WAVE MOTION
2. STRINGS

POINTS TO REMEMBER

1. Frequency of a vibrating string :
   a) The waves formed in a string under tension are transverse stationary and polarized waves.
   b) Always nodes are formed at fixed ends and antinodes at plucked points and free ends.
   c) A string can have number of frequencies depending on its mode of vibration.
2. Fundamental frequency: When a string vibrates in a single loop, it is said to vibrate with fundamental frequency.
   a) Frequency is minimum and wavelength is maximum in this case.
   b) If l is the length of the string $l=\frac{\lambda}{2} \Rightarrow \lambda = 2l$.
   c) The fundamental frequency, $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$ where T=tension, m=linear density.
   d) The fundamental frequency is also given by $n=\frac{1}{2}\sqrt{\frac{T}{Ml}} = \frac{1}{2l}\sqrt{\frac{T}{Ad}} = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}}$.
   e) For small change in tension in string, the fractional change in frequency is $\frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta T}{T}$.
   f) The fundamental frequency is also called the first harmonic.
3. Overtones : If string vibrates with more number of loops, higher frequencies are produced called overtones.
   a) If string vibrates in p loops, it is called p^{th} mode of vibration or p^{th} harmonic or $(p-1)^{th}$ overtone. The corresponding frequency $n_p = \frac{p}{2l}\sqrt{\frac{T}{m}}$ =p.n Hence, for a string, $n_p \alpha p$; $\frac{n_1}{n_2} = \frac{p_1}{p_2}$ when other, quantities are constant.
   b) The fundamental and overtone frequencies are in the ratio 1:2:3:4:….
   c) The wavelength is above case is $\lambda_p = \frac{2l}{p}$ i.e., wavelengths are in the ratio $1:\frac{1}{2}:\frac{1}{3}:...$
4. Laws of transverse waves along stretched string :
   a) Law of length : The frequency of a stretched string is inversely proportional to the length of the string $n \alpha 1/l$ where T & m are constants, nl=constant, $n_1 l_1 = n_2 l_2$.
   b) Law of tension : The frequency of a stretched string is inversely proportional to square root of tension. $n \alpha \sqrt{T}$ when l & T are constant. $\frac{n}{\sqrt{T}}$=constant, $\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}}$.

c) Law of mass : The frequency of a stretched string is inversely proportional to square root of linear density $n \alpha \frac{1}{\sqrt{m}}$ when l & T are constants. $n\sqrt{m}$ =constant; $n_1\sqrt{m_1} = n_2\sqrt{m_2}$.

5. Sonometer is used to determine the velocity of transverse waves in strings and to verify the laws of transverse waves.

LONG ANSWER QUESTIONS

1. Explain the formation of stationary waves in stretched strings and hence deduce the laws of transverse waves in stretched strings.

(March'10,09,08,05)

A. Let a string of length 'l' and linear density 'm' be fixed between two supports with tension 'T'.A transverse waves is produced in the string by plucking it. A stationary wave is formed in the string due to the superposition of the waves. At the points where the string was fixed rigidly nodes are formed. The velocity of transverse vibration in a stretched string is given by $V = \sqrt{\frac{Tension}{Linear\ density}} = \sqrt{\frac{T}{m}}$

When the string is plucked at the middle of the string, it vibrates in a single loop. $\therefore$ Length of the string $l = \lambda / 2\ or\ \lambda = 2l.$ If n_1 is the fundamental frequency of string, then

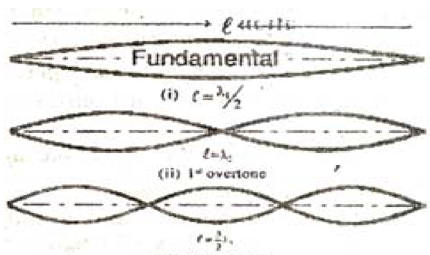
Frequency of vibration $n_1 = \frac{V}{\lambda} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$

n_1 is called fundament frequency or first harmonic .

When the string is plucked at a point at the distance $\frac{l}{4}$ from fixed end, it then vibrates with two loops.

The frequency of vibration , $n_2 = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2\left[\frac{1}{2l}\sqrt{\frac{T}{m}}\right] = 2n_1$

n_2 is called 1st overtone or second harmonic . Similarly if three loops are formed $n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n_1$



Harmonics on stretched strings

n_3 is called2nd overtone or third harmonic . If the number of loops is P then, Fequency of vibration, $n_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$

Hence $n_1 : n_2 : n_3 .... n_p = 1 : 2 : 3 .... p$

Laws of transverse vibrations :

First Law : Fundamental frequency (n) of a string vibrating in one loop is inversely proportional to its length (l),its tension T and linear density m being constant .

