RAY OPTICS
2.REFRACTION
POINTS TO REMEMBER

1. **Refraction:** The bending of light as it passes obliquely from one transparent medium into another is called refraction of light.

2. In refraction. Light rays will travel from one medium to another medium. It follows the relation $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

3. From Snell's law ${}_1\mu^2 = \frac{\mu_2}{\mu_1} = \frac{\sin\theta_1}{\sin\theta_2}$ $for\ air\ \mu_1 = 1$

4. $\mu_{medium} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin i}{\sin r}$

5. Absolute refractive index of a medium, $\mu = \frac{c}{\upsilon}$

6. Relative refractive index of second medium w.r.t first medium,
$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\upsilon_1}{\upsilon_2} = \frac{\lambda_1}{\lambda_2}$$

7. **Total Internal Reflection: (**1)The light ray must travel from denser to rarer medium.(2)The angle of incidence in the denser medium must be greater than the critical angle.

8. If C is the critical angle for a denser medium of absolute refractive index μ separated from vacuum or air then $\mu = \frac{1}{\sin C}$

9. If a denser medium of refractive index μ_2 is separated from a rarer medium of refractive index μ_1 $then \frac{\mu_2}{\mu_1} = \frac{1}{\sin C} = \frac{\upsilon_1}{\upsilon_2}$ where $\upsilon_1\ and\ \upsilon_2$ are the velocities of light in the rarer and denser media respectively.

10. Optical fibre works on the principle of Total Internal Reflection.

11. **Prism:** $\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

12. For a thin prism, $\delta = A(\mu - 1)$

13. The angle of deviation of a prism depends on a) Angle of incidence b) Angle of the prism and c) Refractive index of the material of the prism.

14. **Lens Maker's Formula**
$$\frac{1}{f_{med}} = \left(\frac{\mu_l}{\mu_m} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
R_1, R_2 – Radi of curvatures of the curved surfaces.
$$\frac{1}{f_{air}} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$ This is generally used for paraxial rays.

15. **Focal Power:** It is the reciprocal of focal length expressed in metres.

Focal power (P) = $\frac{1}{f_{(\text{meter})}} = \frac{100}{f_{(cm)}}$ Units: Dioptre (D)

LONG ANSWER QUESTIONS

1. Construct the reflected wave front when a plane wave front is incident on a plane reflecting surface. Hence explain the laws of reflection. Why does an expanding spherical wave continue to expand out ward from the source?

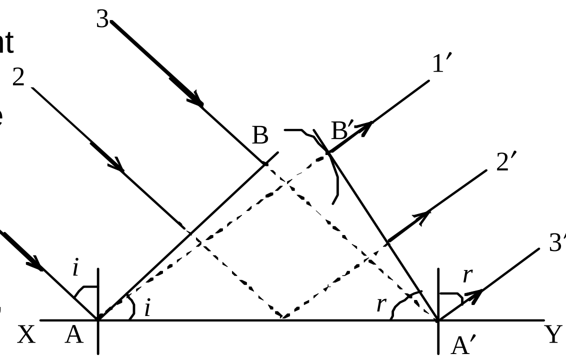
A. **Laws of reflection on the basis of wave theory laws of reflection**

Laws:

1. The angle of incidence and the angle of reflection are equal.

$\angle i = \angle r$

2. The incident light ray, the reflected light ray and the normal drawn to the surface at the point of incidence lie in a single plane.



Explanation: Consider a plane wave front AB incident on a reflecting surface XY at an angle of incidence *i* as shown. According Huygens's principle, every point on the wave front is a source of secondary wavelets. Let the secondary wavelets from B strike the surface XY at A′ in a time *t* so that BA′ = *ct*. During this time interval the secondary wavelets from A reach the point B′. Hence the new wave front of the reflected waves is A′B′. The rays 1, 2, 3 on the incident wave front will be now 1′, 2′, 3′.

In the right angle triangles ABA′ and AB′A′, AA′ is common.

BA′ = AB′ = *ct*

∠ABA′ = ∠AB′A′

Hence these two triangles are called congruent triangles. Therefore ∠BAA′=∠AA′B′, i.e.,

∠*i* = ∠*r*. where *i* and *r* are the angles of incidence and reflection respectively.

This proves the first law of reflection.

Also the incident wave front AB, the reflecting surface XY and the reflected wave front A′B′ are all perpendicular to the plane of the paper. Hence the incident ray normal to the reflecting surface and the reflected ray lie in a single plane.

This proves the second law of reflection

2. Deduce the laws of refraction on the basis of Huygens' wave theory. Why the wavelength of a light wave smaller in a denser medium than in a rarer medium?

A. **Laws of refraction on the Basis of Wave theory**

Laws:

1) The incident light ray, refracted light ray and the normal drawn to the refracting surface at the point of incidence lie in a single plane.

2) The refractive index of the second medium into which the light ray is refracted is given by $\mu = \frac{\sin i}{\sin r} = \frac{V_1}{V_2}$

Where *i* and *r* are the angles of incidence and refraction. V_1 and V_2 are the velocities of light in the rarer and denser media respectively.

Explanation

Consider a case where refraction takes place from a rarer medium to a denser medium.

If V_1 and V_2 are the velocities of light in rarer and denser media respectively then

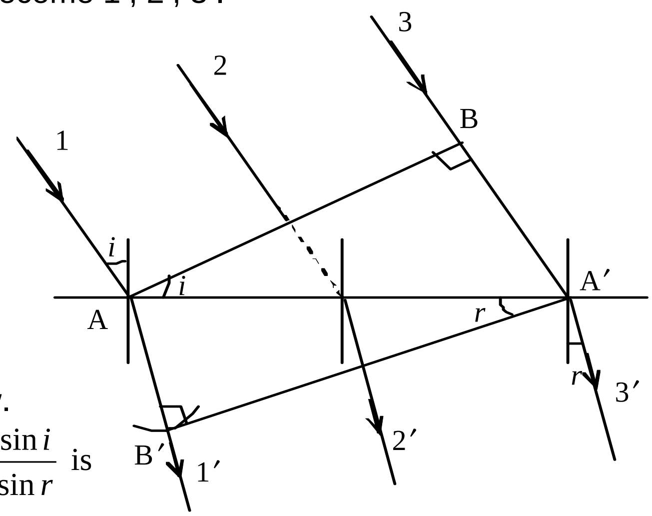
$$V_1 > V_2$$

Consider a plane wave front AB incident on a plane transparent surface XY at an angle of incidence i as shown.

The surface XY separates a denser medium of refractive index μ from a rarer medium.

According to Huygens's principle, every point on the wave front AB is a source of secondary wavelets.Let the secondary wavelets from B reach XY at A′ in a time *t* so that BA′ = $V_1 t$. During this time interval, the wavelets emitted from A travel a distance $V_2 t$.

