RAY OPTICS

4. EYE-PIECES

POINTS TO REMEMBER

1. The defects in the formation of image by lenses are called aberrations. These are of two types 1. Spherical aberration and 2. Chromatic aberration.
2. **Spherical aberration:** The failure of the lens to form a point image of a point object is called spherical aberration. This defect is due to the fact that different annular zones of the lens have different focal lengths. Spherical aberration is due to more deviation of marginal rays than in the paraxial rays.

The minimize chromatic aberration by lesn combination, the separation by lens combination, the separation between lenses must be $d = \frac{f_1 + f_2}{2}$

3. **Chromatic aberration:** The inability of a lens to focus all the colours to a point is called chromatic aberration. This is due to the variation of refractive index of the lens due to different for different colours of light

   Chromatic aberration can be eliminated with a lens combination of $\frac{\omega_1}{\omega_2} = \frac{f_1}{f_2}$.

   Nagative sign shows that one lens in combination must be –ve and ω_1 *and* ω_2 are dispersive power of lenses.
4. **Ramsden's eyepiece:** It consists of two plano - convex lenses whose convex surfaces face each other.The lens nearer to the eye is called eye lens and the lens towards the object is called field lens.If *f* is the focal length of the eye lens, the focal length of the field lens is also *f* and the distance between the lenses is 2*f / 3*. The field lens increases the field of view and the eye lens produces the magnification.

   Equivalent focal length of the eyepiece is , $F = \frac{3f}{4}$

   The distance of the image formed by the objective is at $\frac{f}{4}$ from the field lens and $\frac{11f}{12}$ from the eye lens. At this position cross-wires are arranged.
5. **Huygen's Eyepiece:** It consists of two plano - convex lenses whose convex surfaces face the object.The lens nearer to the eye is called eye lens and the lens towards the object is called field lens.If *f* is the focal length of the eye lens, the focal length of the field lens is 3*f* and the distance between the lenses is 2*f*. The field lens increases the field of view and the eye lens produces the magnification.

   Equivalent focal length of the eye piece is , $F = \frac{3f}{2}$

LONG ANSWER QUESTIONS

1. **What is spherical aberration? Explain the terms: longitudinal spherical aberration, circle of least confusion and lateral spherical aberration. State the different methods to reduce the defect. Is the defect due to the lens?**

A. <u>Spherical Aberration</u>: The inability of a lens to form a point image of a point object is called spherical aberration (or)

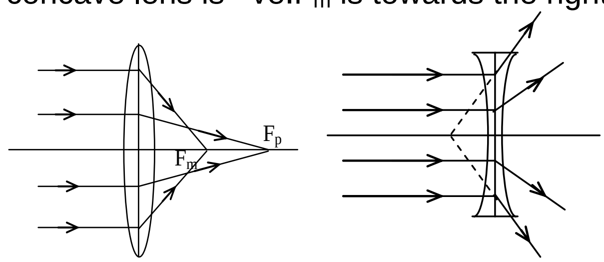
The blurring of image formed by lens due to the focusing of marginal and paraxial rays at different points is known as Spherical aberration

<u>Explanation</u>: If aperture of the lens is large, then the rays of light incident on the lens can be classified into two types

1) Paraxial rays : The rays of light which incident near the principal axis of lens are called paraxial rays

2) Marginal rays : The rays of light which incident near to periphery of the lens are called marginal rays

Paraxial rays form the image at a longer distance than that of marginal rays from the lens. If a screen is placed perpendicular to the axis, a circular patch is formed on the screen .The image I_1 is in the form of a circle bright at the centre and the intensity gradually decreases towards the edge.The image I_2 is a circle faint at the centre and the outer edge is bright.In between I_1 and I_2 there is a circular patch I with almost uniform intensity.This is called the <u>circle of least confusion</u>.It has the nearest approach to a point image to the point object.The radius of the circle of least confusion is a measure of <u>lateral or transverse spherical aberration</u>.Spherical aberration produced by a convex lens is positive.For a convex lens F_m is towards the left of F_P.Spherical aberration for a concave lens is –ve.F_m is towards the right of F_p.



