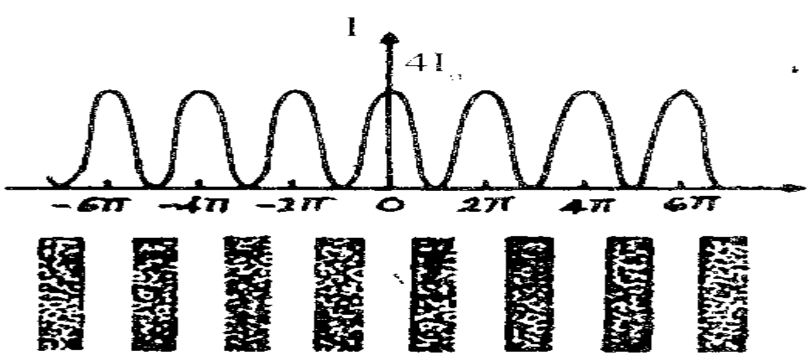
$y = R \sin (\omega t + \theta)$

$R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi$$

If $a_1 = a_2 = a$, then $R = 2a\cos\frac{\phi}{2}$

$$I = 4a^2 \cos^2 \frac{\phi}{2} \quad \text{Or} \quad I = 4I_0 \cos^2 \frac{\phi}{2}$$



Case – I Constructive Interference

$\cos\frac{\phi}{2} = \pm 1$ (or) $\frac{\phi}{2} = n\pi$ (or) $\phi = 2n\pi$

Path difference $(\delta) = n\lambda$

Where n = 0, 1, 2, 3, 4……

$\phi = 0, 2\pi, 4\pi, 6\pi\ldots$

n = 0 refers to the central light band.

$R_{max} = 2a$ and $I_{max} = 4a^2$

Case – II Destructive Interference

$\cos\frac{\phi}{2} = 0$ (or) $\frac{\phi}{2} = (2n-1)\frac{\pi}{2}$

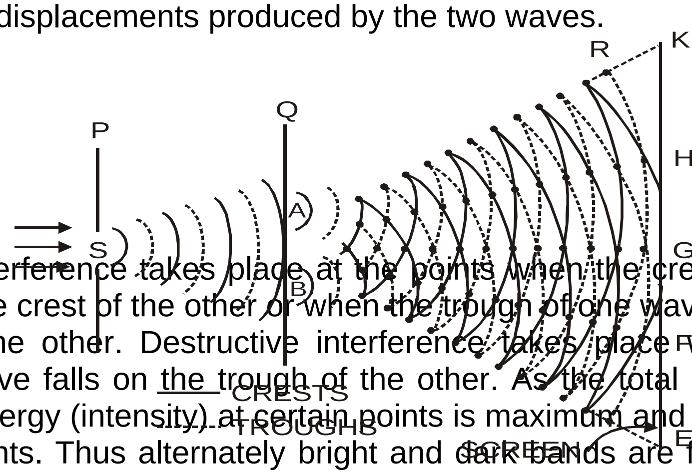
$\phi = (2n-1)\pi$ and $\delta = (2n-1)\frac{\lambda}{2}$

Where n = 1, 2, 3…

$\phi = \pi, 3\pi, 5\pi, \ldots..$

2. Describe Young's double slit experiment and derive an expression for the fringe width of the interference pattern.

A. **Young's double slit experiment:** Consider a source of monochromatic light placed in front of a narrow slit S. The crests of the waves are represented by complete arcs and the troughs are represented by dotted arcs. A and B are identical narrow slits placed very close to the source slit S which behaves like coherent sources.At any particular instant, the displacement at any point is the resultant of the displacements produced by the two waves.



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Experiment: The experimental arrangement of Young's experiment to study the interference on the screen is shown. Let S be a narrow slit illuminated with a source of monochromatic light.$S_1$and$S_2$are the two narrows slits that are very close to each other and equidistant from S. The light waves from S arrive at S_1 & S_2 in the same phase and produce interference on the screen XY.

Let the separation between the two slits S_1 and S_2 be d and the distance of the screen from the plane of the two slits be D.

