WAVE MOTION
3.RESONANCE

POINTS TO REMEMBER

1. **Free (or) Natural vibrations:**

   I. If a body is set into vibrations its natural vibrating conditions, then those vibrations are called natural vibration.

   II. Natural frequency depends on the elasticity, physical conditions of the body like mass, shape etc and mode of vibration.

   III. Force that produces natural vibrations is instantaneous.

2. **Forced vibrations:**

   I. The vibrations of a body under an external periodic force are called forced vibrations.

   II. Ex: When the stem of a vibrating tuning fork is kept on a table, the table vibrates with a forced frequency.

   III. The body vibrates with a frequency of applied force but not with natural frequency.

3. **Damped oscillations:** When dissipative forces such as friction (or) viscous force acts as a naturally vibrating body, the forces offer resistance to the motion and amplitude of vibration gradually decreases and finally the body comes to rest. Such vibrations are called damped oscillation. Ex: The motion of a simple pendulum in a liquid.

4. **Resonance:**

   I. This is a special case of forced vibrations

   II. When the external periodic frequency is equal to the natural frequency of the body, the amplitude of vibration increases rapidly (Theoretically infinity for no damping). This phenomenon is called resonance. Such vibrations are called resonant vibrations (or) symphethetic vibration.

   III. At resonance the amplitude (Intensity) of vibration is maximum and Rapid transfer of energy takes place.

5. **In open pipe,**

   Ist harmonic $n_1 = \frac{\upsilon}{2l}$

   2^{nd} harmonic (or) 1^{st} overtone $n_2 = \frac{2\upsilon}{2l} = 2n_1$

   3^{rd} harmonic (or) 2^{nd} overtone $n_3 = \frac{3\upsilon}{2l} = 3n_1$

6. In an open pipe $n_1 : n_2 : n_3 : .... 1 : 2 : 3 : ....$ Both odd and even harmonics are formed in open pipes.

7. **<u>In closed pipe,</u>**

   1st harmonic or) fundamental mode $n_1 = \frac{\upsilon}{4l}$

   3rd harmonic (or) 1st overtone $n_2 = \frac{3\upsilon}{4l} = 3n_1$

   5th harmonic (or) 2nd overtone $n_3 = \frac{5\upsilon}{4l} = 5n_1$

   In a closed pipe $n_1 : n_2 : n_3 .... = 1 : 3 : 5 : .....$ Only odd harmonics are formed in closed pipes.

8. **<u>Velocity of sound</u>**

   I. Equation for velocity of sound through a medium is $\upsilon = \sqrt{\frac{E}{\rho}}$ where E is modulus of elasticity and ρ is density

   II. Velocity of sound in case of solids $\upsilon = \sqrt{\frac{y}{\rho}}$ where y is Young's modulus

   III. In the case of fluids, velocity of sound $\upsilon = \sqrt{\frac{K}{\rho}}$ where K is Bulk modulus

   IV. Under isothermal conditions, velocity of sound in a gas $\upsilon = \sqrt{\frac{P}{\rho}}$ where P is pressure

   V. According to Laplace's correction under adiabatic conditions, velocity of sound in a gas $\upsilon = \sqrt{\frac{\gamma P}{\rho}}$ where $\gamma =$ ratio of specific heats of gas.

   VI. There is no effect of pressure on velocity of sound as long as temperature remains constant.

   VII. Effect of temperature on velocity of sound $\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{T_1}{T_2}}$ where $\upsilon_1 \; and \; \upsilon_2$ are velocities of sound at absolute temperatures $T_1 \; and \; T_2$ respectively

   VIII. If $\upsilon_0 \; and \; \upsilon_t$ are velocities of sound in air at $0^0 C \; and \; t^0 C$ respectively. Then $\upsilon_t = \upsilon_0 \left[1 + \frac{t}{546}\right]$

   IX. Effect of density $\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{\rho_2}{\rho_1}}$ where $\upsilon_1 \; and \; \upsilon_2$ are velocities of sound at densities $\rho_1 \; and \; \rho_2$ respectively

LONG ANSWER QUESTIONS

1. What are “Harmonics” and “Overtones”? How are they formed in an open pipe? Derive the equations for the frequencies of the harmonics produced in an open pipe. (June2010)

A. Harmonics:

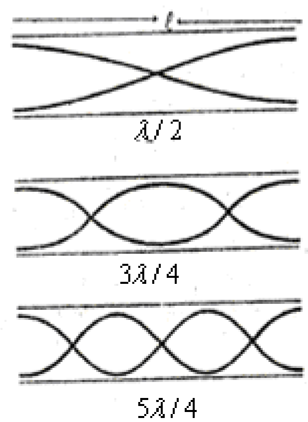
In a given vibrating length ,the possible frequencies in which the standing waves can be formed are called harmonics. A harmonic in a vibrating length with minimum number of nodes and antinodes is known as the fundamental harmonic or the first harmonic.

