WAVE MOTION

1. WAVE MOTION

POINTS TO REMEMBER

1. Sound is a form of energy which gives the sensation of hearing.
2. Audible limits
   Frequency: 20 Hz to 20,000 Hz
   Wave length: 17 m to 0.017m
   Time period: 0.05 to 0.00005 sec
3. The process of transmitting energy through the vibrations of the particles of the medium is known as wave motion.
4. The wave motion in which the particles of the medium vibrate about their mean positions at right angles to the direction of propagation of the wave is called transverse wave.
5. The wave motion in which the particles of the material medium vibrate back and forth about their mean position along the direction of the propagation of wave is called longitudinal wave.
6. Phase difference = $\frac{2\pi}{\lambda}\times$ Path difference Or $\phi=\frac{2\pi}{\lambda}x=Kx$

   Where K is called propagation constant (cm^{-1}) Angular wave number.
7. Phase difference = $\frac{2\pi}{T}\times\text{Time difference}$ Or $\phi=\frac{2\pi}{T}t=wt$

   Where ω is angular frequency.
8. A mathematical representation of the displacements of the particles on a wave along the direction of propagation is known as the wave equation. It can be written as $y(x,t)=A\sin\omega\left(t-\frac{x}{v}\right)$

   $$y(x,t)=A\sin\frac{2\pi}{T}\left(t-\frac{x}{v}\right)=A\sin 2\pi\left(\frac{t}{T}-\frac{x}{vT}\right)$$

   $$=A\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)=A\sin\frac{2\pi}{\lambda}(vt-x)$$

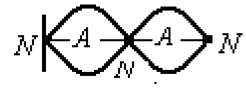
   $$y=A\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)=A\sin\left(\frac{2\pi}{T}t-\frac{2\pi}{\lambda}x\right)$$

   $$\therefore y(x,t)=A\sin(\omega t-kx)$$

   Where ω is called the angular frequency (sec^{-1}) and K is called the propagation constant (cm^{-1}) or angular wave number.

   Wave velocity = $\frac{\omega}{K}=V$
9. If two or more waves propagate simultaneously in a medium the resultant wave is due to the resultant displacement of the particles. This is called the principle of superposition of waves. $\vec{y}=\vec{y}_1+\vec{y}_2+\vec{y}_3+.....$
10. When a progressive waves is reflected at at a fixed end, the incident and reflected waves will be out of phase by π. If the reflection is at an open end, the reflected and incident waves will be in phase.

11. Two waves of same frequency, same amplitude and traveling in opposite directions superpose stationary waves are formed. Since these are confined to a limited region, energy transfer does not take place.



12. Particles in a loop have same phase and the particles in successive loops have a phase difference π radians.

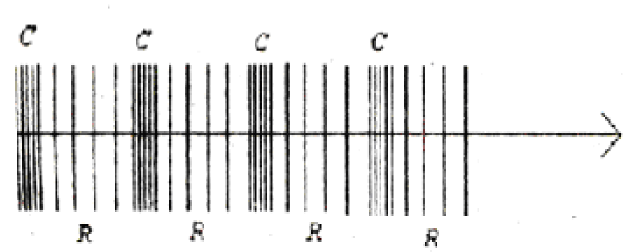
SHORT ANSWER QUESTIONS

1. What are longitudinal waves? Explain with an example.?

Longitudinal waves:

The wave in which the particles of the medium vibrate in the direction of propagation of the wave is called longitudinal wave.

Example:



When a light spring held horizontally is compressed, it produces a pulse of pressure along the spring. this compression tends to release the pressure in the region by pushing the neighbouring layers of the spring forming a rarefaction. Hence this compression is transmitted through the spring. If we push the spring at one end repeatedly at regular intervals of time a periodic longitudinal progressive wave takes place along the length of the spring.

Example : Sound waves in Air.

Characteristics of longitudinal waves:

a) Compressions (C) and Rarefactions (R) are formed alternately.

b) A compression and a rarefaction make up a complete wave.

c) The distance between any two successive compressions or rarefactions is called wavelength (λ) of the wave.

d) All the particles of the medium execute S.H.M with the same amplitude and time period.

e) The phase difference between two successive compressions or rarefactions is 2π radians.

f) Transfer of energy takes place in the direction of wave propagation.

