WAVE MOTION
BEATS AND ECHOS
POINTS TO REMEMBER

1. **BEATS :**
   a) When two sounds of slightly different frequencies superimpose, the resultant sound consists of alternate waxing and waxing. This phenomenon is called beats.
   b) One waxing and one waning together is called one beat.
   c) If simple harmonic progressive waves of frequencies n_1 & n_2 travelling in same direction superimpose, the resultant wave is represented by
   $y=\left\{2a\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t\right\}\sin 2\pi\left(\frac{n_1+n_2}{2}\right)t$.
   d) The amplitude of resultant wave is $2a\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t$.
   e) The frequency of resultant wave is $\left(\frac{n_1+n_2}{2}\right)$.
   f) The number of beats produced per second or beat frequency is equal to the difference of frequencies of nodes producing beats. $n=n_1 \sim n_2$.
   g) The maximum number of beats heard by a person is 10, since persistence of hearing is 1/10 sec.
   h) The time internal between two consecutive maxima or minima is $\frac{1}{(n_1 \sim n_2)}$.
   i) The time interval between consecutive maxima and minima is $\frac{1}{2(n_1 \sim n_2)}$.
   j) When two sound waves of nearly equal (slightly different) frequencies interfere with one another, beats are produced.
   Beat frequency $\Delta n = n_1 \quad n_2 = \upsilon\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]$

2. **ECHO :**
   a) When a sound is produced and a listener hears it after reflection from an obstacle, the reflected sound is called and echo.
   b) If d is the distance between source and the reflecting surface and t is the time taken to hear echo after sound is produced, $t=\frac{2d}{V}$ where V is velocity of sound.
   c) The distance between source and reflecting surface d=Vt/2.
   d) The minimum distance between source and the reflecting surface to hear a clear echo is V/20. It is equal to 16.5 m if V=300 m/s.
   e) If a person standing between two parallel hills fires a gun and hears first echo after t_1 sec and the second echo after t_2 sec, the distance between two hills is $d=\frac{V(t_1+t_2)}{2}$.
   f) If a motor car approaching a cliff with a velocity 'u' sounds the horn and the echo is heard after 't' sec, then the distance between the cliff and the original position of car is $d=\left(\frac{v+u}{2}\right)t$.

g) In the above case, the distance between the cliff and the point where the echo is heard is $d=\left(\frac{v-u}{2}\right)t$.

h) A road runs paralleled to a long vertical line of hills. If a motorist moving with a speed 'u' sounds the horn and hears the echo after 't' sec, then the distance between the road the cliff is
$d=\frac{t}{2}\sqrt{v^2-u^2}$.

i) A road runs midways between two parallel rows of hills. If a motorist moving with speed 'u' sounds the horn and hears echo after 't' sec, then the distance between two rows of hills is
$s=t\sqrt{v^2-u^2}$.

SHORT ANSWER QUESTIONS

1. What are beats? Explain the importance of beats?

A. When two sound waves of slightly different frequencies traveling in the same direction interfere, in the resulting sound waxing and waning is heard at regular intervals of time. This phenomenon is called beats. If n_1 *and* n_2 are frequencies of two bodies, then

Number of beats per second $\Delta n = n_1 \quad n_2$

Uses or importance of Beats:

The phenomenon of beats can be used

1. To detect the presence of dangerous gases in mines.
2. To determine an un-known frequency of a sound note.
3. To determine the frequency of a given tuning fork.
4. To tune a musical instrument it gives a note.
5. To produce special effects in cinematography

VERY SHORT ANSWER QUESTIONS:

1. What are beats? Write the expression for beat frequency.

A. When two sound waves of slightly different frequencies traveling in the same direction interfere, waxing and waning of sound is heard at regular intervals of time. This phenomenon is called beats. If n_1 *and* n_2 are frequencies of two bodies

Number of beats per second $\Delta n = n_1 \quad n_2$ where

2. Mention any two applications of beats.

A. Beats can be used i) to detect the presence of dangerous gases in mines and (ii) to determine un-known frequency of a sound note of a given tuning fork.