A ‘tone’ of sound with a frequency which is an integer multiple of the fundamental frequency is called a harmonic.

Over Tone : The tones of sound higher than the fundamental frequency are known as overtones.

Open Pipe – Modes of Vibrating:

A pipe whose both ends are open is called open pipe. At the open ends of the pipe always antinodes are formed since the particles are free to vibrate.



Case – 1

In the first mode of vibration, the air column vibrates with two antinodes and one node as shown in figure .Since the distance between two successive antinodes is $\frac{\lambda}{2}$

Length of the pipe, $l = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2l$

Fundamental frequency of vibration is given by, $n_1 = \frac{V}{\lambda_1} = \frac{V}{2l}$ ---------(1)

Where n_1 is called fundamental frequency or first harmonic.

Case – II

In the second mode of vibration, the air column vibrates with two nodes and three antinodes as shown in the figure.

Length of the pipe, $l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \lambda_2 \qquad \Rightarrow \lambda_2 = l$

Frequency of vibration is given by, $n_2 = \frac{V}{\lambda_2} = \frac{V}{l} = 2n_1$ ---------(2)

Where n_2 is called second harmonic or first overtone.

Case – III

In the third mode of vibration the air column vibrates with three nodes and four antinodes as shown in the figure.

Length of the pipe, $l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4} = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2l}{3}$

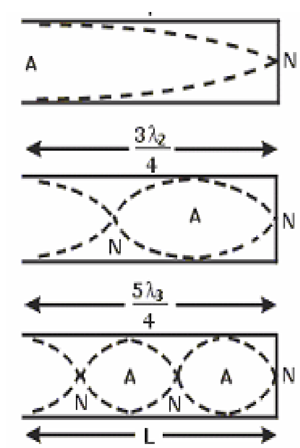
Frequency of vibration is given by, $n_3 = \frac{V}{\lambda_3} = 3\left(\frac{V}{2l}\right) = 3n_1$ ----------(3)

Where n_3 is called third harmonic or second overtone.

Hence from the above it is clear that the frequencies of the harmonics are in the ratio 1 : 2 : 3 : …. i.e the frequencies of the overtones present are multiple integral of the frequency of the fundamental mode of vibration.

2. How are stationary waves formed in a closed pipe ? Explain the various modes of vibrations in a closed pipe and establish the relation between their frequencies.(March2009)

A. **<u>Closed Pipe – Modes of Vibrations :</u>** A Pipe whose one end is closed and the other end is open is called closed pipe. At the closed end of the pipe always a node is formed due to the reflection of the wave and thus a stationary wave is formed. An anti-node is formed at the open end.



<u>Case – I</u>

In the first mode of vibration the air column vibrates with one node and one anti-node as shown in figure. Since the distance between a node and successive antinodes is $\frac{\lambda}{2}$,

Length of the pipe $l = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l$

Frequency of vibration is given by, $n_1 = \frac{V}{\lambda_1} = \frac{V}{4l}$ ---------(1)

where n_1 is called fundamental frequency or first harmonic.

<u>Case – II</u>

In the second mode of vibration the air column vibrates with two nodes and two antinodes as shown in figure.

Length of the pipe $l = \frac{\lambda_3}{2} + \frac{\lambda_3}{4} = \frac{3\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4l}{3}$

Frequency of vibration is given by, $n_3 = \frac{V}{\lambda_3} = 3\left(\frac{V}{4l}\right) = 3n_1.....(2)$

Where n_3 is called third harmonic or first overtone.

<u>Case – III</u>

In the third mode of vibration the air column vibrates with three nodes and three antinodes as shown in figure.

Lngth of the pipe, $l=\frac{\lambda_5}{2}+\frac{\lambda_5}{2}+\frac{\lambda_5}{4}=5\left(\frac{\lambda_5}{4}\right) \quad \Rightarrow \lambda_5=\frac{4l}{5}$

Frequency of vibration is given by, $n_5=\frac{V}{\lambda_5}=5\left(\frac{V}{4l}\right)=5n_1.....(3)$

where n_5 is called fifth harmonic or second overtone.