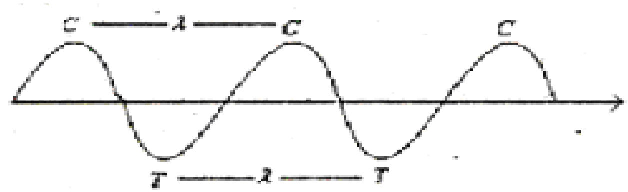
g) These waves can be propagated through solids, liquids and gases.

h) These waves can not be polarized.

2. What are transverse waves? Explain with an example.?

A. **Transverse waves :**

The wave in which particles of the medium vibrate perpendicular to the direction of propagation of the wave is called a transverse wave.



<u>Example:</u> When a horizontal light spring is moved vertically up and down at its one of end periodically, the length of the spring gets into the shape of the curve as shown in figure with ups and down. The ups are the locations of maximum displacements upwards and are known as crests(C). The downs are the maximum displacements in the downward direction and know as troughs (T).Hence the displacement crests and troughs produced at one end vertically, are handed over to the successive turns of the spring producing a transverse progressive wave.

<u>Characteristics of Transverse waves:</u>

a) Crests (C) and troughs (T) are formed alternately

b) A crest and a trough make up a complete wave.

c) The distance between any two successive crests or troughs is called the wavelength (λ) of the wave

d) All the particles in the medium execute S.H.M with the same amplitude and time period

e) The phase difference between any two successive crests or through is equal to 2π radians.

f) Transfer of energy takes place in the direction of wave propagation

g) These waves can be propagated only in solids

h) Transverse waves can be polarized.

3. What is a simple harmonic wave? Deduce progressive wave equation.?

A. The simplest form of a progressive wave which results from the simple harmonic oscillations of the particles of the medium is known as a simple harmonic wave.

Consider a progressive wave traveling with a constant velocity 'v' along positive x- direction from the origin. Let 'T' be the period of the wave and 'λ' be the wavelength. Every particle in the direction of wave motion receives the energy from the previous particle with a slight time delay and hence it differs in phase. Let ϕ be the phase difference. The phase of such a particle at any instant is $(\omega t-\phi)$. Where ω angular speed. The displacement of the particle is given by

$$y = a\sin(\omega t-\phi) \text{------ (1)}$$

Where a' is the amplitude of the wave . But phase difference $\phi = \frac{2\pi}{\lambda}x$

$$\therefore\ y = a\sin\left(\omega t-\frac{2\pi}{\lambda}x\right) \quad \text{Or} \quad y = a\sin(\omega t-kx) \text{--------(2)}$$

Where $k=\frac{2\pi}{\lambda}$ is called the propagation constant.

The wave traveling in the negative X-direction is given by

$$y = a\sin(\omega t - kx) \text{ -------(2)}$$

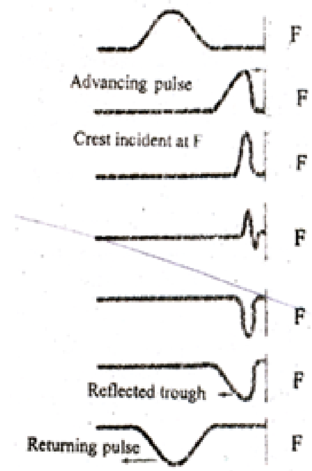
In general a progressive wave can be by,

i) $y = a\sin 2\pi\left(\frac{t}{T} \mp \frac{x}{\lambda}\right) or\ y = a\cos 2\pi\left(\frac{t}{T} \mp \frac{x}{\lambda}\right)$

ii) $y = a\sin\frac{2\pi}{\lambda}(vt \mp x) or\ y = a\cos\frac{2\pi}{\lambda}(vt \mp x)$

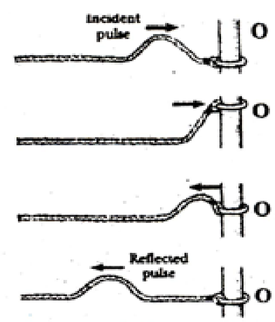
4. **Explain the reflection of waves at closed and open ends.?**

A. **Reflection at closed end :** Consider a string whose one end is attached to a rigid support and a transverse wave pulse is produced along the string by giving a jerk at the other end.



Reflection of waves at fixed end (F)

Suppose the wave pulse travels in the form of a crest from left to right. On reaching the rigid support, the pulse exerts upward force F on the boundary. Since the support is rigid, it remains unaffected but it gives an equal and opposite reaction on the string. This downward reaction reverses the sign of the displacement of the particle of the string and hence an inverted pulse in the form of trough travels from right to left. Thus a phase changes of π takes place in the displacement, when a transverse wave is reflected at the rigid support. Similarly the rarefaction of a longitudinal wave arriving at a rigid end (boundary) is reflected as a rarefaction. Thus the reflection of the waves takes place at the fixed (rigid) end with the phase reverse by π.