Let 'a' be the amplitude of each wave from S_1 and S_2 and ϕ be the phase difference between them at the point P on the screen.The intensity of the individual wave is given by $I_0 = Ka^2$ Where k = constant of proportionality

Band Width:

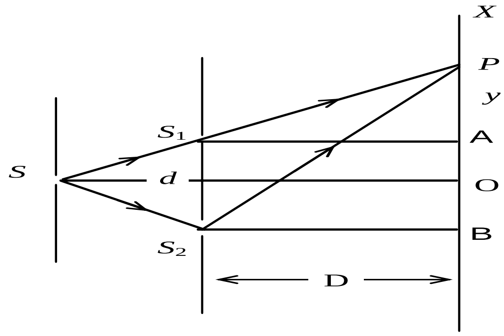
- Let x be the path difference between the waves reaching the point p from S_1 and S_2 path difference $x = S_2P - S_1P$.
- In the right angled ΔS_1AP,

$S_1p^2 = D^2 + \left(y - \frac{d}{2}\right)^2$

In the right angled ΔS_2BP,

$S_2p^2 = D^2 + \left(y + \frac{d}{2}\right)^2$

$(S_2p)^2 - (S_1p)^2 = \left[D^2 + \left(y + \frac{d}{2}\right)^2\right] = \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$



$$(S_2p - S_1p)(S_2p + S_1p) = 2yd$$

But $S_1p \approx S_2p \approx D$

$S_2p - S_1p = \frac{yd}{D}$

Path difference $x = \frac{yd}{D}$

<u>Case – I</u>

For bright band at P, $\frac{yd}{D} = n\lambda$ where n = 0, 1, 2, 3,….

For the nth band, $\frac{yd}{D} = n\lambda$ Or $y = \frac{n\lambda D}{d}$

For the (n +1) th band $\frac{yd}{D} = (n+1)\lambda$ Or $y = \frac{(n+1)\lambda D}{d}$

Band width = y ′ - y $\Rightarrow \beta = \frac{\lambda D}{d}$

<u>Case II</u>

For a dark band at P, $\frac{yd}{D} = (2n-1)\frac{\lambda}{2}$ where n = 1, 2, 3, 4….

For the nth band, $\frac{yd}{D} = (2n-1)\frac{\lambda}{2}$ (or) $y = \frac{(2n-1)}{2}\frac{\lambda D}{d}$

For the (n +1)th band $\frac{yd}{D} = [2n(n+1)-1]\frac{\lambda}{2}$ or $y' = \frac{(2n+1)}{2}\frac{\lambda D}{d}$ 1

Band width = y′ - y $\Rightarrow \beta = \frac{\lambda D}{d}$

Since the band width is independent of n, the interference bands have almost equal widths.

<u>SHORT ANSWER QUESTIONS</u>

1. **Distinguish between geometric and physical (wave) approximations of light.**

A: **In** geometric optics or ray optics the path light rays are represented by straight lines and hence the rectilinear propagation of light is considered.
In physical optics light rays are treated as waves and the phenomena like interference, diffraction and polarization are explained basing on wave theory.
In diffraction, light encroaches into the geometric shadow of an obstacle. If L is the distance between the obstacle and screen, b is the size of obstacle and λ is wavelength of light used, then

If $\frac{b^2}{l\lambda} << 1$ Fraunhofer diffraction is observed

If $\frac{b^2}{l\lambda} \approx 1$ Fresnel diffraction is observed

If $\frac{b^2}{l\lambda} >> 1$ the approximation of geometrical optics is applicable.

Hence for obstacles of very large dimensions compared with wavelength of light, it can be considered as waves traveling in the form of straight lines.

2. **Interference pattern cannot be obtained when two different sources of same wave length are used. Why?**

E. To observe interference pattern of light waves, the two sources should produce waves of same wavelength continuously and the waves must be coherent i.e. they should emit waves of same frequency which are in phase or which maintain a constant phase difference. Light coming from two different sources can never be coherent. Even though wavelengths are equal their amplitudes may not be equal. Since they are two different sources, they cannot maintain constant phase difference between the two waves. Hence interference can not be observed with two light waves coming from different sources.