$n \alpha \frac{1}{l}$ (T and m are constant.) This is called the law of lengths.

Second Law : Fundamental frequency of (n) of a stretched string vibrating in one loop is proportional to square root of tension $\left(\sqrt{T}\right)$ in the string, its length l and linear density m being constant.

$n \alpha \sqrt{T}$ (l and m are constant). This is called the law of Tensions.

Third Law : Fundamental frequency of (n) of a stretched string vibrating in one loop is inversely proportional to the square root of linear density $\left(\sqrt{m}\right)$ of wire used , tension T and length l being constant).

$n \alpha \frac{1}{\sqrt{m}}$ (T and l are constant).This is called the law of linear densities.

2. State laws of transverse vibrations in stretched strings. Explain how do you verify them using sonometer. (March 2011,2007)

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First Law : Fundamental frequency (n) of a string vibrating in one loop is inversely proportional to its length (l),its tension T and linear density m being constant .

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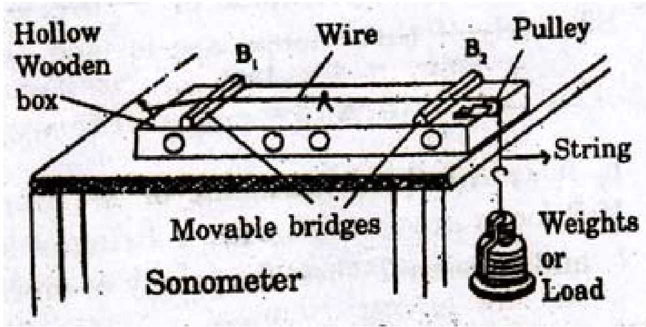
Description of Sonometer :



A sonometer consists of a hollow rectangular wooden box of length about one metre and width about 12 cm with holes at the sides. A thin uniform metal wire is fixed rigidly at one end to a peg and a weight hanger is attached at the other end passing over a smooth pulley. The weight of the load to the hanger gives the tension in the string. The length of vibrating wire segment is adjusted between two movable bridges as shown in figure. A light paper rider of shape '$\wedge$' is placed at the middle of the segment and it flies off at resonance ,Let 'm' be the linear density of string.

To verify first Law: According to first law, the frequency of a stretched string vibrating in one loop is inversely proportional to its length. $n \alpha \frac{1}{t}$ *or* $nl = cons\tan t$

Let a wire of uniform cross-section be fixed on the sonometer and a constant load is applied to the hanger, so that tension in the wire remains constant.

A tuning fork of frequency n_1 is struck gently with a rubber hammer and placed on wooden box. By adjusting the distance between the bridges, the wire is set into resonance, such that the paper rider placed at the middle flutters and flies off. The length 'l_1'of vibrating segment between the bridges is noted.

The experiment is repeated for the various tuning forks of frequencies $n_1, n_2, n_2, ...$ and their corresponding length $l_1, l_2, l_3....$ are noted and tabulated as follows

s.no	Frequency of tuning fork (n)	Length of wire in Resonance I	II		n.l = constant
				Mean(l)	

Thus first law is verified

To verify second Law : The frequency of vibration of stretched string vibrating in one segment is directly proportional to the square root of tension. $n\alpha\sqrt{T}$

But it is difficult to find a tuning fork that suits a particular value of tension and hence this law is verified indirectly by using 1st and 2nd laws.

This law verified as $\frac{\sqrt{T}}{l}$ = constant.

Now a tuning fork of frequency n is vibrated, and the resonating length l_1 is found for tension T_1.

The experiment is repeated for various values of $T_1, T_2, T_3, \ldots$ tensions and corresponding resonating lengths $l_1, l_2, l_3 \ldots$ are noted. The values are tabulated as follows.

<table>
<tr><td rowspan="2">s.no</td><td>Tension applied
T</td><td colspan="3">Resonating length of wire
I II</td><td>$\frac{\sqrt{T}}{l}$ = constant</td></tr>
<tr><td></td><td></td><td></td><td>Mean(l)</td><td></td></tr>
</table>

Thus second law is verified.

<u>To verify third law :</u> The frequency of string is inversely proportional to square root of its linear density. The frequency $n \alpha \frac{1}{\sqrt{m}}$

But it is difficult to find a tuning fork which suits a particular linear density and hence this law is verified by using 1st and 3rd laws.

This law is indirectly verified as $l\sqrt{m}$ = constant with only one tuning fork.