Reflection of waves at open end (O)

Reflection at open end : Consider that one end of the string tied to a mass less ring which is capable of sliding up and down over a smooth rod. When the pulse generated at the free end of the string reaches the ring it exerts an upward force F on the end at 'O' of the string

As the ring is free to slide over the rod, it does not encounter any opposition and hence it rises up. Thus the crest is reflected as a crest and no phase reversal in displacement at the free boundary. Similarly, the rarefaction of a longitudinal wave is also reflected as compression without any change in the phase at a open boundary.

VERY SHORT ANSWER QUESTIONS:

1. Distinguish between Longitudinal waves and transverse waves.

Longitudinal waves	Transverse waves
1. Particles of the medium vibrate in the direction of propagation of the wave.	1. Particles of the medium vibrate in a direction perpendicular to the propagation

2. Compressions and rarefactions are formed in longitudinal waves.	of the wave.
3. These waves cannot be polarized.	2. Crests and troughs are formed in transverse waves.
4. These waves travel in solids, liquids and gases.	3. These waves can be polarized
	5. These waves can be produced only in solids (can be produced partly in liquids)

2. Name the parameter of a progressive wave.

A. The parameters of a progressive wave are (i) amplitude (ii) frequency (iii) time period (iv) wavelength and (v) phase

3. Establish the relation between the wave velocity (v), frequency (n) and the wavelength (λ) of a progressive wave.

A. Let V be the velocity of the wave and n is the frequency of oscillation of the particles of the medium .The wave travels a distance equal to wave length (λ) during the time period (T) of the oscillation of the particles of the medium.

$$\text{Wave Velocity} = \frac{\text{Distance travelled by the wave}}{\text{Time period}} \qquad \text{(Or)} \qquad V = \frac{\lambda}{T}$$

But $n = \frac{1}{T}$

$$\therefore v = n\lambda$$

4. What is the principle of superposition of waves?

A. The principle of superposition of waves states that when two or more waves are simultaneously interfere, the resultant displacement of any particle is equal to the algebraic sum of the displacements of all the waves. If $y_1, y_2, y_3, ...$ are the displacements due to the overlapping of waves, then the resultant displacement of any particle is given by $y = y_1 + y_2 + y_3 + ...$

5. What are the conditions required for a wave to get reflected?

A. Progressive waves traveling in a medium get reflected when the medium ends abruptly at any point. They get reflected even if the density and rigidity of the medium changes at any point. Waves get reflected partially when they refract into another medium.

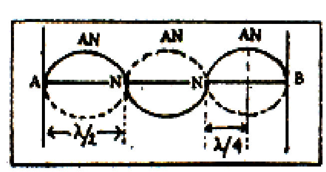
6. What is the phase difference between incident and reflected waves at a) an open end and b) a closed end?

A. a) Phase difference between the incident and reflected waves Is 2π at an open end.

b) Phase difference between the incident and reflected waves is π at a closed end.

7. What is stationary wave? What is the distance between the node and the succeeding anti-node of a stationary wave?

A. A stationary wave is formed due to the superposition of two progressive waves of same



amplitude and frequency traveling in opposite directions along the same straight line.

The distance between a node and the adjacent anti-node is equal to a quarter of wavelength $\left(\frac{\lambda}{4}\right)$.

SOLVED PROBLEMS

1. A longitudinal progressive wave is given by the equation $y = 5\times10^{-2}\sin\pi(400t + x)$ Find i) amplitude ii) frequency iii) wave length and iv) velocity of the wave.

A. The equation of the progressive wave $y = 5\times10^{-2}\sin\pi(400t + x)$

$$= 5\times10^{-2}\sin(400\pi t + \pi x)$$

Comparing with the general equation of the progressive wave , $y = A\sin(\omega t + kx)$

$$\omega = 400\pi \; and \; k = \pi$$

i) Amplitude = $5\times10^{-2} m$

ii) Frequency = $v = \frac{\omega}{2\pi} = \frac{400\pi}{2\pi} = 200Hz$.

iii) Wavelength = $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2m$

iv) Velocity = $v = \frac{\omega}{k} = \frac{400\pi}{\pi} = 400ms^{-1}$

2. A sound wave with amplitude of 3 cm starts towards right from origin and gets reflected at a rigid wall after a second. If the velocity of the wave is $340ms^{-1}$ and it has a wavelength of 2m, deduce the equations of the incident and reflected waves.