3. What is an echo? What is the minimum distance from which an echo can be heard? (Assume velocity of sound in air as $330 ms^{-1}$)(March'06,09)

A. The reflection of sound by an obstacle is called echo. The minimum distance between the source and the reflector is $d=\frac{vt}{2}=\frac{330\times 0.1}{2}=16.5m$

4. What happens to the frequency of a tuning fork when it is loaded with little wax and if when it is filed?

A. When a tuning fork is loaded with wax its frequency decreases due to increase in inertia. When prongs of the fork are filed, its frequency increases due to decreases in inertia.

SOLVED PROBLEMS:

1. A tuning fork 'A' produces 6 beats per second with another fork 'B'. On loading 'B' with a little wax, it produces 5 beats per seconds with 'A'. If the frequency of 'A' is 256 Hz find the frequency of 'B'.

A. Let the frequencies of the forks be n_A *and* n_B respectively. They produce 6 beats per second.

$\therefore v_A - v_B = 6 \quad or \quad v_B - v_A = 6$

The frequency of 'B' decreases on loading i.e., $v'_B < v_B$

The beat frequency after loading is 5.

$\therefore v_A - v'_B = 5 \quad or \quad v'_B - v_A = 5$

If $v_A - v_B = 6$ is correct, then $v_A - v'_B > 6$

But the beat frequency did not increase.

Hence $v_B - v_A = 6$ is correct.

$v_B = v_A + 6 = 256 + 6 = 262Hz$

2. The frequency of a tuning fork 'x' is 5% greater than that of a standard fork of frequency 'K'. The frequency of another fork 'y' is 3% less than that of 'K'. When 'x' and 'y' are vibrated together 4 beats are heard per second. Find the frequencies of x and y. (June2005)

A. The frequency of fork $x = n_x = \frac{105}{100}K$

The frequency of fork $y = n_y = \frac{97}{100}K$

$n_x - n_y = 4$

$\frac{105}{100}K - \frac{97}{100}K = 4 \quad \Rightarrow \; K = 50Hz$

Frequency of $x = \frac{105}{100} \times 50 = 52.5Hz$

Frequency of $y = \frac{97}{100} \times 50 = 48.5Hz$

3. A man fired a bullet in front of a mountain and he heard its echo after 2 seconds. After traveling a distance of 85m towards the mountain, he fired another bullet and heard its echo after 1.5 seconds. Then find the velocity of sound. Also calculate the distance between the mountain and the man when the first bullet was fired.

A. Let the velocity of sound be 'v' and 'x' be the distance between the person and the mountain when the first bullet was fired.

The time taken to hear the first echo of firing the first bullet = 2s

$V = \frac{2x}{2} = x$

The distance traveled by the sound after the second bullet was fire = $(x - 8.5)m$

Time taken to hear the echo = 1.5 s.

In the second case, velocity of sound $V = \frac{2(x-85)}{1.5} \Rightarrow x = 340ms$

$\therefore$ Velocity of sound $V = x = 340ms^{-1}$

UNSOLVED PROBLEM:

1. The height of a cloud above the earth is 100 m. If an observer hears the sound of thunder 0.3 s after the lightening is seen what is the velocity of sound on that day?

A. Distance travelled by sound S = 100 m

Time taken to hear sound t = 0.35 sec

Velocity of sound $= V = \frac{s}{t} = \frac{100}{0.3} = 333.3ms^{-1}$

2. Two sound waves of length 9 m and 10 m produce 34 beats in 9 seconds Find the velocity of sound.

A. $\lambda_1 = 9m \; and \; \lambda_2 = 10m$

Number of beats produced in 1 sec = x = $\frac{34}{9}$

Since $n_1 - n_2 = x \Rightarrow \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = \frac{34}{9} \Rightarrow v\left(\frac{1}{9} - \frac{1}{10}\right) = \frac{34}{9} \Rightarrow v = 340ms^{-1}$

3. Two tuning forks give 4 beats per second simultaneously. The frequency of one of the forks is 384 Hz. When the other fork is loaded with a little wax 6 beats per second are produced. What is the frequency of the second fork?