The new wave front will be A′B′ which is the refracted wave front.The incident light rays 1, 2, 3 will now become 1′, 2′, 3′.



In the right angle triangle ABA′,

$$\sin i = \frac{A'B}{AA'} = \frac{V_1 t}{AA'}$$

In the right angle triangle AB′A′ =

$$\sin r = \frac{AB'}{AA'} = \frac{V_2 t}{AA'}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \mu$$

which proves the Snell's law.

Experimentally it is proved that $\frac{\sin i}{\sin r}$ is greater than one

$$\therefore \frac{V_1}{V_2} > 1 \Rightarrow V_1 > V_2$$

Hence according to the Huygen's wave theory of light, light travels very fast in the rarer medium and very slow in the denser medium. Hence the wavelength of a light wave smaller in a denser medium than in a rarer medium.

Also the reflected light ray, refracted light ray and normal drawn to the refracting surface at the point of incidence lie in the same plane. This is the second law of refraction.

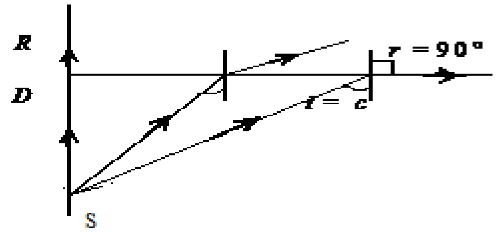
SHORT ANSWER QUESTIONS

1. A beam of white light passing through a hollow prism gives no spectrum Explain?

A. Spectrum is the combination of different coloured light rays. It is produced due to different deviations for different colours.As the hollow prism contains air inside it, the deviation is same for all colours. Hence spectrum is not produced when a white light passes through the hollow prism.

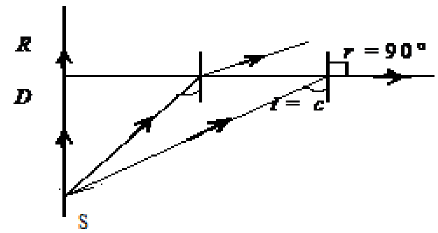
2. Define critical angle and explain it with a neat ray diagram.

A. Critical angle: When a light ray travels from a denser medium to a rarer medium, the angle of incidence in the denser medium for which the angle of refraction in rarer medium becomes 90^0 is defined as the critical angle of the medium.



3. Deduce the relation between critical angle and refractive index.

A. When a light ray travels from a denser medium to a rarer medium, the angle of incidence in the denser medium for which the angle of refraction in rarer medium becomes 90^0 is defined as the critical angle of the medium.



According to Snell's law, refractive index of denser medium with respect to the rarer medium is given by

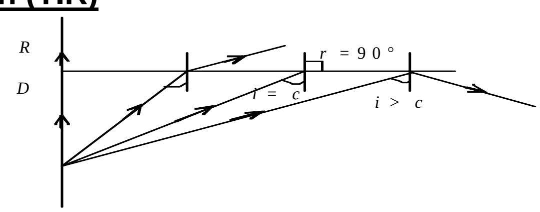
Refractive index, $_D\mu_R = \frac{\sin C}{\sin 90^0} \Rightarrow {}_D\mu_R = \frac{1}{\sin C}$

4. What is total internal reflection? State the conditions to be fulfilled for total internal reflection.

A. The phenomena of reflection of light when light traveling in a denser medium strikes the interface of denser – rarer medium at an angle greater than critical angle is called total internal reflection.

<u>Conditions for total Internal Reflection (TIR)</u>

1. The light ray must travel from denser to rarer medium.
2. The angle of incidence in the denser medium must be greater than the critical angle.



5. Describe the construction and working of an optical fibre. State its uses. (March2010)

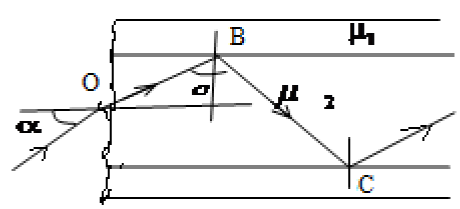
A. **<u>Construction:</u>** An optical fibre consists of three parts:
i) core ii) cladding and iii) coating or buffer.

The central part of the fibre made of glass or plastic with radius of the order of micromere is called core. The core is surrounded by a layer made of glass or plastic called cladding. The cladding material 1) reduces the loss of light from the core to the surrounding medium 2) reduces the scattering loss from the surface of the core 3) protects the fibre from surface contamination and.4) gives the mechanical strength to the fibre.

The refractive index of the cladding material is less than the refractive index of the core material, so that total internal reflection takes place inside the core.

The cladding is enclosed in another layer made of plastic called the coating or buffer is used to protect the fibre from physical damage. It prevents wearing by friction and scattering losses.

Principle : When light is incident at one end of the core at an angle called launching or glancing angle α.as shown, it gets refracted along OB into the core. If it is incident on core - the cladding interface at an angle greater then



the critical angle, the light undergoes total internal reflection along BC. Thus light travels only along the core of the fibre.

Uses: 1) Optical fibres are used to transmit communication signals through light pipes. There will not be cross talk and signals are transmitted with more clarity.

2.Optical fibres are used in laproscopy, endoscopy etc.

3.The optical fibre sensors are used to measure temperature and pressure.

4.The optical fibre photometric sensors are used for measuring the blood flow in the heart.

6. An iron ball covered with soot and held under wafer presents the appearance of a ball of polished silver. Explain?

A. When a ball coated with lamp black is immersed in water the lamp black does not get wet completely. A thin layer of air is formed between the sphere and water. When light ray passing through water and incident on water – air interface, it suffers total internal reflection hence the lamp coated sphere appears silvery.

7. An empty test tube in a beaker of water illuminated from one side present a silvery appearance. Explain ?

A. When an empty test tube is dipped in water, a light ray incident on the water – air interface suffers total internal reflection when the angle of incidence is greater than critical angle. As a result the test tube appears silvery when viewed from a suitable direction.

8. A bubble of air coming out through water in a glass vessel appear silvery to an observer standing by the side. Explain?

A. When a light ray passing through water incident on water air interface, it suffers total internal reflection for an angle of incidence greater than critical angle. The light rays inside the bubble reflected from the curved surface of the bubble which behaves as a mirror. Hence an air bubble inside water shines.

9. Looking obliquely at a crack in a plane of glass, the crack appears silvery. Explain?

A. The crack on a glass sheet consists of air in it. When it is viewed at certain angle the light ray traveling through glass incident on the glass –air interface suffers total internal reflection. Hence the crack appears shining.

