EXERCISE – 9(B)

I. Find the values of the following integrals.

1. $\int_0^{\pi/2} \sin^{10} x dx$

Sol. n=10 even

$$\therefore \int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot....\frac{1}{2}\cdot\frac{\pi}{2}$$

$$\int_0^{\pi/2} \sin^{10} x dx = \frac{9}{10}\cdot\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \frac{63\pi}{512}$$

2. $\int_0^{\pi/2} \cos^{11} x\, dx$

Sol. n=11 is odd

$$\therefore \int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot....\frac{2}{3}\cdot 1$$

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}...\frac{2}{3}$$

$$= \frac{11-1}{11}\cdot\frac{11-3}{9}\cdot\frac{11-5}{7}...\frac{2}{3} = \frac{256}{693}$$

3. $\int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$

Sol. $I = \int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$ **, m=2, n=7**

$\int_0^{\pi/2} \sin^m x \cdot \cos^n x dx$ **Here m is even, n is odd**

$$= \frac{n-1}{m+n}\cdot\frac{n-3}{m+n-2}...\frac{2}{m+3}\cdot\frac{1}{m+1}$$

$$= \frac{7-1}{9}\times\frac{7-3}{7}\times\frac{7-5}{5}\times\frac{1}{2+1}$$

$$= \frac{6}{9}\cdot\frac{4}{7}\cdot\frac{2}{5}\cdot\frac{1}{3} = \frac{16}{315}$$

4. $\int_0^{\pi/2} \sin^4 x \cdot \cos^4 x \cdot dx$

Sol. $\int_0^{\pi/2} \sin^m x \cdot \cos^n xdx$. **m=n=4**

Here m, n even

$$= \frac{(n-1)(n-3)...}{(m+n)(m+n-2)...}\frac{1}{2}\cdot\frac{\pi}{2}$$

$$= \frac{(4-1)(4-3)(3\pi)}{8\cdot 6\cdot 4\cdot 2\cdot 2} = \frac{3\pi}{256}$$

5. $\int_0^{\pi} \sin^3 x \cos^6 xdx$

Sol. let $f(x) = \sin^3 x \cos^6 x \Rightarrow f(\pi - x) = \sin^3(\pi - x)\cos^6(\pi - x)$

$= \sin^3 x \cos^6 x = f(x)$

$$\therefore \int_0^{\pi} \sin^3 x \cos^6 xdx = 2\int_0^{\frac{\pi}{2}} \sin^3 x \cos^6 xdx$$

$$= \frac{(n-1)}{m+n}\frac{(n-3)}{m+n-2} --- \frac{m-1}{m}\frac{m-3}{m-2} --- \frac{2}{3}1$$

$$= 2.\frac{5}{9}.\frac{3}{7}.\frac{1}{5}.\frac{2}{3} = \frac{4}{63}$$

6. $\int_0^{2\pi} \sin^2 x \cos^4 x\, dx$

$f(x) = \sin^2 x \cos^4 x$

Sol. $\Rightarrow f(2\pi - x) = \sin^2(2\pi - x)\cos^4(2\pi - x)$ $\quad I = 4\int_0^{\frac{\pi}{2}} \sin^2 x \cos^4 x\, dx$

$\sin^2 x \cos^4 x = f(x)$

and $f(\pi - x) = \sin^2 x \cos^4 x = f(x)$

$$= \frac{4(4-1)(4-3)\frac{\pi}{2}}{6\cdot 4\cdot 2} = \frac{4.3\pi}{6\cdot 4\cdot 2\cdot 2} = \frac{\pi}{8}$$

7. $\int_{-\pi/2}^{\pi/2} \sin^2\theta \cdot \cos^7\theta\, d\theta$

Sol. $\sin^2\theta \cos^7\theta$ is even function

$f(\theta) = \sin^2\theta \cos^7\theta\, d\theta$

$f(-\theta) = \sin^2(-\theta)\cos^7(-\theta) = f(\theta)$

$$= 2\int_0^{\pi/2} \sin^2\theta \cdot \cos^7\theta\, d\theta$$

$$\int_0^{\pi/2} \sin^m x \cdot \cos^n x\, dx$$

n is odd, n = 7

$$= 2\frac{n-1}{m+n} \cdot \frac{n-3}{m+n-2} \ldots \frac{2}{m+3} \cdot \frac{1}{m+1}$$

$$= 2 \cdot \frac{7-1}{9} \cdot \frac{7-3}{9-2} \cdot \frac{7-5}{9-4} \cdot \frac{1}{3}$$

$$= 2 \cdot \frac{6}{9} \cdot \frac{4}{7} \cdot \frac{2}{5} \cdot \frac{1}{3} = \frac{32}{315}$$