A. The amplitude of the wave $A = 3\times10^{-2} m$

Velocity of the wave $v = 340ms^{-1}$.

The wavelength of the wave $\lambda = 2m$

The incident wave towards right is given by

$$y = A\sin\frac{2\pi}{\lambda}(vt - x) = 3\times10^{-2}\sin\pi(340t - x)$$

The reflection at the rigid wall will produce a phase difference π.

Hence the reflection wave is given by $y = 3\times10^{-2}\sin(340\pi t + \pi x + \pi)$

$y = -3\times10^{-2}\sin\pi(340t + x)$ (Towards left)

UNSOLVED PROBLEM

1. A progressive transverse wave is given by the equation

$y = 0.01\sin\frac{10\pi}{3}(x + 330t)$ Where y, x and t are mentioned in S.I units .Find its

i) Amplitude ii) frequency iii) velocity and iv) wavelength.

A. $y = 0.01\sin\left[\frac{10\pi x}{3} + \frac{3300\pi}{3}t\right]$

Comparing with, $y = A\sin(kx + \omega t)$

i) A = amplitude = 0.01 m

$\omega = \frac{3300\pi}{3} rad\sec^{-1}$ and $K = \frac{2\pi}{\lambda} = \frac{10\pi}{3}$

i) Frequency = $n = \frac{\omega}{2\pi} = \frac{1100\pi}{2\pi} = 550Hz$

ii) Velocity $= \upsilon = \frac{\omega}{K} = \frac{3300\pi/3}{10\pi/3} = 330ms^{-1}$

iv) Wavelength = $\lambda = \frac{2\times3}{10} = \frac{6}{10} = 0.6\ m$

2. Two progressive transverse waves given by $y_1 = 0.07\sin\pi(12x - 500t)$ and $y_2 = 0.07\sin\pi(12x + 500t)$ traveling along a stretched string form nodes and anti-nodes where y_1, y_2, x and t are taken in SI units. What is the displacement at the i) nodes ii) antinodes. What is the wavelength?

A. $y_1 = 0.07\sin(12\pi x - 500\pi t)$ ------ (1)

$y_2 = 0.07\sin(12\pi x + 500\pi t)$ -------- (2)

Resultant displacement $= Y = y_1 + y_2$

$y = 0.07\sin(12\pi x - 500\pi t) + \sin(12\pi x + 500\pi t)$

$$= (0.07)(2)\sin\left(\frac{24\pi x}{2}\right)\cos\left(\frac{1000\pi x}{2}\right)$$

$y = 0.14\sin(12\pi x)\cos(500\pi t)$ …. (3)

Comparing (3) with, $y = A\sin(kx)\cos(\omega t)$

$A = 0.14m$

$K = \frac{2\pi}{\lambda} = 12\pi \Rightarrow \lambda = \frac{2\pi}{12\pi} = \frac{1}{6}m$

At Nodes, Displacement = 0

At Antinodes, Displacement = A = 0.14 m.

ASSESS YOURSELF

1. How are sound waves different from light waves?

A. Sound waves are longitudinal mechanical waves and light waves are transverse electromagnetic waves.

2. Can compressions and rarefactions take place through mercury?

A. Yes. Longitudinal waves can travel through liquids.

3. What is the distance between two crests with a phase difference of 4π ?

A. 2λ.

4. What is the phase difference between a compression and next rarefaction?

A. π.

5. How many times a particle will reach maximum displacement during the time taken by the wave to advance by one wave length?

A. Twice.

6. How can you say that the equation $y = A\sin(\omega t + kx)$ represents a progressive wave?

A. The given equation is periodic both in time and position.

7. What is the significance of $\frac{\omega}{k}$ in the case of progressive wave given by $y = A\sin(\omega t - kx)$?

A. $\frac{\omega}{k} = \frac{2\pi n}{2\pi / \lambda} = n\lambda = V \; (Velocity\ of\ the\ wave)$

8. What is the resultant displacement of the particles when a compression falls on a rarefaction?

A. Minimum.

9. What happens when a crest falls on the crest during super position of waves?

A. The resultant displacement becomes maximum.

10. The crest of a water wave strikes a rigid wall. Do you get a crest or trough on reflection at the wall?

A. Trough.