10. A glass tumbler partially filled with water is held above the level of the eye and then tilted. Beyond a certain tilt the surface will appear glittering. Explain.

A. When the partially filled glass tumbler is tilted, beyond a certain limit the angle of incidence becomes greater than critical angle, as the light ray travels from denser to rarer medium total internal reflection takes place. Hence the surface appear glittering

VERY SHORT ANSWER QUESTIONS:

1. Define absolute refractive index. What is the relation between the relative refractive index between two media and the absolute reactive index of the media?

A. **Absolute refractive index** (μ) : It is defined as the ratio of velocity of light in vacuum to the velocity of light in the medium. $\mu = \frac{c}{V}$

When a light ray travels from a medium to the medium B, then the relative refractive index of the medium B with respect to the medium A is defined as the ratio of absolute refractive index of medium B to the absolute refractive index of medium A.

$$_{A}\mu_{B} = \frac{\mu_B}{\mu_A} = \frac{C/V_B}{C/V_A} = \frac{V_A}{V_B}$$

Where $V_A \; and \; V_B$ are velocities of light in the media A and B respectively and C is velocity of light.

2. Give the ratio of velocities of two light waves traveling in vacuum and having wavelengths 5000 A^0 and 6000 A^0

A. The velocity of light in vacuum is independent of wave length. $C_1 : C_2 = 1:1$

3. For the same angle of incidence, the angles of refraction in three different media A, B and C, are $30^0, 40^0, and\ 45^0$ respectively. In which medium will the velocity of light be maximum?

A. $\mu_A = \frac{\sin i}{\sin 30^0} = 2\sin i$; $\mu_B = \frac{\sin i}{\sin 40^0} = 1.55\sin i$; $\mu_C = \frac{\sin i}{\sin 45^0} = 1.41\sin i$

$\Rightarrow \mu_C < \mu_B < \mu_A \; and \; v \propto \frac{1}{\mu}$ and hence $V_C > V_B > V_A$

Hence velocity of light is maximum in the medium C.

4. Can absolute refractive index of a medium be less than unity?

A. $\mu_{absolute} = \frac{Velocity\ of\ light\ in\ vacuum}{velocity\ of\ light\ in\ medium} = \frac{C}{V}$

Since $C > V$, absolute refractive index of a medium is always greater than one.

5. Can relative refractive index between two media be less than unity?

A. For a pair of two media media A and B,

Relative refractive index of A with respect to B is $_{B}\mu_{A} = \frac{\mu_A}{\mu_B}$

Relative refractive index of B with respect to A is $_{A}\mu_{B} = \frac{\mu_B}{\mu_A}$

If $_{B}\mu_{A} > 1, then \; _{A}\mu_{B} < 1$

6. On what factors does the critical angle depend?

A. Critical angle depends on the refractive index of that medium. As refractive index (μ) increases, critical angle (C) decreases.

7. State the consequences of total internal reflection.

A. 1) The sparkling nature of diamond is due to total internal reflection.
2) The formation of mirages in desert is due to total internal reflection.
3) Optical fibre works on the principle of total internal reflection.

8. What is an optical fibre? State its principle.(June2010)

A. Optical fibre is a transmitting medium to carry the optical signal without any appreciable energy loss. It works on the principle of total internal reflection

9. Why do diamonds sparkle?

A. The sparkling nature of diamond is due to total internal reflection. The faces of a diamond are cut so that light entering into it suffers total internal reflection repeatedly at different faces before emerging out of it.

10. Light travels from denser medium to rarer medium. When the emergent ray grazes the separating surface what is the angle of deviation?

A. When the emergent ray grazes the separating surface ,then the angle of refraction in the rarer medium is $r = 90^0$ and angle incidence in the denser medium is equal to the critical angle (C) of the medium.

$\therefore$ Angle of deviation $D = r - i = \frac{\pi}{2} - C$

11. When light undergoes total internal reflection, what is the angle of deviation of the reflected ray?

A. When light undergoes total internal reflection,

Angle of deviation (D) = $\left(180^0 - 2i\right) = \pi - 2i$

12. A ray of light while traveling from a denser to rarer medium grazes the separating surface. Express the critical angle in terms of the speed of light in the two media.

A. According to Snell's law, reflective index of denser medium with respect to the rarer medium, ${}_r\mu_d = \frac{\mu_d}{\mu_r} = \frac{\sin r}{\sin i}$

Here $i = C \quad and \quad r = 90^0$

$\therefore \frac{\mu_d}{\mu_r} = \frac{\sin 90}{\sin C} = \frac{1}{\sin C}$

But, $\mu \alpha \frac{1}{v} \Rightarrow \frac{\mu_d}{\mu_r} = \frac{v_r}{v_d}$

$\therefore \frac{v_r}{v_d} = \frac{1}{\sin C} \Rightarrow \sin C = \frac{v_d}{v_r} \Rightarrow C = \sin^{-1}\left(\frac{v_d}{v_r}\right)$

13. Mention any three uses of optical fibres.

A. 1) They are used in communication network

2) They are used in medical equipments such as laproscope and endoscope.

3) They are used as sensors in industry.

<u>SOLVED PROBLEMS</u>

1. If the refractive index of glass is 1.5, find the velocity of light in glass (Velocity of light in vacuum = $3\times10^8 m/s$)

A. $\mu = 1.5; c = 3\times10^8 m/s$

The absolute refractive index of glass. $\mu = \frac{c}{v}$ (or) $1.5 = \frac{3\times10^8}{v} \Rightarrow v = 2\times10^8 m/s$

2. The refractive index of glass with respect to water is 9/8. If the velocity and wavelength of light in glass are $2\times10^8 m/s$ and $4000A^0$ respectively, find the velocity and wavelength of light in water.

A. ${}_w\mu_g = \frac{9}{8}; v_g = 2\times10^8 m/s; \lambda_g = 4000A^0; v_w = ? : \lambda_w = ?$ and $\mu = \frac{c}{v}$

a) The relative refractive index of glass w.r.t water,

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{v_w}{v_g}$$

$$\therefore \frac{9}{8} = \frac{v_w}{2\times10^8} \Rightarrow v_w = 2.25\times10^8 m/s$$

b) ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\lambda_w}{\lambda_g}$ $\left[\mu_g = \frac{\lambda}{\lambda_g} \ and \ \mu_w = \frac{\lambda}{\lambda_w}\right]$

$$\therefore \frac{9}{8} = \frac{\lambda_w}{4000} \Rightarrow \lambda_w = 4500A^0$$

3. A ray of light is incident on a glass slab of refractive index 1.5 making an angle of 40^0 with the surface. Find the angle of refraction in glass. (Velocity of light in vacuum = $3\times10^8 m/s$).