The distance I_m I_p is called <u>longitudinal spherical aberration.</u>

<u>Reducing Methods</u>

1. Spherical aberration can be minimized by using stops which reduce the effective lens aperture. This is used to stop mainly the marginal rays

2. By using the two plano convex lenses separated by a distance equal to the difference in their focal length $I_m\ and\ (d = f_1 \quad f_2)$

3. A crossed lens in which $\left(\frac{R_1}{R_2}\right) = \left(\frac{-1}{6}\right)$ should be used to minimize the longitudinal spherical aberration. Here R_1, R_2 are the radii of curvature of two surfaces of convex lens.

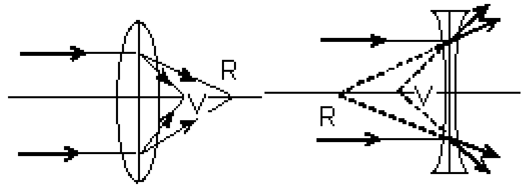
4. Plano convex lens has more advantages compared to crossed lens in minimizing the spherical aberration. The curvature surface must face the incident beam.

5. Since spherical aberration is positive for convex and negative for concave lenses, a suitable combination must be chosen to minimize the aberration.

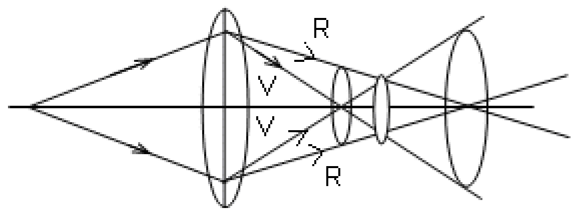
2. What is chromatic aberration? How does this defect in image formation arise? Explain the terms : longitudinal chromatic aberration, circle of least confusion and lateral chromatic aberration. State the different methods to reduce the defect.

A. **<u>Chromatic Aberration</u>**

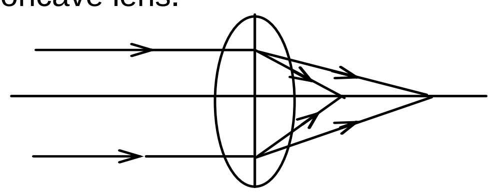
The inability of a lens to focus all the colours to a point is called chromatic aberration.If a parallel beam of white light is incident on a convex lens, violet rays are brought to focus at F_v and red rays are brought to focus at F_R as shown in the diagram.



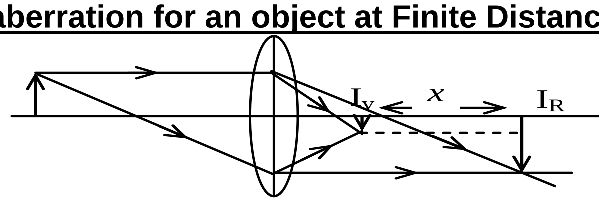
When white light rays from a point object are incident on a convex lens, a coloured image spread along the axis from I_1 to I_2 is formed on the screen. If a screen is placed perpendicular to the principle axis, the image I_1 is in the form of a circle with red at the centre and periphery inlet.If the screen is at I_2, the image is circular with violet at the centre and red at the periphery, other colours being in between.A circular path I with almost uniform intensity of different colurs is <u>called circle of least confusion</u>.It has the nearest approach to a point image of a point object.



For an extended object the images formed by different colours of different sizes are shown.The distance I_vI_p is called <u>axial or longitudinal chromatic aberration</u>.The difference in the sizes of the images I_v and I_p is called <u>lateral or transverse chromatic aberration.Chromatic aberration</u> is +ve for convex lens and –ve for a concave lens.



<u>Longitudinal chromatic aberration for an object at Finite Distance.</u>



<u>Reducing Methods</u>

1. When two or more lenses are combined together in such a way, the combination is free from chromatic aberration; such combination is called achromatic combination of lenses.
2. The condition for a chromatic for two thin lenses in contact is given by

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

One of the two lenses should be convex and the other concave. In general convex lens is made of crown glass and concave is made of flint glass.The materials of the lenses must be different.

If the materials are same, i.e., if $\omega_1 = \omega_2$ (dispersive powers), then $\frac{1}{f_1} + \frac{1}{f_2} = 0$.

Hence the combination behaves like a glass plate.This combination is achromatic doublet.