($\therefore n$ is constant)

A wire of linear density $'m_1'$ is fixed to the sonometer, then resonating length l_1 is noted by placing vibrating tuning fork on the box .

The experiment is repeated with another wire of linear density m_2 by placing same weight ie. under the same tension. Let the resonating length l_2, is found. The values are tabulated as follows.

<table>
<tr><td rowspan="2">s.no</td><td>Linear density</td><td colspan="3">Resonating length of wire</td><td>$1\sqrt{m}$ = constant</td></tr>
<tr><td>If wire (n)</td><td>I</td><td>II</td><td>Mean (l)</td><td></td></tr>
</table>

Thus thir law is verified.

<u>Precautions:</u>

1. Paper rider must be placed on the wire exactly at the centre of the two bridges.
2. Weight hanger should not touch the experimental bench.
3. Tension applied should be within the elastic limit of the string.

` 4. The tuning fork must always be held at its stem only.

<u>SHORT ANSWER QUESTIONS</u>

1. State and explain the laws of transverse waves along a stretched string.

A. <u>Laws of transverse vibrations :</u>

<u>First Law</u> : Fundamental frequency (n) of a string vibrating in one loop is inversely proportional to its length (l),its tension T and linear density m being constant .

$n \alpha \frac{1}{l}$ (T and m are constant.) This is called the law of lengths.

<u>Second Law</u> : Fundamental frequency of (n) of a stretched string vibrating in one loop is proportional to square root of tension $\left(\sqrt{T}\right)$ in the string, its length l and linear density m being constant.

$n \alpha \sqrt{T}$ (l and m are constant). This is called the law of Tensions.

Third Law : Fundamental frequency of (n) of a stretched string vibrating in one loop is inversely proportional to the square root of linear density $\left(\sqrt{m}\right)$ of wire used , tension T and length l being constant).

$n \alpha \frac{1}{\sqrt{m}}$ (T and l are constant).This is called the law of linear densities.

2. Explain the harmonics on a stretched string with suitable illustrations.

A. When the string is plucked at the middle of the string, it vibrates in a single loop. $\therefore$ Length of the string $l = \lambda / 2 \; or \; \lambda = 2l.$ If n_1 is the fundamental frequency of string, then

Frequency of vibration $n_1 = \frac{V}{\lambda} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$

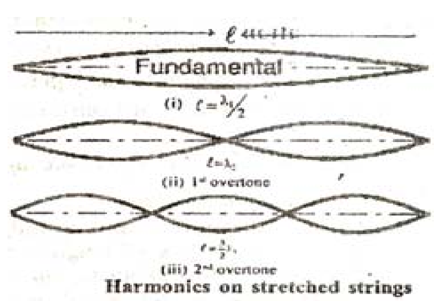
n_1 is called fundament frequency or first harmonic .

When the string is plucked at a point at the distance $\frac{l}{4}$ from fixed end, it then vibrates with two loops.

The frequency of vibration , $n_2 = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2\left[\frac{1}{2l}\sqrt{\frac{T}{m}}\right] = 2n_1$

n_2 is called 1st overtone or second harmonic . Similarly if three loops are formed

$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n_1$



Harmonics on stretched strings

n_3 is called2nd overtone or third harmonic . If the number of loops is P then,

Fequency of vibration, $n_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$

Hence $n_1 : n_2 : n_3 .... n_p = 1:2:3....p$.

VERY SHORT ANSWER QUESTIONS

1. A wire of length 'l' is vibrating in three segments. What is the wavelength of the note emitted?

A. The distance between two successive nodes is $\lambda / 2$.

$$\therefore l = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2l}{3}$$

2. What is the ratio of the frequency of fourth overtone to the fundamental frequency of a stretched string?

A. If fundamental frequency is n_1 Fourth overtone is equal to 5^{th} harmonic.

Frequency of 5^{th} harmonic is $5n_1$

$\therefore$ Ratio of frequencies is $5:1$

3. What happens to the fundamental frequency of stretched string when the tension is quadrupled?

A. The fundamental frequency of a stretched string becomes doubled when tension is quadrupled.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T}{4T}} = \frac{1}{2} \Rightarrow n_2 = 2n$$

4. What happens to the fundamental frequency of a stretched string when the linear density becomes 1/4 of its initial value?(March2006)

A. $$\frac{n_2}{n_1} = \sqrt{\frac{m_1}{m_2}} \quad Given \quad m_2 = \frac{m_1}{4}$$

$$\therefore \frac{n_2}{n_1} = \sqrt{4} \Rightarrow n_2 = 2n_1$$

Hence the Fundamental frequency of a string is doubled.