A. $i = 90^0 - 40^0 = 50^0$; $\mu = 1.5$

$$\mu = \frac{\sin i}{\sin r} \Rightarrow 1.5 = \frac{\sin 50^0}{\sin r} \Rightarrow r = 30^0 43'$$

4. Find the critical angle of a denser medium of refractive index 1.65 for its interface with air.

A. $\mu = 1.65,$ but $\mu = \frac{1}{\sin C}$

$$\Rightarrow \sin C = \frac{1}{\mu} = \frac{1}{1.65} = 0.6061 \Rightarrow \quad C = 37^0 18'$$

5. Find the critical angle for glass – water interface if the refractive index of glass is 1.50 and that of water is 1.33

A. $\mu_1 = 1.50, \mu_2 = 1.33; C = ?$

But, $\frac{\mu_2}{\mu_1} = \frac{1}{\sin C}$

$$\Rightarrow \sin C = \frac{\mu_2}{\mu_1} = \frac{1.33}{1.50} = 0.8867 \quad \Rightarrow \quad C = 68^0 28'$$

6. If the refractive indices of glass and water with respect to air are 3/2 and 4/3 respectively, calculate the velocity of light in glass and water and from the result, calculate the refractive index of glass with respect to water. (Velocity of light in air = $3\times10^8 m/s$)

A. $\mu_g = \frac{3}{2}; \mu_w = \frac{4}{3}; c = 3\times10^8 m/s; v_g = ?, v_w = ?$

The absolute refractive index of glass, $\mu_g = \frac{c}{v_g}$

$$\frac{3}{2} = \frac{3\times10^8}{v_g} \Rightarrow v_g = 2\times10^8 m/s$$

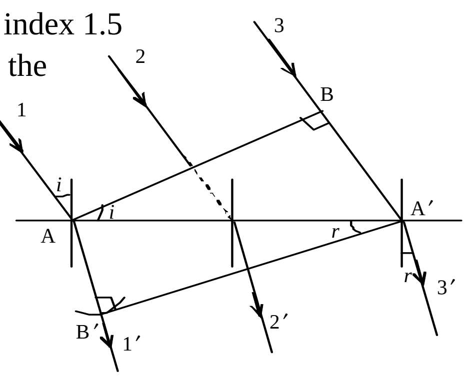
The absolute refractive index of water, $\mu_w = \frac{c}{v_w}$

$$\frac{4}{3} = \frac{3\times10^8}{v_w} \Rightarrow v_w = 2.25\times10^8 m/s$$

Refractive index of glass with respect to water, ${}_\omega\mu_g = \frac{\mu_g}{\mu_w} = \frac{v_w}{v_g}$

$$= \frac{2.25\times10^8}{2\times10^8} = 1.125$$

7. A beam of light is incident on a glass slab of refractive index 1.5 making an angle 60^0 with the surface. Find the ratio of the widths of the incident beam of light to the refracted beam of light.



A. $i = 90^0$; Angle made by the light ray with the surface $= 90^0 - 60^0 = 30^0$

From Snell's law , $\mu = \frac{\sin i}{\sin r}$

Angle of refraction, $r = 90^0 28'$

From figure, $\frac{width\, of\ incident\, beam}{Width\, of\ refracted\, beam} = \frac{AB}{A^1B^1}$

$$\cos i = \frac{AB}{AA^1}; AB = AA^1 \cos i$$

$$\cos r = \frac{A^1B^1}{AA^1}; A^1B^1 = AA^1 \cos r$$

$$\therefore \frac{Width\, of\ incident\, beam}{Width\, of\ refracted\, beam} = \frac{\cos i}{\cos r} = \frac{\cos 30^0}{\cos 19^0 28'} = \frac{0.8660}{0.9428} = \frac{0.9185}{1}$$

8. Find the time taken by light to travel a distance of 3m in water. (Velocity of light in vacuum = $3 \times 10^8 m/s$, Refractive index of water = 4/3.)

A. But $\mu = \frac{C_o}{C_m} \Rightarrow C_m = \frac{C_0}{m}$

$$\therefore t = \frac{d}{C_0} m \quad \Rightarrow t = \frac{3}{3 \times 10^8} \times \frac{4}{3} = 1.33 \times 10^{-8}$$

9. The optical path of a monochromatic light is the same if it goes through 4.00 m of glass or 4.50 m of a liquid. IF the refractive index of glass is 1.5. What is the refractive index of the liquid?

A. When light travels a distance x in a medium of refractive index μ , the distance 'd' that it can travel in vacuum during the same time is given by

$$\mu = \frac{c}{v} = \frac{d/t}{x/t} = \frac{d}{x} \text{ Or } d = \mu x$$

This distance traveled in vacuum is called optical path.

For two different media, $\mu_1 x_1 = \mu_2 x_2$

$$1.5 \times 4 = \mu_2 \times 4.5$$

Refractive index of water, $\mu_2 = \frac{1.5 \times 4}{4.5} = 1.333$

10. Find the thickness of a transparent plastic plate which will produce a change in optical path equal to the wavelength λ of the light passing through it normally the refractive index of the plastic plate is μ .

A. When light travels a distance x in a medium of refractive index μ , its optical path $= \mu x$

Change in optical path = $\mu x - x = (\mu - 1)x$

$$\therefore \lambda = (\mu - 1)x$$

Thickness of the plate, $x = \frac{\lambda}{\mu - 1}$

11. The wavelength of light in vacuum is $6000 A^0$. When it travels normally through glass of thickness 2.0 cm, find the number of waves of light in 2.0 cm thickness of glass.(Refractive index of glass = 1.5).

A. $l = 2.0cm = 2.0 \times 10^{-2} m; \mu = 1.5; \lambda = 6000 A^0 = 6000 \times 10^{-10} m$

Number of waves in a thickness l of a medium of refractive index μ is

Number of waves = $\frac{thickenss}{wavelength} = \frac{l}{\lambda_m} = \frac{l\mu}{\lambda}$ $\left(\therefore \lambda_m = \frac{\lambda}{\mu}\right)$

Where λ is the wavelength of light in vacuum and $\lambda\mu$ is the wavelength in the medium

The number of waves $= \frac{2.0\times10^{-2}\times1.5}{6000\times10^{-10}} = 5\times10^4$

12. When light of wavelength $5000A^0$ in vacuum travels through same thickness in glass and water, the difference in the number of waves is 400. Find the thickness. (Refractive indices glass and water are $\frac{3}{2}$ *and* $\frac{4}{3}$ respectively.)

A. Difference in number of waves = $\frac{l}{\lambda}\left(\mu_g - \mu_w\right)$ where μ_g *and* μ_w are the refractive indices of glass and water respectively.