Hence from the above it is clear that the frequencies of the harmonics are in the ratio 1 : 3 : 5 ……

i.e. the frequencies of the overtones present are odd multiples of the frequency of the fundamental mode of vibration.

SHORT ANSWER QUESTIONS:

1. Explain the characteristic of a sound note.

A. The characteristics of sound notes are (i) Pitch, ii) Loudness and iii) Quality (or) timbre.

i) Pitch : The pitch of the note depends on frequency. The pitch of the buzzing of a bee is higher that the roar of a lion. The lower the frequency the lower is the pitch. Pitch is the subjective sensation of hearing while frequency is physically measurable quantity.

ii) Loudness : The loudness of note depends on the intensity of the wave. The greater the intensity the great the loudness. The roar of a lion is louder than the buzz of a bee. The intensity of the wave is directly proportional to the square of the amplitude of the wave. Loudness is the subjective sensation of hearing while intensity is physically measurable quantity.

iii) Quality or Timbre : It is the quality of a note which enables us to distinguish a note produced on one musical instrument from a note of the same pitch and intensity produced on another. This is due to the difference in number, order and relative intensities of the overtones which accompany the note. One can distinctly recognize the voice of a person due to the timbre of the characteristic notes from his vocal chords

2. What are free and forced vibrations? Explain with examples.

A. **Free vibrations :**

If an impulse is given to a body and if it is left free to itself, it vibrates with a finite frequency. The frequency of vibrations depends on the elastic constants the dimensions and the mode of vibration of the body, such vibrations are known as free vibrations or natural vibrations of the body.

Example : When a tuning fork is struck with a rubber hammer ,it vibrates with its natural frequency. This frequency of the fork depends on the elastic nature of the material, the mass distribution and the dimensions of the prongs of the fork.

Forced Vibration :

When a body is vibrated by an external periodic force such that the body vibrates with the frequency of the periodic force acting on it, then such oscillations are called forced vibrations.

Example : When the stem of a vibrating tuning fork is placed on the top of a table,a louder sound is heard due to forced vibrations

3. What is resonance? Explain with examples.

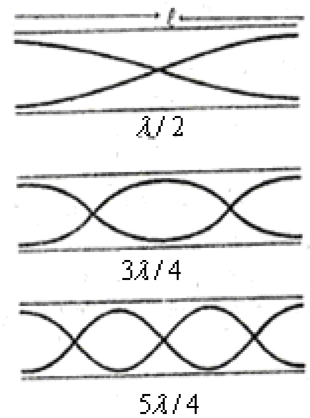
Resonance: If the frequency of forced vibrations is equal to the frequency of the free or natural vibrations resonance occurs and the amplitude of vibration increases enormously.

Resonant forced vibrations are also known as sympathetic vibrations.

Example : Consider a child in a swing. On giving a periodic push to the swing in the same direction every time it passes the extreme position, the amplitude of the swing goes on increasing and soon becomes very large. Since the natural frequency of the swing is equal to the frequency of the periodic force, resonance occurs between the two and hence the amplitude increases.

4. Explain the modes of vibrations in an open pipe with suitable illustrations.

A. **Modes of Vibration in an open Pipe :**



A pipe whose both ends are open is called open pipe. At the open ends of the pipe always antinodes are formed since the particles are free to vibrate.

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Fundamental frequency of vibration is given by, $n_1 = \frac{V}{\lambda_1} = \frac{V}{2l}$ ---------(1)

Where n_1 is called fundamental frequency or first harmonic.

Case – II

In the second mode of vibration, the air column vibrates with two nodes and three antinodes as shown in the figure.

Length of the pipe, $l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \lambda_2$ $\Rightarrow \lambda_2 = l$

Frequency of vibration is given by, $n_2 = \frac{V}{\lambda_2} = \frac{V}{l} = 2n_1$ ---------(2)

Where n_2 is called second harmonic or first overtone.

Case – III

In the third mode of vibration the air column vibrates with three nodes and four antinodes as shown in the figure.

Length of the pipe, $l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4} = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2l}{3}$

Frequency of vibration is given by, $n_3 = \frac{V}{\lambda_3} = 3\left(\frac{V}{2l}\right) = 3n_1$ ----------(3)

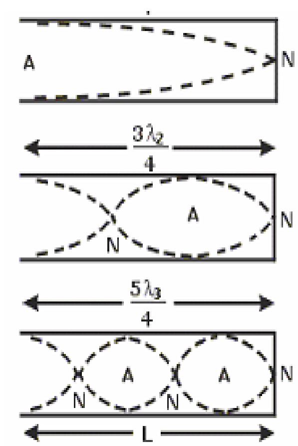
Where n_3 is called third harmonic or second overtone.