5. A tuning fork is in unison with a stretched wire length 50cm and linear density 1 g/m. What is the frequency of the tuning fork if the tension in wire is 40 N?

A. l = 50 cm = 0.5 m and T = 40 N

m = 1 gm/m = $1\times10^{-3} kg/m$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times0.5}\sqrt{\frac{40}{1\times10^{-3}}} \Rightarrow n = 200\,Hz$$

SOLVED PROBLEMS

1. A metal wire of length 1 m has a mass of 33g. Find the tension required to stretch the wire to propagate a transverse wave along its length with a speed equal to $1/10^{th}$ of the speed of sound in air at $0^0 C$. (Velocity of sound in air at $0^0 C = 330 ms^{-1}$)

A. Linear density of the wire $= m = \frac{33\times10^{-3}}{1} = 33\times10^{-3} kgm^{-1}$

Velocity of sound in air at $0^0 C = 330 ms^{-1}$

Velocity of the transverse wave $v=\left(\frac{1}{10}\times 330\right)=33ms^{-1}$

But $v=\sqrt{\frac{T}{m}}$

$\therefore v=\sqrt{\frac{T}{33\times 10^{-3}}}=33 \quad \Rightarrow \quad T=(33)^{3}\times 10^{-3}=35.937N$

2. A wire of length 1 m and mass 20 g is stretched with a force of 800 N. Find the fundamental frequency. Also find the frequencies of the first two overtones. (March2011)

A. Length of the wire (l) = 1m

Tension in the string (T) = 800 N.

Mass per unit length of the wire $(m)=\frac{mass}{length}=\frac{20\times 10^{-3}}{1}=20\times 10^{-3}kgm^{-1}$

Fundamental frequency $v=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2\times 1}\sqrt{\frac{800}{20\times 10^{-3}}}=100Hz$

The frequency of 1st overtones $v_1=2v=2\times 100=200Hz$

The frequency of 2nd overtone $v_2=3v=3\times 100=300Hz$

<u>UNSOLVED PROBLEM</u>

1. The third overtone produced by a vibrating string 2m long is 1200 Hz. What is the velocity of propagation of the wave?

A. Length of string $l=2m$

Frequency of 3rd overtone = 4th harmonic

$n_4=1200Hz$

$\Rightarrow n_4=4n_1=1200$

$\Rightarrow n_1=\frac{1200}{4}=300 \quad \Rightarrow \frac{1}{2l}\sqrt{\frac{T}{m}}=300(or)\frac{1}{(2\times 2)}\times v=300$

$\therefore v=1200ms^{-1}$

2. In a sonometer a stretched wire is observed to vibrate with a frequency of 30 Hz in the fundamental mode, when the distance between bridges is 0.6m. If the string has a linear mass of $0.05kgm^{-1}$, find (a) the velocity of propagation of transverse wave in the string and (b) the tension in the string.

A. n = 30 Hz, l = 0.6 m.

$\lambda=2l=1.2m$ and m = 0.05 kg/m

a) Velocity $v=n\lambda$

$\therefore v=30\times 1.2=36m/\sec$

b) From $v=\sqrt{\frac{T}{m}}$ or $T=m.v^2$

$\therefore T = 0.05 \times (36)^2 = 84.8N$

3. Two wires are fixed on a sonometer wire. Their tensions are in the ratio 8 : 1, the length are in the ratio 36 : 35, the diameters in the ratio 4 : 1 and densities in the ratio 1 :2. If the string of higher pitch has a frequency of 360 Hz. Calculate the frequency of the other string?

A. $\frac{T_1}{T_2} = \frac{8}{1}; \frac{l_1}{l_2} = \frac{36}{35}, \frac{r_1}{r_2} = \frac{4}{1}\, and\, \frac{d_1}{d_2} = \frac{1}{2}$

$$\frac{n_1}{n_2} = \frac{l_2}{l_1}.\frac{r_2}{r_1}\sqrt{\frac{T_1}{T_2}.\frac{d_2}{d_1}} = \frac{35}{36}.\frac{1}{4}\sqrt{8\times 2} = \frac{35}{36}\times\frac{1}{4}\times 4 \Rightarrow \frac{n_1}{360} = \frac{35}{36}$$

$$n_1 = 350Hz$$

4. Two strings ‘x’ and ‘y’ on a Veena playing the same note are slightly out of tune and produce 6 beats per second. The tension in ‘x’ is slightly decreased and it is found that the beats fall to 3 per second. If the original frequency of ‘x’ is 324 Hz what is the frequency of ‘y’?