$\mu_g = \frac{3}{2}; \mu_w = \frac{4}{3}; \lambda = 5000A^0 = 5000\times10^{-10}m$

$\therefore 400 = \frac{l}{5000\times10^{-10}}\left(\frac{3}{2} - \frac{4}{3}\right) \quad = \frac{l}{5\times10^{-7}}\times\frac{1}{6}$

The thickness $l = 400\times5\times6\times10^{-7} = 1.2mm$

13. The refractive index of a denser medium with respect to a rarer medium is 1.60. The difference between the velocities of light in the two media is $0.75\times10^8 m/s$. Find the velocities of light in the two media and their refractive indices. (Velocities of light in vacuum $= 3.0\times10^8 m/s$)

A. Relative refractive index of second medium w.r.t first medium

${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$ where μ_1 *and* μ_2 are the refractive indices, v_1 *and* v_2 are the velocities of light in rarer and denser media respectively.

${}_1\mu_2 = \frac{v_1}{v_2} = 1.6 \quad and \quad v_1 - v_2 = 0.75\times10^8 m/s$

Solving, we get ,

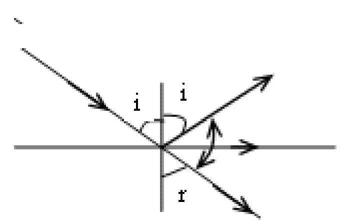
$v_2 = 1.25\times10^8 m/s \quad and \quad v_1 = 2\times10^8 m/s$

Absolute refractive index of the first medium, $\mu_1 = \frac{c}{v_1} = \frac{3\times10^8}{2\times10^8} = 1.5$

Absolute refractive index of the second medium, $\mu_2 = \frac{c}{v_2} = \frac{3\times10^8}{1.25\times10^8} = 2.4$

14. Find the angle at which light is to be incident on the surface of water so that the angle between the reflected and refracted rays is equal to 90^0. (Refractive index of water $= \frac{4}{3}$)

A. From the figure it can be seen that



$i + r + 90^0 = 180^0 \quad \Rightarrow \quad r = 90^0 - i$

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

$\mu = \frac{\sin i}{\sin\left(90^0 - i\right)} = \frac{\sin i}{\cos i} = \tan i$

$\Rightarrow \quad i = \tan^{-1}(\mu) = \tan^{-1}\left(\frac{4}{3}\right) = 53^0 4'$

15. A parallel sided glass slab of thickness 4 cm is made of material of refractive index $\sqrt{3}$. When light is incident on one of the parallel faces at an angle of 60^0 it emerges from the other parallel face. Find the lateral displacement of the emergent beam.

A. **Lateral Shift:**

d = i – r and RS = x

sind = $\frac{x}{PR}$ (or)

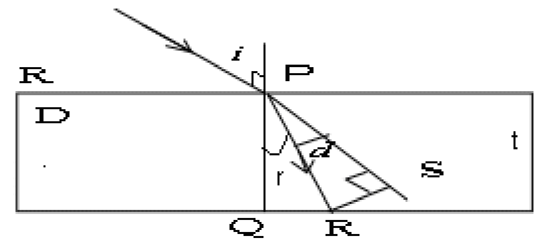
x = PR sin(i – r)…….(1)

cos r = $\frac{PQ}{PR} = \frac{t}{PR}$

t = PR cosr……………(2)

Dividing eq (1) by eq (2)

$\frac{x}{t} = \frac{\sin(i-r)}{\cos r}$



$x = \frac{t\sin(i-r)}{\cos r}$

From Snell's law of refraction. $\mu = \frac{\sin i60}{\sin r} \Rightarrow r = 30^0$

Now, Lateral shift $= \frac{t}{\cos r}\sin(i-r) = \frac{4}{\cos 30^0}\sin(60^0 - 30^0) = 2.309cm$

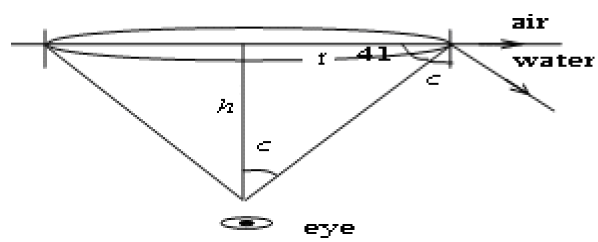
16. A liquid of refractive index 1.5 is poured into a cylindrical jar of radius 20 cm up to a height of 20 cm. A small bulb is lighted at the centre of the bottom of the jar. Find the area of the liquid surface through which the light of the bulb passes into air.

A. $\mu = \frac{1}{\sin C}$

$\sin C = \frac{1}{\mu}$

$\cos C = \sqrt{1 - \frac{1}{\mu^2}}$

$TanC = \frac{1}{\sqrt{\mu^2 - 1}}$



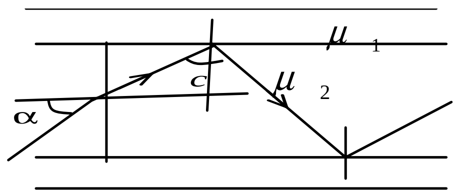
But from the diagram, $TanC = \frac{r}{h}$

Or $r = hTanC \qquad \Rightarrow r = \frac{h}{\sqrt{\mu^2 - 1}}$

Area of the circular base = $\pi r^2 = \frac{\pi h^2}{\mu^2 - 1} = \frac{\pi \times (0.20)^2}{1.5^2 - 1} = 0.1005m^2$

17. An optical fibre is made of glass fibre of refractive index 1.68. The outer coating of the glass is made of material fo refractive index 1.44. What is the range of angles of the incident rays with the axis of the pipe for which total internal reflection inside the optical fibre takes place ?

A. ***Launching Angle or Glancing Angle***



$$\mu_2 = \frac{\sin\alpha}{\sin r}$$

$\sin\alpha = \mu_2 \sin r$

But r + C = 90

(or) r = 90 – C

$\therefore \sin\alpha = \mu_2 \sin(90 - C)$

$\sin\alpha = \mu_2 \cos C$

$$\frac{\mu_2}{\mu_1} = \frac{1}{\sin C} \Rightarrow \sin C = \frac{\mu_1}{\mu_2}$$

$$\cos C = \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}} = \frac{\sqrt{\mu_2^2 - \mu_1^2}}{\mu_2}$$

$$\sin\alpha = \mu_2 \frac{\sqrt{\mu_2^2 - \mu_1^2}}{\mu_2}$$

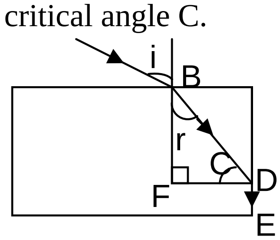
$$\Rightarrow \sin\alpha = \sqrt{\mu_2^2 - \mu_1^2} = \sin^{-1}\left(\sqrt{(1.68)^2 - (1.44)^2}\right) \qquad (\therefore \mu_2 = 1.68; \mu_1 = 1.44)$$

$$\sin^{-1}(0.8653) = 59^0 55'$$

This is the maximum value of i. All light rays with angle of incidence between 0^0 and $59^0 55'$ will undergo total internal reflection.