3. **Explain why two waves of significantly different frequencies can not be coherent?**

A: In order to observe a sustained interference pattern one has to use two coherent light sources of same wavelength. A laser beam is an example of coherent light LASER-an acronym for light amplification by stimulated emission of radiation). Such coherent light sources can be obtained by illuminating a screen containing two narrow slits by a monochromatic light source. So the coherence can obtain only by one frequency. Due to the two waves of significantly different frequencies cannot be coherent.

3. **What are the applications of interference of light? (March2009)**

A. Applications of interference phenomenon :

Interference of light is used

i) to determine the wavelength of a monochromatic light and the difference between the wavelengths of two closely – spaced spectral lines.
ii) to determine the thickness of a thin transparent material.
iii) to determine the refractive index of a liquid or a gas.
iv) to test the flatness of surfaces
v) to test the reflectivity of the surfaces of lenses and prisms.

VERY SHORT ANSWER QUESTIONS

1. **How many times will the distance between adjacent interference bands increase on the screen in Young's experiment if a red light filter $\lambda = 6.5 \times 10^{-5}$ cm is used instead of green one with $\lambda = 5 \times 10^{-5}\, cm$?**

Sol: $\beta = \frac{\lambda D}{d} \Rightarrow \beta \alpha \lambda$

Where λ wavelength of light

D = distance to the screen from the coherent sources

d = distance between the coherent sources

From the above equation,

$$\therefore \frac{\beta_{red}}{\beta_{green}} = \frac{\lambda_{red}}{\lambda_{green}} = \frac{6.5 \times 10^{-5}}{5 \times 10^{-5}} = 1.3$$

$$\beta_{red} = (1.3)\,\beta_{green}$$

Hence the fringe width with red filter is 1.3 times the fringe width with green filter

2. **If we move from one bright fringe in a two slit interference pattern to the next one further out, how does the path difference ΔL change? Does it increase or decrease? If so, how much in terms of wavelength λ ?**

A: Path difference = $n\lambda$

Where n = 0, 1, 2, 3…………. And $\lambda \rightarrow$ is wave length

For the next bright fringe (n +1), the path difference = $(n+1)\lambda$

$\therefore$ Change in path difference $(n+1)\lambda - n\lambda = \lambda$

3. **How does the spacing between fringes in Young's double slit experiment change**

a) if the slit separation is increased ?

b) if the colour of the light is changed from red to blue?

A: $\beta = \frac{\lambda D}{d}$

Where λ wavelength of light

D = distance to the screen from the coherent sources

d = distance between the coherent sources

i) $\beta \propto \frac{1}{d}$ Hence if slit separation increases, spacing between the fringes decreases.

ii) $\beta \alpha \lambda$ Hence if colour of light changes from red to blue, wavelength decreases and spacing between the fringes decreases.

4. **If slits in Young's double slit experiment are illuminated with white light which colour-blue or red fringe will be closer to the central maxima?**

A: In Young's double slit experiment if the slits are illuminated with white light, blue fringe will be closer to the central fringe.

5. **Two waves of same frequency have amplitudes 1.00 and 2.00. They interfere at a point where their phase difference is 60.0^0.What is the resultant amplitude?**

A: $a_1 = 1$; $a_2 = 2$ and $\phi = 60^0$

$\therefore$ Resultant amplitude $a_{res} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{1 + 4 + 2(1)(2)\left(\frac{1}{2}\right)} = \sqrt{7}$

6. **Can Young's double slit experiment be conducted with sound? How can we carry out this experiment?**

A: Young's double slit experiment cannot be conducted with sound with narrow slits. For observing interference with sound slit width should be increased.

7. The maximum intensity at bright fringe is 4 times that of either wave. Does this violate energy conservation? If not, why?

A: In interference, there is no loss or gain of energy i.e., total energy remains constant. In interference, redistribution of energy takes place.

8. Can the head lights of a distant car produce interference pattern? If so, how it is observed? If not, why?

A: For a distant car, the head lights are broad. Interference cannot be obtained with two broad sources. A broad source of light is composed of a large number of narrow sources lying side by side. Each set of sources will produces its own interference pattern which will overlap and resulting no net interference pattern .