3. Chromatic aberration can also be eliminated by using two plano convex lenses of focal lengths f_1, f_2 separated by a distance d given by

$d = \frac{f_1 + f_2}{2}$ (For same material) $d = \frac{\omega_1 f_2 + \omega_2 f_2}{\omega_1 + \omega_2}$ (For different materials)

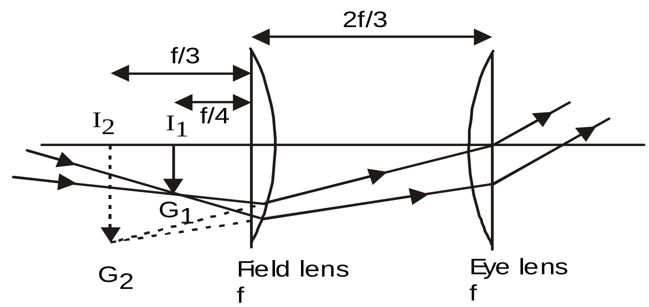
3. Explain the construction and working of a Ramsden's eyepiece with a neat ray diagram. Can cross- wires be used with such an eyepiece? If so, where?(March2011, March2009))

A. **Construction**

It consists of two plano - convex lenses whose convex surfaces facing each other.

The lens nearer to the eye is called eye lens and the lens towards the object is called field lens.If f is the focal length of the eye lens, the focal length of the field lens is also f and the distance between the lenses is $2f/3$. The field lens increases the field of view and the eye lens produces the magnification.

Working



Equivalent focal length of the eyepiece is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{f} + \frac{1}{f} - \frac{2f}{3ff}$$

$$\frac{1}{F} = \frac{4}{3f} \quad \text{Or} \quad F = \frac{3f}{4}$$

- The distance of the equivalent lens from the field lens is given by

$$x = \frac{Fd}{f_e} = \frac{3f}{4} \times \frac{2f}{3} \times \frac{1}{f} \quad \Rightarrow x = \frac{f}{2}$$

- Distance of the equivalent lens from the eye lens is $\frac{f}{6}$
- In order to form the final image at infinity, the image formed by the objective must be at the focus of the equivalent lens, i.e., the distance of the image formed by the objective is at $\frac{f}{4}$ from the field lens and $\frac{11f}{12}$ from the eye lens.At this position cross-wires are arranged.
- The distance of the image formed by the field lens is given by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{4}{f} + \frac{1}{v} \Rightarrow v = \frac{-f}{3}$$

- This image acts like an object for the eye lens.

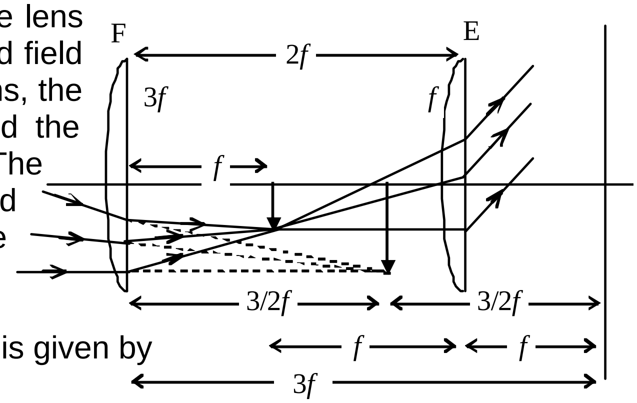
- For the eye lens, the above image is at its focus. Hence the final image is formed at infinity. This eyepiece is known as a positive eyepiece since the image formed due to the objective is in front of the field lens.
- Due to the presence of cross wires this eyepiece is used for taking measurements in physical science laboratories.
- This eyepiece does not satisfy the condition for minimum spherical and chromatic aberrations.
- The spherical aberrations are minimized since the deviation produced is shared by all the four surfaces.
- Chromatic aberration is also reduced.

4. Explain the construction and working of a Huygens eyepiece with a neat ray diagram. Compare it with Ramsden's eyepiece.(May2009)

A. **Construction**

It consists of two plano - convex lenses whose convex surfaces face the object.

The lens nearer to the eye is called eye lens and the lens towards the object is called field lens.If *f* is the focal length of the eye lens, the focal length of the field lens is 3*f* and the distance between the lenses is 2*f*. The field lens increases the field of view and the eye lens produces the magnification.