18. A ray of light incident on the horizontal surface of a glass slab at 70^0 just grazes the adjacent vertical surface after refraction. Compute the critical angle and refractive index of glass.

A. Let AB be the horizontal surface of a glass slab on which the light ray incident at an angle of incidence i and r be the angle of refraction. The light ray after refraction along BD grazes the vertical face and emerges along DE. At the vertical face the angle of incidence on the glass – air interface is the critical angle C.



In the triangle BFD, $r + C = 90^0 \Rightarrow C = 90^0 - r$

But, $\mu = \frac{1}{\sin C} = \frac{1}{\sin(90^0 - r)} = \frac{1}{\cos r} \Rightarrow \cos r = \frac{1}{\mu}$

$$\sin r = \sqrt{1 - \cos^2 r} = \sqrt{1 - \left(\frac{1}{\mu}\right)^2} = \frac{\sqrt{\mu^2 - 1}}{\mu}$$

Then Snell's law, $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r = \frac{\mu\sqrt{\mu^2 - 1}}{\mu} \Rightarrow \mu = \sqrt{1 + \sin^2 i}$

$$(or)\, \mu = \sqrt{1 + \sin^2(70^0)} = \sqrt{1 + (09397)^2} = 1.372$$

$$\therefore \sin C = \frac{1}{\mu} = \frac{1}{1.372} = 0.7289 \Rightarrow C = 46^0 47'$$

UNSOLVED PROBLEMS :

1. White light forms a continuous spectrum of minimum wavelength $3800A^0$ c corresponding to violet light and maximum wavelength $7800A^0$ corresponding to red light. Find the minimum and maximum frequencies of white light $\left(C = 3\times10^8 m/s\right)$.

A. Minimum wavelength = $\lambda_{\min} = 3800A^0 = 38\times10^{-8} m$

Maximum wavelength $= \lambda_{\max} = 7800A^0 = 78\times10^{-8} m$

Velocity of light $= C = 3\times10^8 ms^{-1}$

But, $C = n\lambda \Rightarrow n = \dfrac{c}{\lambda}$

Maximum frequency = $n_{\max} = \dfrac{C}{\lambda_{\min}} = \dfrac{3\times10^8}{38\times10^{-8}} = 7.895\times10^{14} Hz$

Minimum frequency $= n_{mn} = \dfrac{C}{\lambda_{\max}} = \dfrac{3\times10^8}{78\times10^{-8}} = 3.846\times10^{14} Hz$

2. If the refractive index of diamond is 2.4 find the velocity of light in diamond. $(C = 3\times10^8 m/s)$

A. Refractive index of diamond = $\mu_d = 2.4$

Velocity of light in air $= C = 3\times10^8\ ms^{-1}$

Velocity of light in diamond = V_d

$$\mu_d = \frac{C}{V_d} \Rightarrow V_d = \frac{c}{\mu_d} = \frac{3\times10^8}{2.4} = \frac{30}{24}\times10^8 = 1.25\times10^8 ms^{-1}$$

3. When light of wavelength $5000A^0$ in vacuum passes through water. what will be its wavelength and frequency if the reactive index of water is 4/3?

A. Refractive index of water $\mu_w = \dfrac{4}{3}$

Wavelength of light in vacuum = $\lambda_a = 5000A^0 = 5000\times10^{-10} m$

Wavelength of light in water = λ_w

$$From\, \mu_w = \frac{\lambda_a}{\lambda_w} \Rightarrow \lambda_w = \frac{\lambda_a}{\lambda_w} = \frac{5000\times10^{-10}}{4/3} = 3.75\times10^{-7} m \ = \ 3750A^0$$

Frequency of light in water $= n_w$

Velocity $= v_w = \dfrac{3\times10^8}{4/3} = \dfrac{9}{4}\times10^8 ms^{-1}$

$$\therefore n_w = \frac{v_w}{\lambda_w} = \frac{9}{4}\times\frac{10^8}{3.75\times10^{-7}} = 6\times10^{14} Hz$$

4. The wavelength of blue light in air is $4500A^0$. what is its frequency? If the refractive index for blue light is 1.55 in glass ,what will be the wavelength of blue light in glass. $\left(c = 3\times10^8 m/s\right)$

A. Wavelength of blue light in air $\lambda_b = 4500A^0 \ = \ 4500\times10^{-10} m$

Velocity of light = $C = 3\times10^8 ms^{-1}$

Frequency of blue light $= n_b From\, C = n_b\lambda_b$

$$\Rightarrow n_b = \frac{C}{\lambda_b} = \frac{3\times10^8}{4500\times10^{-10}} = 6.667\times10^{14}\,Hz$$

Refractive index of blue light in glass $= \mu_b = 1.55$

Wavelength of blue light in glass = λ_g

$$\therefore \mu_b = \frac{\lambda_b}{\lambda_g} \Rightarrow \lambda_g = \frac{\lambda_b}{\mu_b} = \frac{45\times10^{-8}}{1.55} = 29.03\times10^{-8}\,m = 2903A^0$$

5. A ray of light is incident on a glass slab making an angle of 60^0 with the surface. Calculate the angle of refraction in glass slab and the velocity of light in glass if the refractive index of glass and the velocity of light tin air are 1.5 and $3.0\times10^8\,m/s$ respectively.