Hence from the above it is clear that the frequencies of the harmonics are in the ratio 1: 2: 3: …. i.e the frequencies of the overtones present are multiple integral of the frequency of the fundamental mode of vibration.

5. Explain the formation of standing wave pattern in a closed pipe with suitable figures.

A. **Modes of Vibrations in a closed Pipe**

: A Pipe whose one end is closed and the other end is open is called closed pipe. At the closed end of the pipe always a node is formed due to the reflection of the wave and thus a stationary wave is formed. An anti-node is formed at the open end.



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Frequency of vibration is given by, $n_5 = \frac{V}{\lambda_5} = 5\left(\frac{V}{4l}\right) = 5n_1.....(3)$

where n_5 is called fifth harmonic or second overtone.

Hence from the above it is clear that the frequencies of the harmonics are in the ratio 1 : 3 : 5 ……

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VERY SHORT ANSWER QUESTIONS:

1. What is Newton's formula for velocity of sound in a gas? How is it corrected by Laplace?

A. **Newton's formula :** The speed of longitudinal wave in a medium of elasticity E and density ρ is given by , $V = \sqrt{\frac{E}{\rho}}$ Where V is velocity of sound.

Newton assumed that when sound travels through a gas ,it undergoes an isothermal change. Isothermal elasticity of a gas is equal to its pressure.

$$\therefore V = \sqrt{\frac{P}{\rho}}$$

For air at NTP, $v = \sqrt{\frac{1.013 \times 10^5}{1.293}} \approx 280\, m/s$.

This value is about 16% less than the actual value which is about 332 m/s.

Laplace correction:

Laplace assumed that the propagation of sound in a gas as adiabatic.

According to Laplace, when sound waves travel through air, temperature does not remain constant and the process is adiabatic. The adiabatic elasticity of a gas is γ times the pressure of the gas.

$$\therefore V = \sqrt{\frac{\gamma P}{\rho}}\ where\ \gamma = \frac{C_p}{C_\gamma}$$

2. Distinguish between free and forced vibrations.

A. **Natural or free vibrations :** If an impulse is given to a body and if it is left free to itself, it vibrates with a finite frequency. Such vibrations are known as free vibrations or natural vibrations of the body Eg : The vibrations of a vibrating tuning fork.

Forced vibrations : When a body is set into vibration with the help of an external periodic force then the vibrations of the body are called forced vibrations Eg : When a vibrating tuning fork is placed on a wooden table, the later also vibrates. The vibrations of the table are called forced vibrations

3. Soldiers marching on a bridge often ordered to go out of step. Why?

A. Soldiers marching on suspended bridge are often ordered to go out of step. If they were to go marching in step, and if the natural frequency of the bridge happens to be equal to the frequency of their marching, resonance occurs and the amplitude of vibration may increase enormously and the bridge may collapse.

4. What is the ratio of the frequencies of harmonics in an air column of same length in a) a closed pipe and b) an open pipe?

A. a) In a closed pipe only odd harmonics are formed. The ratio of frequencies of harmonics in an air column of a closed pipe of length 'l' is

$$n_1 : n_2 : n_3 : n_4 ..... = \frac{V}{4l} : \frac{3V}{4l} : \frac{5V}{4l} : \frac{7V}{4l} .... = 1 : 3 : 5 : 7 : ....$$

b) In an open pipe, all harmonics are formed. The ratio of frequencies of harmonics in an air column of an open pipe of length ‘l’ is

$$n_1 : n_2 : n_3 : n_4 ..... = \frac{V}{2l} : \frac{2V}{2l} : \frac{3V}{2l} : \frac{4V}{2l} .... = 1 : 2 : 3 \quad ....$$

5. What is end correction in resonating air column?

A. Due to the pressure of the medium out side an organ pipe, anti-node is formed nearer the open end instead at the open end. Hence the length which is to be added to the resonating length is called end correction. If d is the diameter of the organ pipe, then end correction (e) = 0.3d.

SOLVED PROBLEMS

1. Find the frequencies of the first three harmonics of an open pipe of length 165 m, given the velocity of sound in air is equal to $330ms^{-1}$. What will be the first three frequencies if the pipe is closed at one end?

A. Velocity of sound in air $v = 330ms^{-1}$

Length of the pipe $l = 1.65m$

Frequency of the fundamental harmonic in the open pipe

$$v_1 = \frac{v}{2l} = \frac{330}{2\times1.65} = 100Hg$$

Frequency of the second harmonic = $2v_1 = 2\times100 = 200Hz$

Frequency of the third harmonic $3v_1 = 3\times100 = 300Hz$

If the pipe is closed at one end, only odd harmonics are possible.