Working

Equivalent focal length of the eye piece is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} = \frac{1}{3f}+\frac{1}{f}-\frac{2f}{3f\times f}$$

$$\frac{1}{F}=\frac{2}{3f} \text{ Or } F=\frac{3f}{2}$$

Distance of the equivalent lens from the field lens

$$x=\frac{Fd}{fe}=\frac{3f}{2}\times\frac{2f}{f} \quad \Rightarrow x=3f$$

- Distance of the equivalent lens from the eye lens is *f*.
- In order to form the final image at infinity the image formed by the objective must be at the focus of the equivalent lens, i.e., the distance of the image formed by the objective is at $\frac{f}{2}$ from the eye lens and $\frac{3f}{2}$ from the field lens.
- Since the image due to the objective is in between the lens cross wires cannot be arranged.
- This eyepiece is called a –ve eyepiece.
- The distance of the image formed by the field lens is given by

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \text{ Or } \frac{1}{3f}=\frac{-2}{3f}+\frac{1}{v} \text{ Or } v=f$$

Since this image is at the focus of the eye lens, the final image is formed at infinity.

- This eyepiece is used only for observation purposes.

- This minimizes the spherical and chromatic aberrations.

Comparision of Huygen's and Ramsden's Eyepieces(May2009)

Huygen's eyepiece	Ramsden's eyepiece
1. The focal length of th field lens is 3f and where as the focal length of eye lens is f 2. The distance between the field and eye lens is 2f 3. The image due to objective is formed at a distance of $\frac{3f}{2}$ 4. It reduces the spherical and chromatic aberrations 5. It cannot be used for measurements	1. The focal length of the field lens and eye lens is same 2. The distance between the field and eye lens is $\frac{2f}{3}$ 3. The image due to objective is formed at a distance fo $\frac{f}{4}$ 4. It reduces the spherical aberration 5. It can be used for measurement.

SHORT ANSWER QUESTIONS

1. What are lens aberrations? Give reasons for them.

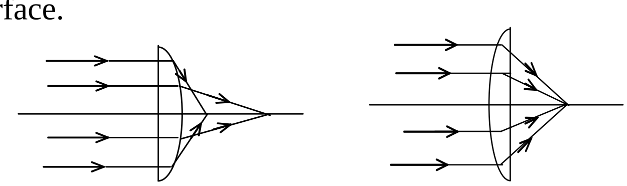
A. The defects in the formation of image by lenses are called aberrations. These are of two types 1. Spherical aberration and 2. Chromatic aberration.

1. Spherical aberration: The failure of the lens to form a point image of a point object is called spherical aberration. This defect is due to the fact that different annular zones of the lens have different focal lengths. Spherical aberration is due to more deviation of marginal rays than in the paraxial rays.

2. Chromatic aberration: The inability of a lens to focus all the colours to a point is called chromatic aberration. This is due to the variation of refractive index of the lens due to different for different colours of light

2. When a plano-convex lens is used to reduce the spherical aberration, the convex surface should face the incident rays. Why?

A. Spherical aberration is due to more deviation of marginal rays than in the paraxial rays. If the plane surface faces the incident beam, the deviation is produced only at the spherical surface.



If the curved surface faces the incident beam, the deviation will be at two faces.

3. Find the effective focal lengths of i) Ramsden's eyepiece and ii) Huygen's eyepiece.

A. For Rams den's eye piece, $\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} = \frac{1}{f} + \frac{1}{f} - \frac{2f}{3ff} = \frac{1}{f} + \frac{1}{f} - \frac{2}{3f}$

$$\therefore f_e = \frac{3f}{4}$$

For Huygen's Eyepiece, $\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3ff}$

$$\therefore f_e = \frac{3f}{2}$$

Where f_e is the effective focal length

f_1 is the focal length of field lens and f_2 is the focal length of eye lens.

VERY SHORT ANSWER QUESTIONS:

1. What are marginal and paraxial rays? How do they effect the image formation in lenses?

A. The rays that are near to the principal axis of the lens are called paraxial rays. The rays that are near to the periphery of the lens are called marginal or peripheral rays. Due to spherical aberration in lenses marginal rays are focused nearer to the optic centre of the lens than paraxial rays.