A. Angle of incidence $= i = 90 - 60 = 30^0$

Angle of refraction = r

Refractive index of glass = $\mu_g = 1.5$

Velocity of light in air = $v_a = 3\times10^8\,ms^{-1}$

From $\mu_g = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu_g} = \frac{\sin 30^0}{1.5} = 0.3333 \Rightarrow r = 19^0 28'$

From $\mu_g = \frac{v_a}{v_g} \Rightarrow v_g = \frac{v_a}{\mu_g} = \frac{3\times10^8}{1.5} = 2\times10^8\,ms^{-1}$

6. The refractive indices of glycerine and diamond with respect to air are 1.4 and 2.4 respectively. Calculate the speed of light in glycerine and diamond. From these results calculate the refractive index of diamond with respect of glycerine $\left(c = 3\times10^8\,m/s\right)$

A. Velocity of light in air = $C = 3\times10^8\,ms^{-1}$

Refractive index of glycerine $= \mu_g = 1.4$

Refractive index of diamond $= \mu_d = 2.4$

Velocity of light in glycerine = v_g

Velociyt of light in diamond = v_d

From, $v_g = \frac{C}{\mu_g} = \frac{3\times10^8}{1.4} = 2.143\times10^8\,ms^{-1}$

From , $v_d = \frac{C}{\mu_d} = \frac{3\times10^8}{2.4} = 1.25\times10^8\,ms^{-1}$

Refractive index of diamond w.r.t glycerine = ${}_g\mu_d = \frac{\mu_d}{\mu_g} = \frac{2.4}{1.4} = 1.714$

7. The refractive indices of glass and water are $\frac{3}{2}\,and\,\frac{4}{3}$ respectively. Find the refractive index of water with respect to glass.(June2010)

A. Refractive index of glass $= \mu_g = \frac{3}{2}$

Refractive index of water $= \mu_w = \frac{4}{3}$

Refractive index of water w.r.t glass = ${}_g\mu_w = \frac{\mu_w}{\mu_g} = \frac{8}{9} = 0.8889$

8. The refractive index of glass with respect to water is $\frac{9}{8}$. If the velocity and wavelength of light in water are $2.25\times10^8 m/s$ and $5400A^0$ respectively ,find the velocity and wavelength of light in glass.

A. Refractive index of glass w.r.t water ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{9}{8}$

Velocity of light in water $= v_w = 2.25\times10^8 ms^{-1}$

Wavelength of light in water $= \lambda_w = 5400A^0 = 5400\times10^{-10} m$

Velocity of light in glass = v_g

Wavelength of light in glass = λ_g

But, $\frac{\mu_g}{\mu_w} = \frac{\lambda_w}{\lambda_g} = \frac{v_w}{v_g}$

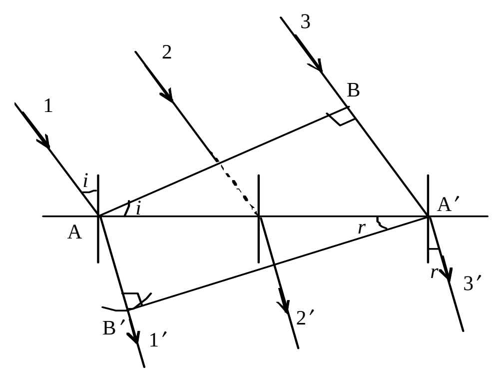
$\frac{9}{8} = \frac{5400\times10^{-10}}{\lambda_g} \Rightarrow \lambda_g = 48\times10^{-8} m = 4800A^0$

Also, $\frac{9}{8} = \frac{2.25\times10^8}{v_g} \Rightarrow v_g = 2\times10^8 ms^{-1}$

9. A beam of light is incident on a glass slab of refractive index 1.5 at an angle of incidence of 45^0. Find the ratio of the width of refracted beam to the incident beam.

A. Refractive index of glass = 1.5

Angle of incidence = i = 45^0



From $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu} = \frac{\sin 45^0}{1.5}$

$= \frac{1}{\sqrt{2}}\times\frac{2}{3} = \frac{\sqrt{2}}{3} = \frac{1.414}{3}$

$\Rightarrow \sin r = 0.4713 = \sin 28^0 7' \Rightarrow r = 28^0 7'$

From figure, $\frac{width\ of\ incident\ beam}{Width\ of\ refracted\ beam} = \frac{AB}{A^1B^1}$

$\cos i = \frac{AB}{AA^1}; AB = AA^1\cos i$

$\cos r = \frac{A^1B^1}{AA^1}; A^1B^1 = AA^1\cos r$

$\therefore \frac{Width\ of\ incident\ beam}{Width\ of\ refracted\ beam} = \frac{\cos i}{\cos r} = \frac{\cos 28^0 7'}{\cos 45^0} = \frac{0.8222}{1/\sqrt{2}} = \frac{1.247}{1}$

10. Find the time taken by light to travel a distance of 3 cm in glass. ($C = 3\times10^8 m/s$ and refractive index of water = 4/3)

A. Refractive index of water = $\mu_w = \frac{4}{3}$

Velocity of light in air = C = $3\times10^8 ms^{-1}$

Distance traveling in glass = $x_g = 3cm = 3\times10^{-2} m$

Time taken to travel in glass = t_g

Velocity of light in glass = $v_g = \frac{x_g}{t_g}$

From $\mu_w = \frac{v_a}{v_w} = \frac{C}{x_g / t_g} = \frac{C \times t_g}{x_g} \Rightarrow t_g = \frac{\mu_w \times x_g}{C} = \frac{4}{3} \times \frac{3 \times 10^{-2}}{3 \times 10^{8}} = 1.333 \times 10^{-10}$ sec

11. Find the ratio of the time taken by light to travel a distance of 100 m in air and water. $C = 3 \times 10^8 m/s$; $\mu_{air} = 1.0008$; $\mu_{water} = 1.33$)

A. Refractive index of air $= \mu_a = 1.0008$

Refractive index of water $= \mu_w = 1.33$

Velocity of light in air = $v_a = \frac{x_a}{t_a} = \frac{100}{t_a}$

Velocity of light in water $= v_w = \frac{x_w}{t_w} = \frac{100}{t_w}$

From, $\frac{\mu_w}{\mu_a} = \frac{v_a}{v_w} = \frac{t_w}{t_a} \Rightarrow \frac{t_a}{t_w} = \frac{\mu_a}{\mu_w} = \frac{1.0008}{1.33}$ = 0.7525 : 1

12. The optical path of a monochromatic light is the same if it goes through 2.00 cm of glass or x cm of ruby. If the refractive index of glass is 1.510 and that of ruby is 1.760, find the value of x.

A. $x_1 = 2cm$, $x_2 = x$

$\mu_g = 1.51, \mu_r = 1.76$

From $\frac{\mu_g}{\mu_r} = \frac{v_r}{v_g} = \frac{x_2 / t_2}{x_1 / t_1}$ $here\, t_1 = t_2$

$\therefore \frac{1.510}{1.760} = \frac{x}{2} \Rightarrow x = 1.716cm$

13. The wavelength of light in vacuum is $5000A^0$. When it travels normally through diamond of thickness 1.0 mm, find the number of waves of light in 1.0 mm of diamond (Refractive index of diamond 2.417)

A. Wavelength of light in vacuum $= \lambda_a = 5000A^0 = 5000 \times 10^{-10} m$

Distance traveled in diamond $= x = 1mm = 10^{-3} m$

Refractive index of diamond = $\mu_d = 2.417$

From $\frac{\mu_d}{\mu_a} = \frac{\lambda_a}{\lambda_d} \Rightarrow \frac{2.417}{1} = \frac{5000 \times 10^{-10}}{\lambda_d} \Rightarrow \lambda_d = 2068A^0$