9. Why is it impossible to obtain interference fringes in a double slit experiment if the slit separation is less than the wavelength of light being used?

A: $\beta = \frac{\lambda D}{d}$, when the slit separation is less than the wavelength of the light used, i.e. if $d < \lambda$, then fringe width will be greater than the distance to the screen i.e., $\beta > D$. Hence fringe pattern will not be visible.

10. What change will occur in a Young's double slit experiment if the whole apparatus is immersed in water rather than air?

A: $\beta = \frac{\lambda D}{d}$. When Young's double slit experiment is conducted in water, wavelength (λ) decreases. Hence fringe width (β) decreases.

11. If Young's two slit interference experiment is done with white light what would be observed?

A: If white light is used as source in Young's double slit experiment, then coloured fringes of unequal width are formed. Fringe nearest to either side of the central white fringe is violet and the fringe farthest from the central white fringe is red.

12. Young's double slit experiment is known for interference. Is it also a diffraction experiment? If yes, why?

Sl: The light waves arriving at each slit are diffracted and the diffracted waves interfere and finally given an interference pattern. Hence in Young's double slit experiment both diffraction and interference of light are involved.

13. Is it possible to have coherence between light sources emitting light of different wavelengths?

A: When the two slits in Young's double slit experiment are illuminated by two independent sources, then interference bands are not obtained on the screen because, the sources cannot be coherent.

14. In the analysis of double slit experiment which approach Fraunhofer or Fresnel is suitable to be considered?

A: In Young's double slit experiment the wave front coming from a source is divided into two parts. This type of interference is called division of wave front. Hence Fresnel approach is considered.

15. If the slit separation d in Young's experiment is doubled how must the distance D, of the viewing screen be changed to maintain the same fringe spacing?

A: Slit separation (d) is doubled. $\beta = \frac{\lambda D}{d}$

For getting the same fringe width, the distance to the screen (D) should also be doubled.

16. Find the slit separation of a double slit arrangement that will produce interference fringes 0.01784 rad apart on a distance screen when sodium light of $\lambda = 589\,nm$ is used.

A: $\lambda = 589\,nm\ = 5890\times10^{-10}\,m$

Angular fringe width $W_a = \dfrac{\beta}{D} = 0.01784\,rad$; d = ?

$$\Rightarrow d = \frac{\lambda}{W_a} = \frac{5890\times10^{-10}}{0.01784} = 330156.9\times10^{-10}\,m$$

SOVLED PROBLEMS

1. Suppose that the young's double slit experiment the waves emerging from the slits S_1 and S_2 have unequal amplitudes a_1 and a_2. Derive the expression for resultant intensity (i) Discuss the cases where the phase difference is 0^0 and 90^0 and 180^0. (ii) Discuss about intensity when the waves from S_1 and S_2 are incoherent and coherent.

Sol: $y_1 = a_1 \sin\omega t$ and $y_2 = a_2\sin(\omega t + \phi)$

$y = y_1 + y_2 = a\sin\omega t + a_2\sin(\omega t+\phi)$

$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$

$= \sin \omega t\, [a_1 + a_2 \cos \phi] + a_2 \cos \omega t \sin \phi$

Let $R \cos\theta = (a_1 + a_2\cos \phi)$ & $R \sin \theta = a_2\sin \phi$

$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$

$y = R \sin (\omega t + \theta)$

$R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

If $\varphi = 0$ $\quad I = I_1 + I_2 + 2\sqrt{I_1I_2} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

If $\varphi = 90^0$ $\quad I = I_1 + I_2$

If $\varphi = 180^0$ $\quad I = I_1 + I_2 - 2\sqrt{I_1I_2} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

ii) The difference φ of incoherent waves varies continuously and can take any value with equal probability. Hence the time averaged value of $\cos\varphi$ equals zero. Square of resultant amplitude $R^2 = a_1^2 + a_2^2$. As $I \propto a^2 = (amplitude)^2$ we can conclude that intensity resulting from the superposition of incoherent waves equals the sum of the intensities produced by each of the waves individually.