2. Can we minimize spherical and chromatic aberrations simultaneously? If so how?

A. When two lenses of If f_1 *and* f_2 are the focal lengths of the plano – convex lenses separated by a distance'd'

To minimize the spherical aberration, $d = f_1 - f_2$

To minimize the chromatic aberration, $d = \frac{f_1 + f_2}{2}$

3. What are the advantages of Ramsden's eyepiece?

A. 1) It is used for measurement purpose due to the presence of cross wires.
2) Its field of view is high.

4. What are the demerits of Huygen's eyepiece?

A. 1) It cannot be used for measurement purpose. 2) It cannot be used to examine directly an object or real image formed by the objective. 3) It field of view is less.

5. What are the disadvantages of a single converging lens used as eyepiece?

A. A single converging lens in an eyepiece,
1. The field of view is reduced and intensity is not uniform
2. Spherical and chromatic aberrations may arise.

6. State the functions of an eye-piece.

A. An eye-piece increases the field of view and makes the intensity uniform. It minimizes spherical and chromatic aberrations.

SOLVED PROBLEMS

1. An achromatic doublet consists of two lenses whose materials have dispersive power 0.018 and 0.027. Calculate the focal lengths of two lenses if the combination is to be a converging lens of focal length 150 cm.

A. Let f_1 *adn* f_2 be the focal lengths of the two lenses. When these lenses are kept in contact the equivalent focal length of the combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{150} = \frac{1}{f_1} + \frac{1}{f_2}$$

But $\omega_1 = 0.018 \, and \, \omega_2 = 0.027$. The condition for an achromatic doublet is given by

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{0.018}{f_1} + \frac{0.027}{f_2} = 0 \Rightarrow f_1 = 50cm \quad and \quad f_2 = 75cm$$

UNSOLVED PROBLEMS :

1. Two plano-convex lenses are separated by a distance of 2 cm to minimize spherical aberration. If the equivalent focal length of the combination is 6 cm find the focal lengths of the lenses.

A. Distance between lenses = d = 2 cm

Foal length of combination = F = 6 cm

$d = f_1 - f_2 = 2$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{6} = \frac{f_1 + f_2}{f_1 f_2} - \frac{f_1 - f_2}{f_1 f_2} \Rightarrow \frac{1}{6} = \frac{2 f_2}{f_1 f_2}$$

$\therefore \quad f_1 = 12cm \quad and \quad f_2 = 10cm$

2. A convex lens of focal length 10 cm is combined with a concave lens of focal length 20 cm to form an achromatic combination. The convex lens is made of a material of dispersive power 0.016. Find the dispersive power of the material used for concave lens.

A. $f_1 = +10$ cm (for convex lens) and $f_2 = -20$ cm (for concave lens)

$\omega_1 = 0.016$ (Dispersive power of convex lens)

For achromatic combination

$$\frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} \Rightarrow \frac{0.016}{w^2} = \frac{-(10)}{-20} = \frac{1}{2}$$

$\omega_2 = 0.016 \times 2 = 0.032$

3. An achromatic doublet is a converging lens of equivalent focal length 100 cm. If the dispersive powers of the material of the lenses are in the ratio 1:2, find the focal lengths of the two lenses.

A. Focal length = F = 100 cm

$$\frac{\omega_1}{\omega_2} = \frac{1}{2}$$

But, $\frac{w_1}{w_2} = \frac{-f_1}{f_2} = \frac{1}{2} \Rightarrow f_2 = -2 f_1$

From $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{100} = \frac{1}{f_2} - \frac{1}{2 f_1} = \frac{1}{2 f_1}$

$$\Rightarrow f_1 = \frac{100}{2} = 50cm$$

$f_2 = -2 f_1 = -2(50) = -100cm$

ASSESS YOURSELF

1. Can mirrors give rise to spherical and chromatic aberration?

A. Mirrors can show spherical aberration.

2. Do you think that the aberrations are due to defects in lens making?

A. No. These are due to the approximations in deriving the lens formulae.

3. If two Plano-convex lenses are used to minimize both spherical and chromatic aberration, what are their focal lengths and the separation between the lenses?

A. $f_1 = 3f \;\; and \;\; f_2 = f \;\; and$ separation between the lenses d = 2f

4. What is the function of field lens in an eyepiece?

A. It increases the field of view.

5. What is the function of eye lens in an eyepiece?

A. It gives the magnification.

6. What is the distance of the image due to the objective from the eye lens of Ramsden's eyepiece?

A. $\frac{11f}{12}$

7. What is the distance of the cross- wires from the field lens of Ramsden's eyepiece?

A. $\frac{f}{4}$