$\therefore$ Number of waves $= n = \frac{x}{\lambda_d} = \frac{10^{-3}}{2068 \times 10^{-10}} = 4834$

14. When light of wavelength $4000A^0$ in vacuum travels through the same thickness in air and vacuum, the difference in the number of water is one. Find the thickness $(\mu_{air} = 1.0008)$

A. $\mu_a = 1.0008, \mu_v = 1, \lambda = 4000A^0 = 4000 \times 10^{-10} m$ and $\Delta n = 1$

Difference in number of waves, $\Delta n = \frac{t}{\lambda}(\mu_a - \mu_v) \Rightarrow 1 = \frac{t}{4000 \times 10^{-10}}[1.0008 - 1]$

$\therefore$ t = 0.5mm

15. The refractive index of denser medium with respect to rarer medium is 1.125. The difference between the velocities of light in the two media is $0.25\times10^{8} m/s$. Find the velocities of light in the two media and their refractive indices. $\left(c=3\times10^{8} m/s\right)$

A. $\dfrac{\mu_d}{\mu_r}=1.125$

$\dfrac{\mu_d}{\mu_r}=\dfrac{v_r}{v_d}\Rightarrow 1.125=\dfrac{v_r}{v_d}\Rightarrow V_r=V_d\left(1.125\right)$

But, $v_r-v_d=0.25\times10^{8} ms^{-1}$

$\left(1.125\right)v_d-v_d=0.25\times10^{8}$

$v_d=2\times10^{8} ms^{-1}$ and

$v_r=2.25\times10^{8} ms^{-1}$

Also, Absolute refractive index of denser medium= $\mu_d=\dfrac{c}{c_d}=\dfrac{3\times10^{8}}{2\times10^{8}}=1.5$

Absolute refractive index of rarer medium $=\mu_r=\dfrac{c}{v_r}=\dfrac{3\times10^{8}}{2.25\times10^{8}}=1.333$

16. Find the angle of incidence on the surface of diamond of refractive index 2.417, if the angle between the reflected and refracted rays is 90^{0}.

A. μ of diamond = 2.417

Reflected and refracted rays are perpendicular.

$\therefore \mu=\tan i\Rightarrow \tan i=2.417\Rightarrow i=67^{0}31'$

17. The refractive index of water relative to air is 1.33 and of glass relative to water is 1.166. What are the critical angles for water and glass relative to air?

A. ${}_a\mu_g=\dfrac{\mu_w}{\mu_g}=1.33\, and\ {}_w\mu_g=\dfrac{\mu_g}{\mu_w}=1.166$

$\Rightarrow \dfrac{\left(\dfrac{\mu_g}{\mu_a}\right)}{\left(\dfrac{\mu_w}{\mu_w}\right)}=1.166\dfrac{\mu_g}{\mu_w}\times\dfrac{\mu_w}{\mu_a}=1.66\times1.33\left(or\right)\dfrac{\mu_g}{\mu_a}=1.55$

$\Rightarrow \dfrac{\mu_w}{\mu_a}=\dfrac{1}{\sin C_w}\Rightarrow \sin C_w=\dfrac{1}{\left(\dfrac{\mu_w}{\mu_a}\right)}=\dfrac{1}{1.33}\quad\Rightarrow C_w=48^{0}45^{1}$

Similarly, $\dfrac{\mu_g}{\mu_a}=\dfrac{1}{\sin C_g}\Rightarrow 1.55=\dfrac{1}{\sin C_g}\left(or\right)\sin C_g=\dfrac{1}{1.55}=0.645$

$\Rightarrow \sin C_g=\sin 40^{0}12^{1}\quad\Rightarrow C_g=40^{0}12^{1}$

18. The critical angle for refraction from glass to air is 42^{0} and that from water to air is 48^{0}. Find the critical angle for refraction from glass to water.

A. $C_g=42^{0} C_w=48^{0}$

But, $\mu = \dfrac{1}{\sin c}$

$\mu_g = \dfrac{1}{\sin C_g}$ And $\mu_w = \dfrac{1}{\sin C_w}$

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\sin C_w}{\sin C_g}$$

$${}_w\mu_g = \frac{\sin 48}{\sin 42} = \frac{0.7431}{0.6691}$$

$${}_w\mu_g = \frac{7431}{6691}$$

$$\therefore \sin C = \frac{1}{{}_w\mu_g} = \frac{6691}{7431}$$

$$\sin C = 0.904 \Rightarrow C = 61^0 12^1$$

19. A light ray is traveling from a denser medium into rarer medium. The velocities of light in the two media are $2\times10^8 m/s\, and\, 2.5\times10^8$ m/s respectively. What is the critical angle at the interface of the two media?

A. Velocity of light in rarer medium $= v_r = 2.5\times10^8 ms^{-1}$

Velocity of light in denser medium $= v_d = 2\times10^8 ms^{-1}$

From $\mu = \dfrac{1}{\sin C} \Rightarrow \dfrac{\mu_d}{\mu_r} = \dfrac{1}{\sin C} \Rightarrow \dfrac{v_r}{v_d} = \dfrac{1}{\sin C}$

$$\Rightarrow \frac{2.5\times10^8}{2\times10^8} = \frac{1}{\sin C} \Rightarrow \sin C = \frac{2}{2.5} = 0.8$$

$$\therefore C = 53^0 8^1$$

20. Calculate the refractive index of glass with respect to air, if its critical angle is $40^0 12^1$

A. $C = 40^0 12^1$

But, $\mu = \dfrac{1}{\sin c} \Rightarrow \mu_g = \dfrac{1}{\sin 40^0 12^1} = \dfrac{1}{0.6455} = 1.549$

ASSESS YOURSELF

1. When light travels from a rarer medium to a denser medium ,why does light bend towards the normal?

A. This is because velocity of light is more in rarer medium than in denser medium.

2. Is the refractive index for violet light greater than or less than or equal to the refractive index for red light in glass?

A. Refractive index for violet light greater than the refractive index for red light in glass.

3. Can absolute refractive index of a medium be less than unity?

A. Since the velocity of light is maximum in vacuum ,absolute refractive index of a medium will be more than unity

4. When does a light ray travel undeviated?

A. When light incident normally on the boundary separating the two media or the light ray passes through the same medium, it undergoes no deviation.

5. Which colour of light has least critical angle?

A. Violet.

6. When a bubble of air rises through a column of water placed in strong diffuse light, the bubble appears shining. Why?

A. Total internal reflection.

7. How does a crack in a pane of glass look when seen obliquely?

A. Due to total internal reflection, crack in a pane of glass looks shining (silvery).