$\therefore I = I_1 + I_2$

For coherent waves $\cos\varphi$ does not vary with time (may have different value at different points in space)

$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\varphi$

2. A beam of light consisting of two wavelength $6500\,\overset{0}{A}$ and $5200\,\overset{0}{A}$ is used to obtain interference fringes in a Young's double slit experiment. (i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength $6500\,\overset{0}{A}$. (ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Distance between the slits is 2mm, distance between the slits and the screen D = 120 cm.

Sol: i) The distance of the m^{th} bright fringe from the central maximum

$$y_m = \frac{m\lambda D}{d}; y_3 = \frac{3\lambda D}{d} = \frac{3\times(6500\times10^{-10})\times1.20}{2\times10^{-3}} = 1.17mm$$

ii) Let the n^{th} bright fringe of wavelength λ_n and m^{th} bright fringe of wavelength λ_m coincide at a distance y from the central maximum then $\lambda = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$

$$\therefore \frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{6500}{5200} = \frac{5}{4}$$

i.e., 5th bright fringe of wavelength $5200\overset{0}{A}$ coincides with the 4th bright fringe of wavelength $6500\overset{0}{A}$ $\therefore y_m = \frac{m\lambda_m D}{d} = \frac{5(5200\times10^{-10})\times1.20}{2\times10^{-3}} = 1.56\,mm$.

3. In an arrangement of double slit experiment the slits are separated by d = 0.250 cm. An interference pattern is formed on the screen D = 120 cm away from the slits which are illuminated by a coherent light of wavelength $\lambda = 600\,nm$. Calculate the distance x' above the central maximum for which the intensity on the screen is 75.0% of the maximum.

Sol: $\lambda = 600nm$; L = 120 cm; and d = 0.250cm

$$I = 4I_0\cos^2\left(\frac{\delta}{2}\right) \quad \Rightarrow\ 75 = 100\cos^2\left(\frac{\delta}{2}\right)$$

$$\Rightarrow\ \cos\frac{\delta}{2} = \frac{\sqrt{3}}{2} \quad \Rightarrow\ \delta = 60^0 \text{ or } \frac{\pi}{3} \text{ radian.}$$

Path difference $d\sin\theta = \frac{\lambda}{2\pi}$ (phase difference)

$$\therefore d\sin\theta = \frac{\lambda}{2\pi}\left(\frac{\pi}{3}\right) = \frac{\lambda}{6}.$$

As θ is very small, $\sin\theta \approx \tan\theta \;\; \frac{x}{D}$.

$$\therefore d.\frac{x}{D} = \frac{\lambda}{6} \quad \Rightarrow\ x = \frac{\lambda D}{6d} = 48\mu n.$$

4. In Young's double slit experiment a thin glass plate of $\mu = 1.5$ is placed in the path of one of interfering rays. This causes the central bright fringe to shift by five bright fringes (not counting the central one). The ray falls on the plate perpendicularly. The wavelength is 600 nm. What is the thickness of the plate?

Sol: **A**dditional difference in paths between interfering rays due to the presence of a glass plate $\Delta x = t(\mu - 1)$ where t is the thickness of the plate and μ the refractive index of the plate.

If the number of fringes shifted are m the additional path difference is $m\lambda.\ \therefore t(\mu-1) = m\lambda$.

$$\therefore t = \frac{m\lambda}{\mu - 1} = \frac{5\times600\times10^{-9}}{(1.5-1)} = 6\times10^{-6}\,m.$$

<u>UNSOVLED PROBLEMS</u>

1. Monochromatic green light of wavelength 550nm illuminates two parallel narrow slits $7.7\mu m$ apart. Calculate the angular deviation θ of third order (for m = 3) bright fringe a) in radian and b) in degree.

Sol: $\lambda = 550nm = 550\times10^{-9}\ m$

d = $7.7\mu m = 77\times10^{-7}\ m$; n = 3 ;

Angular deviation $\theta = \frac{n\lambda}{d} = \frac{3\times550\times10^{-9}}{77\times10^{-7}} = 0.216\,\text{radian}$

b) Angular deviation θ $= 0.216\times\frac{\pi}{180} = 12.4^0\ (nearly)$

2. A Young's interference experiment is performed with monochromatic light. The separation between the slit is 0.500mm, and the interference pattern on a screen 8.50m away shows the first side maximum at 3.40mm from the centre of the pattern. What is the wavelength?

