

WAVE MOTION
5. DOPPLER EFFECT

POINTS TO REMEMBER

1. DOPPLER EFFECT :

The apparent change in frequency due to relative motion between the source and the listener is called Doppler Effect.

2. Let V_O and V_s represents the velocities of a listener and a source respectively. Let V be the velocity of sound and n and n_1 be the true and apparent frequencies of the sound. Then if

a) the source alone is in motion towards the observer, $n_1 = n \left(\frac{V}{V - V_s} \right)$.

b) the source alone is in motion away from the observer, $n_1 = n \left(\frac{V}{V + V_s} \right)$.

c) the observer alone is in motion towards the source, $n_1 = n \left(\frac{V + V_O}{V} \right)$.

d) the observer alone is in motion away from the source, $n_1 = n \left(\frac{V - V_O}{V} \right)$.

e) the source and the observer both are in motion towards each other, $n_1 = n \left(\frac{V + V_O}{V - V_s} \right)$.

f) the source and the observer both are in motion away from each other,
 $n_1 = n \left(\frac{V - V_O}{V + V_s} \right)$.

g) the source and the observer both are in motion, source following the observer, $n_1 = n \left(\frac{V - V_O}{V - V_s} \right)$.

h) the source and the observer both are in motion, observer following the source, $n_1 = n \left(\frac{V + V_O}{V + V_s} \right)$.

i) the source, observer and the medium all are moving in the same direction as the sound, $n_1 = n \left(\frac{V + V_w - V_O}{V + V_w - V_s} \right)$ where V_w = velocity of wind.

j) the source and the observer are moving in the direction of the sound but the direction of wind is opposite to the direction of the propagation of sound,
 $n_1 = n \left(\frac{V - V_w - V_O}{V - V_w - V_s} \right)$.

LONG ANSWER QUESTIONS:

1. **What is Doppler Effect? Find an expression for the apparent frequency heard when the source is in motion and the listener is at rest. What is the limitation of Doppler Effect?**

- A. **Doppler effect:** Doppler's effect is the phenomenon of change in apparent pitch of the sound due to relative motion between the source of sound and the listener

Apparent frequency when the source is in motion and observer is at rest:

Source is moving towards the listener.

Consider that a source of sound 'S' is emitting the sound waves of frequency n . Let The velocity of these waves in the medium be v .

Let the source is moving towards a stationary observer with velocity v_s . During the time period T the source will travel a distance $v_s T$.

Now successive compression and rarefaction produced in time period T lies with in a distance

$$\lambda' = \lambda - v_s T. \quad \text{But } T = \frac{1}{n}$$

$$\therefore \lambda_1 = \lambda - \frac{v_s}{n}$$

Due to the motion of source wavelength is shortened.

So apparent frequency of sound heard $n' = \frac{v}{\lambda_1}$

$$\therefore n' = \frac{v}{\lambda - \frac{v_s}{n}} \quad \text{But } \lambda = \frac{v}{n}$$

$$\therefore n' = \frac{v}{\frac{v}{n} - \frac{v_s}{n}} = \frac{v \cdot n}{v - v_s}$$

$$\text{Or apparent frequency } n' = \left[\frac{v}{v - v_s} \right] n$$

Hence when source is moving towards stationary observer apparent frequency increases.

Source is moving away from listener.

During the time T the source will move away by a distance $v_s T$. During this time period T the distance between successive compression and rarefaction is equals to new wavelength λ''

$$\text{Now } \lambda'' = \lambda + v_s T \quad \text{But } T = \frac{1}{n}$$

$$\therefore \lambda'' = \lambda + \frac{v_s}{n}$$

So apparent frequency of sound heard $n'' = \frac{v}{\lambda''}$

$$\therefore n'' = \frac{v}{\lambda + \frac{v_s}{n}} \quad \text{But } \lambda = \frac{v}{n}$$

$$\therefore n'' = \frac{v}{\frac{v}{n} + \frac{v_s}{n}} = \frac{v \cdot n}{v + v_s}$$

Or apparent frequency $n'' = \left[\frac{v}{v + v_s} \right] n$

Hence when source is moving away from listener apparent frequency decreases.