A. $n_1 = 384\,Hz; n_2 =$ frequency of second fork

Given $n_1 \quad n_2 = 4 \Rightarrow n_1 - n_2 = 4 \; or \; n_2 - n_1 = 4$

When n_2 is loaded with wax, let n_2^1 be its frequency $n_2^1 < n_2$

But, $n_2^1 - n_1 = 6 \Rightarrow n_1 - n_2^1 = 6 \; or \; n_2^1 - n_2 = 6$

$\therefore n_2 = 380Hz \; or \; 388\,Hz \; and \; n_2^1 = 378Hz \; or \; 390Hz$

As $n_2^1 < n_2; n_2 = 380Hz$

4. When two tuning forks A and B are sounded at the same time 3 beats are heard per second. If the frequency of 'A' is 512 Hz and when second fork 'B' is filed the beat frequency reduces to 2. Find the frequency of the fork 'B'.

A. $n_A = 512Hz \;;\; n = 3, \quad n_B = ?$

$n_A - n_B = n \Rightarrow 512 - n_B = 3 or\, n_B - 512 = 3Hz$

$\therefore n_B = 509Hz\, or\, 515Hz$

Let n_B be the frequency of B after filing in $n_B > n_B^1$

But, $n_A = 512Hz$

$n_B^1 - n_A = 2 or\, n_A - n_B^1 = 2$

$\therefore n_B^1 = 510Hz\, or\, 514Hz$

As $n_B^1 > n_B, n_B = 509Hz$

5. The frequency of a tuning fork 'A' is 2% greater than that of a standard fork 'K'. The frequency of another tuning fork 'B' is 3% less than 'K'. When 'A' and 'B' are vibrates together 6 beats per second are heard per second. Find the frequencies of 'A' and 'B'.

A. Frequency of fork $A = K\left(1 + \frac{2}{100}\right) = 1.02K$

Frequency of fork B = $K\left(1 - \frac{3}{100}\right) = 0.97K$

Number of beats heard $\Delta n = 6$

$A : B = 6 \Rightarrow 6 = [1.02 - 0.97]K$ Or $K = \frac{6}{0.05} = \frac{600}{5} = 120Hz$

Frequency of $A = 120 \times 1.02 = 122.4Hz$

Frequency of B = $120 \times 1.97 = 116.4Hz$

6. A man standing between two parallel cliffs fires a gun. He hears the first echo after 2 seconds and the next after 5 seconds. What is the distance between the cliffs? Velocity of sound in air is 350 ms^{-1}.

A. $x_1 = \frac{vt}{2} = \frac{350 \times 2}{2}$ and $x_2 = \frac{vt}{2} = \frac{350 \times 5}{2}$

Distance between the cliffs = $x_1 + x_2 = \frac{350}{2} \times 7$ = 1225 m

7. A motor car approaching cliff with a velocity of 90 kmph sounds the horn and the echo is heard after 20 seconds. Assuming the velocity of sound in air to be $332ms^{-1}$. Calculate the distance between the car and the cliff when the horn is sounded?

A. Velocity of car $u = 90kmph = 25m/s$

Velocity of sound = 332 m/s

Distance traveled by sound wave is d + (d – x_)

$t = \frac{2d - x}{v} = \frac{2d - ut}{v}$

$$d = \left(\frac{v+u}{2}\right)t = \left(\frac{32+25}{2}\right)20 \Rightarrow d = 5370m$$

ASSESS YOURSELF

1. A man standing at certain distance blows a horn towards a big wall. He hears the echo after 5 s. If the velocity of sound in air is $340ms^{-1}$, what is the distance between the man and the wall?

A. Time taken to hear an echo t = 5 s

Velocity of sound in air $c = 340ms^{-1}$

Distance between the wall and the man, d = $\frac{Vt}{2} = \frac{340 \times 5}{2} = 850m$

2. What is the condition under which beats are formed?

A. When two sound waves of slightly different frequencies traveling in the same direction interfere, waxing and waning of sound is heard at regular intervals of time. This phenomenon is called beats.

3. Two waves of frequencies 256 & 250 are forming beats. What will be the beat frequency?

A. 6 beats per second.