Sol: d = 0.5mm $= 5\times10^{-4} m$; D = 3.5m

Distance of 1st maximum from centre of screen $y_1 = 3.4mm = 3\times10^{-4}\ m$

From $y_1 = \frac{D\lambda}{d} \Rightarrow \lambda = \frac{y_1 d}{D} = \frac{34\times10^{-4}\times5\times10^{-4}}{3.5} = = 4857A^0 (or) 486\,nm$

3. Young's double slit experiment is performed with 589 nm light with a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

Sol: $\lambda = 589\,nm = 589\times10^{-9}\ m$; D =2m

Distance of 10th minima from central maximum $y_{10} = 7.26mm = 726\times10^{-5}\ m$

$y_{10} = \frac{21D\lambda}{2d} \Rightarrow d = \frac{21D\lambda}{2\times y_{10}} \Rightarrow d = \frac{21\times2\times589\times10^{-9}}{2\times726\times10^{-5}} = 1.704mm$

4. In a double slit arrangement, the slits are separated by a distance equal to 100 times the wavelength of the light passing through the slits. (a) What is the angular separation in radian between the central maximum and an adjacent maximum? (b) What is the distance between these maxima on a screen 50.0 cm from the slits?

Sol: d = 100λ

a) Angular separation $= \theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{\lambda}{100\lambda} = \frac{1}{100} = 0.01\text{radian}$

b) D = 50cm $= \frac{1}{2}m$

Distance between maxima $= \beta = \frac{D\lambda}{d} = \frac{1}{2}\times\frac{1}{100}$ = 0.05 radian

5. In an young's double slit experiment, light with $\lambda = 546\,nm$ is used. The first order maximum is found to be at 2.5 (a) what is the slit separation? (b) At what angle will the second order maximum occur?

Sol: $\lambda = 546\,nm = 546\times10^{-9}\ m$

Angular separation of 1st maxima $= \theta = 2.5^0 = \frac{2.5\times\pi}{180}rad$

a) Separation between slits = d

But, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

$$\Rightarrow d = \frac{\lambda}{\theta} = \frac{546\times10^{-9}}{\frac{2.5}{180}\times\pi} = \frac{98280}{7.85}\times10^{-9} \quad = 1.25\times10^{-3}\,cm$$

b) Angular separation for 2nd maximum $\quad 2\theta = 2\times2.5 = 5^0$

6. In a double slit experiment, the slit separation is 0.20cm and the slit to screen distance is 100cm. Find the positions of the first three minima, if wavelength of the source is 500 mm.

Sol: d = 0.2 cm $= 2\times10^{-3}\,m$; D = 100 cm = 1m ; $\lambda = 500\,nm \; = 5\times10^{-7}\,m$

Distance of 1st minimum $= y_1 = \frac{D\;x}{2d} = \frac{1\times5\times10^{-7}}{2\times2\times10^{-3}} \; = \pm0.025\,cm$

Distance of 2nd minimum $= y_2 = 3\left(\frac{D\lambda}{2d}\right) = 3(0.0125) = \pm0.0375\,cm$

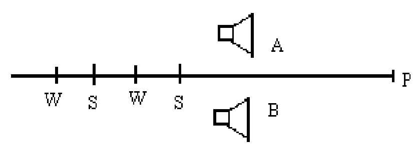
Distance of 3rd minimum $= y_3 = 5\left(\frac{D\lambda}{2d}\right) = 5(0.0125) = \pm0.0625\,cm$

7. In Young's double slit experiment, blue-green light of wavelength 500 nm is used. The slits are 1.20mm apart and the viewing screen is 5.0m away from the slits. What is the fringe width?

Sol: $\lambda = 500\,nm \; = 5\times10^{-7}\,m$; d = 1.2 nm $= 12\times10^{-4}\,m$; D = 5.m

$$\beta = \frac{D\lambda}{d} = \frac{5.4\times5\times10^{-7}}{12\times10^{-4}} = 2.25\times10^{-3}\,m$$

8. Two loud speakers A and B are situated at the same point vertically one above the other. As one of the speakers is moved away from the other along a perpendicular straight line an observer hears a strong tone when the speaker is at S, a weak tone at W. How far apart are W and S if the observer is far away at P?