Limitations of Doppler effect: Doppler effect is not applicable when the velocity of source (v_s) or velocity of listener (v_0) are equal to the velocity of Sound 'v'

2. **What is Doppler's Shift? Derive an expression for the apparent frequency heard by a moving listener when the source of sound is at rest. What is the effect of the moving medium on the Doppler's effect? (March 2010)**

- A. **Doppler's shift :** The change in the frequency of sound produced (n) and apparent frequency of sound heard (n^1) by the listener is called Doppler's shift

Doppler's shift $\Delta n = n - n^1$

source is at rest and listener is in motion:

Listener is moving towards from the source:

Let a listener is moving towards a stationary source with a velocity v_0 and the frequency of sound note produced by source is 'n'. Since listener is moving towards the source, he will receive more number of waves per second. The additional number of waves received due to his motion is $\frac{v_0}{\lambda}$

Apparent frequency $n^1 = n + \frac{v_0}{\lambda}$ But $\lambda = \frac{v}{n}$

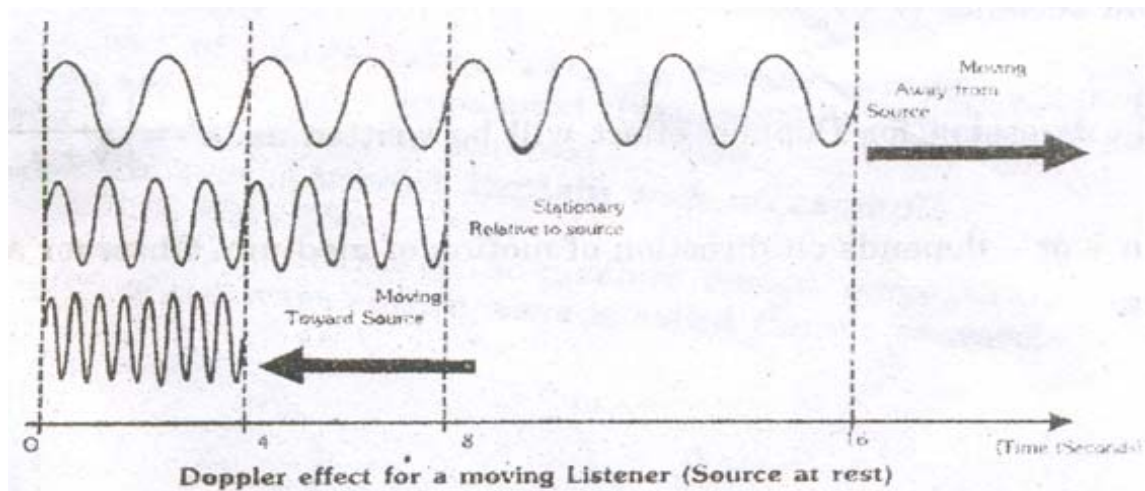
$$\therefore n^1 = n + \frac{v_0 \cdot n}{v} \Rightarrow n^1 = n \frac{(v + v_0)}{v}$$

When listener is moving towards a stationary source apparent frequency increases.

Listener is moving away from the source:

Let the listener is moving away from the source with a velocity v_0 . Since the listener is moving away from source he will receive less number of waves per second

Less number of waves received per second = $\frac{v_0}{\lambda}$



Apparent frequency of sound heard $n'' = n - \frac{v_0}{\lambda}$ but $\lambda = \frac{v}{n}$

$$\therefore n'' = n - \frac{v_0 \cdot n}{v} \Rightarrow n'' = n \frac{(v - v_0)}{v}$$

Hence when listener is moving away from source apparent decreases.

Effect of motion of the medium on apparent frequency :

Let the medium is moving with a velocity v_m towards the listener then velocity of sound becomes $(v + v_m)$. If the medium is moving away from the listener then velocity of the sound becomes $(v - v_m)$

$$\therefore n' = \left[\frac{v \pm v_m \pm v_0}{v \pm v_m \mp v_s} \right] n$$

Here the sign + or – depends on direction of motion of medium, Observer and source.

SHORT ANSWER QUESTIONS

1. What are the applications of Doppler’s effect?

A. **Applications of Doppler’s effect :**

1. This effect is used in accurate navigation of aircraft and it is the basis of accurate target bombing techniques developed recently.
2. This principle is used in measuring the speeds of the automobiles by traffic police.
3. This effect is used in determining the velocities of stars relative to the earth.
4. The rotation of Saturn’s rings and of double stars has been detected using Doppler’s effect.
5. This principle is used for tracking an earth satellite.

VERY SHORT ANSWER QUESTIONS

1. What is Doppler Effect? Write its limitations.

- A. **Definition:** The apparent change in the frequency of sound heard by an observer due to relative motion between the source of sound and the observer is called Doppler Effect.

Limitations:

1. The Velocity of source of sound must be less than that of velocity of source $V_s < V$.
2. If $V_s > v$, then due to shock waves (Waves traveling with velocity greater than that of sound are called shock waves or supersonic waves) the wave front gets distorted and Doppler effect is not observed.
3. Velocity of the observer must be less than that of sound $V_o < V$ For example, if $V_o > V$ and if the observer is receding the source, the waves never reach the observer and Doppler effect is not observed.
4. Source and observer should not move perpendicular to each other.
5. Doppler Effect is not applicable if the observer is moving along with the source.
6. Doppler Effect is not applicable when the medium alone moves.