Fundamental Frequency $v_1 = \frac{v}{4l} = \frac{330}{4\times16.5} = 50Hz$

Frequency of the 3rd harmonic = $3v_1 = 3\times50 = 150Hz$

Frequency of the 5th harmonic = $5v_1 = 5\times50 = 250Hz$

2. An open pipe and a closed pipe are in resonance with each other with their first overtones. Find the ratio of their lengths.(June2010)

A. Let the lengths of the open pipe are crossed pipes be l_1 and l_2 respectively.

Velocity of sound in air = v

Frequency of the first overtone of the open pipe $v_0 = \frac{2v}{2l_1}$

Frequency of the first overtone in closed pipe $v_c = \frac{3v}{4l_2}$

At resonance $v_0 = v_c$ *i.e.* $\frac{2v}{2l_1} = \frac{3v}{4l_2} or \frac{l_1}{l_2} = \frac{4}{3}$

UNSOLVED PROBLEM

1. The frequency of the fundamental note of a tube closed at one end is 200 Hz. What will be the frequency of the fundamental note of a similar tube of same length but open at both ends? (May2009)

A. For closed tube, Fundamental frequency $= n_c = \frac{v}{4lc} = 200Hz$

For open tube, Fundamental frequency $= n_o = \frac{v}{2l_o}$ where $l_0 = l_c$

$$\Rightarrow \frac{n_0}{n_c} = \frac{\left(\frac{v}{2l_0}\right)}{\left(\frac{v}{4l_c}\right)} = \frac{4}{2} = 2 \qquad \Rightarrow n_0 = 2x\, n_c = 2x\, 200 = 400Hz$$

2. Two tuning forks A and B give 6 beats per second. A resonates with a closed column of air 15 cm long and B with an open column 30.5 cm long in their fundamental harmonics. Calculate their frequencies.

A. Length of closed air column = $l_c = 15cm$

Length of open air column = $l_0 = 30.5cm$

Frequency of $A = n_A = n_c = \frac{v}{4l_c}$

Frequency of $B = n_B = n_0 = \frac{\upsilon}{2l_c}$

Number of beats per second = $n_A - n_B = 6$

$$\Rightarrow \frac{v}{4l_c} - \frac{v}{2l_0} = 6 \qquad \Rightarrow v\left[\frac{1}{4(15)} - \frac{1}{2(30.5)}\right] = 6$$

$$\therefore \upsilon = 219.6ms^{-1}$$

$$n_A = \frac{v}{4l_c} = \frac{219.6}{4\times15\times10^{-2}} = 366Hz$$

$$n_B = \frac{v}{2l_0} = \frac{219.60}{2\times30.5\times10^{-2}} = 360Hz$$

ASSESS YOURSELF

1. A sound wave traveling along an air column of a pipe gets reflected at the open end of the pipe. What is the phase difference between the incident and reflected waves at the open end?

A. 2 π.

2. Why sound cannot travel in vacuum, while light can?

A. Sound waves being mechanical require a material medium for their propagation.

3. What is the characteristic of sound which distinguishes a male voice and a female voice?

A. Quality or timber.

4. What is the range of musical sounds?

A. 30Hz to 5000Hz

5. Three identical sound waves sent through an air column, a brass rod and an oil pipe of same length. In which of the three will it take the least time to reach the other end?

A. Brass rod. Velocity of sound is maximum in solids.

6. Sound travels in two gases with densities $\rho_1 \text{ and } \rho_2$ under identical conditions. What is the ratio of the velocities of sound in the two?

A. $v \alpha \frac{1}{\sqrt{\rho}} \Rightarrow v_1 : v_2 = \sqrt{\rho_2} : \sqrt{\rho_1}$

7. What is the influence of resonance on the forced vibrations?

A. The amplitude of vibration increases.

8. What is the condition for resonance to occur?

A. When the natural frequency becomes equal to the forced frequency, resonance occurs.

9. What is the difference between the harmonics produced in closed pipe and open pipe?

A. In an open pipe all harmonics are possible where in a closed pipe only odd harmonics are possible.

10. What is the distance between the closed end and open end of a pipe vibrating in the 7^{th} harmonic?

A. $7\lambda/4$.

11. What is the distance between two ends of open pipe vibrating in the 3^{rd} over tone?

A. 2λ.