Sol: The two positions of the speakers correspond to maximum and minimum. The path difference for the sound wave between the two positions should be $\frac{\lambda}{2}$, where λ is the wavelength of sound wave.

9. Light from a mercury arc lamp is passed through a filter that blocks every thing except for one spectrum line in the green region of the spectrum. It then falls on two slits separated by 0.6nm. In the resulting interference pattern on a screen 2.5mm away, adjacent bright fringes are separated by 2.27mm. What is the wavelength?

Sol: d = 0.6 mm = $6\times10^{-4}m$; D= 2.5m

Fringe width = $= 2.27\,mm = 227\times10^{-5}\,m$

$$\beta = \frac{D\lambda}{d} \Rightarrow \lambda = \frac{\beta d}{D} = \frac{227\times10^{-5}\times6\times10^{-4}}{2.5} = 5.8 \text{ nm}$$

10. Two wavelength λ_1 and λ_2 are used in double slit experiment. If one is 430 nm, what value must the other have for the fourth order bright fringe of one to fall the sixth order bright fringe of the other?

Sol: $\lambda_1 = 430\,nm$; $n_1 = 6$; $n_2 = 4$

$$n_1\lambda_1 = n_2\lambda_2 \quad \Rightarrow \lambda_2 = \frac{n_1\lambda_1}{n^2} \quad = \frac{6\times430}{4} = 645\,nm$$

11. A flake of glass of index of refraction 1.6 is placed over of the openings of double slit apparatus. There is a displacement of the interference pattern

through eight successive maxima toward the side where the flake was placed. If the wavelength of the light used is $\lambda = 540\,nm$, calculate the thickness of the flake?

Sol: $\mu = 1.6$; n = 8 ; $\lambda = 50\,nm = 5\times10^{-8}\,m$

$$n\lambda = (\mu - 1)t \Rightarrow t = \frac{n\lambda}{\mu - 1} = \frac{8\times5\times10^{-8}}{1.6-1} = 7.2\times10^{-6}\,m$$

12. In double slit experiment, a light of wavelength $\lambda = 600\,nm$ is used. When a film of material $3.6\times10^{-3}\,cm$ thick was placed over one of the slits, the fringe pattern was displaced by a distance equal to 30 times that between two adjacent fringes. What is the refractive index of the material?

Sol: $\lambda = 600\,nm = 6\times10^{-7}\,m$; t = $3.6\times10^{-3}\,cm = 36\times10^{-6}\,m$; n = 30

$$n\lambda = (\mu - 1)t \quad \Rightarrow 30\times6\times10^{-7} = (\mu - 1)36\times10^{-6} \quad \Rightarrow \mu = 1.5$$

ASSESS YOURSELF

1. Can we say that light sources emitting waves of different wavelength coherent?

A. Light sources emitting waves of different wavelength are not coherent.

2. Suppose that in double –slit experiment, filters are placed over the slits so that $\lambda = 436nm$ (blue) light goes through one slit and $\lambda = 546nm$ (green) light goes through the other. Will it be possible to see an interference pattern on the screen?

A. Since the waves are not coherent it will not be possible to see an interference pattern on the screen.

3. How does the fringe width of interference pattern change if red and green colour lights are used?

A. $\beta = \frac{\lambda D}{d}$. Fringe width is more for red than for green because more wave length for red.

4.. Consider a dark fringe in double-slit experiment at which no light energy is arriving. Waves from both slits travel to this point, but the waves cancel. Where does the energy go?

A. According to the law of conservation energy, in the interference phenomenon the energy is redistributed. The average energy which is absent in the site of dark fringe is adjusted at the site of bright fringe.

6. What changes you observe in the interference pattern when a piece of red glass and another of blue glass are placed over the slits, one after another separately, between the plane of the two slits and the screen in young's double slit experiment arrangement?

A. $\beta = \frac{\lambda D}{d}$. Fringe width is more for red than for blue because more wave length for red.