2. Write any two applications of Doppler Effect

- 1) Doppler's effect is used in accurate navigation of aircraft and it is the basis at accurate target bombing techniques developed recently
- 2) Doppler's effect is used to estimate the velocity of vehicles by traffic police

SOLVED PROBLEMS:

1. A train is traveling at 120 kmph and blows a whistle of frequency 1000 Hz. What will be the frequency of the note heard by a stationary observer?

1) If the train is approaching him?

b) If the train is moving away from him (The velocity of sound in air is 330 ms^{-1})

- A. Frequency of the whistle $n = 1000 \text{ Hz}$

$$\text{Velocity of the train } V_s = 120 \text{ kmph} = \frac{100}{3} \text{ ms}^{-1}$$

$$\text{Velocity of sound in air } v = 330 \text{ ms}^{-1}$$

i) When the train approaches him,

$$\text{Apparent frequency } v' = \left(\frac{V}{V - V_s} \right) v = \left[\frac{330 \times 1000}{330 - 100/3} \right] = 1112 \text{ Hz}$$

ii) When train is moving away from him,

$$\text{Apparent frequency } v^n = \left(\frac{V}{V + V_s} \right) v = \frac{330 \times 1000}{[330 + 100/3]} = 908 \text{ Hz}$$

Exercise 1

1. A fire engine with its bell ringing with a frequency of 200 Hz is moving with a velocity of 54 kmph towards an observer at rest near a hut on fire. What is the apparent frequency of sound heard by the observer? (Velocity of sound in air = 300 ms^{-1}) (March 2010)

A. Velocity of source $v_s = 54 \text{ kmph} = 15 \text{ ms}^{-1}$

Velocity of observer $v_o = 0$

Velocity of sound $v = 300 \text{ ms}^{-1}$

Frequency of sound $n = 200 \text{ Hz}$

$$\text{Apparent frequency of sound } n^1 = \left(\frac{v}{v - v_s} \right) n \Rightarrow n^1 = \left(\frac{300}{300 - 15} \right) 200$$

$$n^1 = 210.5 \text{ Hz}$$

2. **Two aeroplanes A and B are moving away from one another with a speed of 720 kmph. The frequency of the whistle emitted by 'A' is 1100 Hz calculate the apparent frequency of the whistle as heard by the passenger of the aeroplane 'B' velocity of sound in air is 350 ms^{-1}**

A. Velocity of source (A) = $v_s = 720 \text{ kmph} = 200 \text{ ms}^{-1}$

Velocity of observer (B) = $v_o = 200 \text{ ms}^{-1}$

Velocity of sound $v = 350 \text{ ms}^{-1}$

Frequency of sound $n = 1100 \text{ Hz}$.

$$\text{Apparent frequency of sound, } n^1 = \left(\frac{v - v_o}{v + v_s} \right) n \Rightarrow n^1 = \left(\frac{350 - 200}{350 + 200} \right) 1100$$

$$\therefore n^1 = 300 \text{ Hz}$$

3. **A tuning fork of frequency 328 Hz is moved towards a wall at a speed of 2 ms^{-1} . An observer standing on the same side as the fork hears two sounds, one directly from the fork and the other reflected from the wall. How many beats per second can be heard? (Velocity of sound in air 330 ms^{-1})**

A. Frequency of tuning fork = $n = 328 \text{ Hz}$.

Velocity of source = $v_s = 2 \text{ ms}^{-1}$

Velocity of observer = $v_o = 0$

Velocity of sound = $v = 330 \text{ ms}^{-1}$

For direct sound heard by observer, the source is moving away from him

$$\therefore \text{Apparent frequency heard} = \left(\frac{v}{v + v_s} \right) n \Rightarrow n_1 = \left(\frac{1}{1 + \frac{v_s}{v}} \right) n = \left(v - \frac{v_s}{v} \right) n$$

For reflected sound heard by observer, the source appears to be moving towards him

$$\text{Apparent frequency heard} = n_2 = \left(\frac{v}{v - v_s} \right) n \Rightarrow n_2 = \left(\frac{1}{1 - \frac{v_s}{v}} \right) n = \left(v + \frac{v_s}{v} \right) n$$

$$\begin{aligned} \text{No. of beats heard by observer} &= n_2 - n_1 = n \left(v + \frac{v_s}{v} - v + \frac{v_s}{v} \right) \\ &= \frac{2v_s n}{v} = \frac{2 \times 2 \times 328}{330} = 3.97 = 4 \text{ beats / sec (nearly)} \end{aligned}$$

ASSESS YOURSELF

1. **Why do we hear a higher frequency apparently when we approach a stationary sounding railway engine?**
 - A. Due to the increase in the relative velocity of the waves received.

2. **What is the reason for hearing a higher frequency when a source of sound moves towards a stationary listener?**
 - A. Due to the decrease in the apparent wavelength.

3. **What happens when the velocity of sound approaches with the velocity of the source or the velocity of listener?**
 - A. Doppler's effect is not applicable.