A. Frequency of x is $n_x = 324Hz$

Frequency of y is $n_y = ?$

No of beats $\Delta n = n_x \quad n_y = 6 \Rightarrow \Delta n = n_x^1 \quad n_y = .3$.

But $n \alpha \sqrt{T}$. Thus the frequency of ‘x decreases when tension in it decreases.

If $\Delta n = n_y - n_x$ then the number of beats should increase. But here the beats are decreased and hence.

$$\Delta n = n_x - n_y \Rightarrow n_y = n_x - \Delta n$$

$$\therefore n_y = 324 - 6 = 318Hz$$

5. A tuning fork is set vibrating on the sound box of a sonometer. 3 beats are heard in 10 seconds when the tension in the wire is 20 N and 20.1 N. What is the frequency of the tuning fork?

A. $n \propto \sqrt{T} \quad \Rightarrow \quad \frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta T}{T}$

Let n be the frequency of the tuning fork and n_1, n_2 be the frequencies of the sonometer wires

$$n - n_1 = 0.3$$

$$n_2 - n = 0.3$$

$$n_2 - n_1 = \Delta n = 0.6$$

$$\therefore \frac{0.6}{n} = \frac{1}{2}\frac{0.1}{20} \Rightarrow n = 240Hz$$

6. A tuning fork is set vibrating on the sound box of a sonometer. 5 beats per second are heard when the length of the wire is 20 cm and 21 cm keeping tension constant. What is the frequency of the tuning fork?

A. Let n be the frequency of the tuning fork

$$n_1 - n = 5$$

$$\frac{n - n_2 = 5}{n_1 - n_2 = 10}$$

and $\frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{n_1}{n_2} = \frac{21}{20} \quad \Rightarrow n_1 = \frac{21}{20} n_2$

$n_2 = 200Hz \quad \Rightarrow \quad n = 205Hz$

7. A wire 90 cm long and of mass 1.2 g is making 256 vibrations per second under a tension supplied by a brass weight hanging vertically in air. On immersing the weight in water, the vibrating length of the wire is to be decreased by 5.4cm to regain its original frequency. What is the density of brass?

A. $n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$

In air, $n_1 = \frac{1}{2\times 90}\sqrt{\frac{d_B}{m}}$

In a liquid, $n = \frac{1}{2\times 84.6}\sqrt{\frac{d_B - d_l}{m}}$

But , $n_1 = n_2$

$$\frac{1}{2\times 90}\sqrt{\frac{d_B}{m}} = \frac{1}{2\times 84.6}\sqrt{\frac{d_B - d_l}{m}} \quad \Rightarrow \quad \frac{\sqrt{d_B}}{90} = \frac{\sqrt{d_B - dl}}{84.6}$$

$$d_B = 8.591\times 10^3 kg / m^3$$

8. A stretched sonometer wire is in unison with a tuning fork. When the length is increased by 2% the number of beats per second is 5. Find the frequency of the fork.

A. $n \propto \frac{1}{l} \Rightarrow \frac{\Delta n}{n} = \frac{\Delta l}{l}$

$\frac{5}{n} = \frac{2}{100} \quad \therefore n = 250Hz$

9. Find the velocity of propagation of a transverse wave, if a string of length 0.5. produces a fundamental note frequency300Hz. (March 2009)

A. $n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{V}{2l} \Rightarrow V = 2l\,\mathrm{n} = 2\times 0.5\times 300 = 300m / s$

ASSESS YOURSELF

1. Which type of waves are formed due to vibrations of stretched strings?

A. Transverse stationary waves.

2. When stretched string vibrates in two segments, how many nodes and antinodes will be there?

A. 3 nodes and 2 antinodes.

3. What is the frequency of the 9th overtone on a stretched string of length 'l'; and linear density 'm' when the tension is $"T"$?

A. $v = \frac{10}{2l}\sqrt{\frac{T}{m}} = \frac{5}{l}\sqrt{\frac{T}{m}}$

4. Two identical wires on a sonometer are stretched with the same tension $"T"$ If their lengths are in the ratio 1 : 2, what is the ratio of their frequencies?

A. $v \alpha \frac{1}{l} \Rightarrow v_1 : v_2 = 1 = 2 : 1$

5. When do the paper riders on a sonometer wire fly off?

A. Due to resonance, when the string vibrates with maximum amplitude, the paper rider on the sonometer wire flies off.

6. A stretched string is plucked and it vibrates transversely. Are the vibrations free or forced?

A. Free vibrations.