8. $\int_{-\pi/2}^{\pi/2} \sin^3\theta \cos^3\theta\, d\theta$

Sol. $f(\theta) = \sin^3\theta \cos^3\theta$

$f(-\theta) = \sin^3(-\theta)\cos^3(-\theta)$

$= -\sin^3\theta \cos^3\theta = -f(\theta)$

$f(\theta)$ is odd

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^3\theta \cos^3\theta\, d\theta = 0$$

9. $\int_0^a x(a^2 - x^2)^{7/2} dx$

Sol. $x = a\sin\theta$

$dx = a\cos\theta d\theta$

$$= \int_0^{\pi/2} a\sin\theta(a^2 - a^2\sin^2\theta)^{7/2} a\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} a^9 \cos^8\theta \sin\theta d\theta = a^9 \int_0^{\pi/2} \cos^8\theta \sin\theta \cdot d\theta$$

$$= a^9\left(\frac{-\cos^9\theta}{9}\right)_0^{\pi/2} = a^9\left(-0+\frac{1}{9}\right) = \frac{a^9}{9}$$

10. $\int_0^2 x^{3/2}\sqrt{2-x}\,dx$

Sol. x = 2 cos^2 θ, dx = - 4 cos θ sin θ dθ

$$I = \int_{\pi/2}^{0} (2)^{3/2}\cos^3\theta$$

$$\sqrt{2-2\cos^2\theta}(-4\cos\theta\sin\theta)d\theta$$

$$= 4\int_0^{\pi/2} 2^{3/2}\cdot 2^{1/2}\cdot\cos^4\theta\cdot\sin^2\theta d\theta$$

$$\left[16\cdot\int_0^{\pi/2}\cos^4\theta\sin^2\theta d\theta\right]$$

n = even, m = even

$$= 16\left[\frac{4-1}{6}\cdot\frac{4-3}{5-2}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right] = \frac{\pi}{2}.$$

II. Evaluate the following integrals

1. $\int_0^1 x^5(1-x)^{5/2}dx$

Sol. given integral is $I = \int_0^1 x^5(1-x)^{5/2}dx$

Put x = sin^2 θ

dx = 2sin θ cosθ dθ

$U.L.X = 1 \Rightarrow \theta = \frac{\pi}{2}$

$L.L$ X=O $\Rightarrow \theta$=0

$$I = \int_0^{\pi/2}\sin^{10}\theta(1-\sin^2\theta)^{5/2}2\cos\theta\sin\theta d\theta$$

$$= 2\int_0^{\pi/2}\sin^{11}\theta\cos^6\theta d\theta$$

m=11 odd and n=6 even

$$= \frac{n-1}{m+n}\cdot\frac{n-3}{m+n-2}\cdots\frac{1}{m+2}\,\frac{m-1}{m}\cdots\frac{2}{3}$$

$$I = 2\cdot\frac{5}{17}\cdot\frac{3}{15}\cdot\frac{1}{13}\cdot\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3} = \frac{512}{153153}$$

2. $\int_0^4 (16 - x^2)^{5/2}\,dx$

Sol. $I = \int_0^4 (16 - x^2)^{5/2}\,dx$

Put x = 4 sinθ

dx = 4 cosθ dθ

U.L. x=4 ⇒ θ = π/2

L.L X=0 ⇒θ=0.

$$I = \int_0^{\pi/2} (16 - 16\sin^2\theta)^{5/2}\cdot 4\cos\theta\cdot d\theta$$

$$= \int_0^{\pi/2} (4)^5\cdot\cos^5\theta\cdot d\theta = (4)^6\int_0^{\pi/2}\cos^6\theta\cdot d\theta$$

$$= (4)^6\cdot\frac{6-1}{6}\cdot\frac{6-3}{6-2}\cdot\frac{6-5}{6-4}\cdot\frac{\pi}{2}$$

$$= (4)^6\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = (4)^4\cdot\frac{5}{2}\cdot\pi = 640\pi$$

3. $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$

Sol. $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}} = \int_0^1 \frac{1}{\sqrt{x+1}+\sqrt{x}}\,\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx$

$$\int_0^1 \frac{(\sqrt{x+1}-\sqrt{x})}{x+1-x}\,dx$$

$$= \int_0^1 (\sqrt{x+1}-\sqrt{x})dx = \left[\frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2}\right]_0^1$$

$$= \left(\frac{2}{3}(2)^{3/2} - \frac{2}{3}\right) - \left(\frac{2}{3}\right) = \frac{4}{3}(\sqrt{2}-1)$$

4. $I = \int_0^{\pi/4} \sin^3 \theta d\theta$

Sol. $\int_0^{\pi/4} \frac{3\sin\theta - \sin 3\theta}{4} d\theta$

$= \frac{3}{4}\int_0^{\pi/4} \sin\theta\, d\theta - \frac{1}{4}\int_0^{\pi/4} \sin 3\theta\, d\theta$

$= \frac{3}{4}[-\cos\theta]_0^{\pi/4} - \frac{1}{4}\cdot\frac{1}{3}[-\cos 3\theta]_0^{\pi/4}$

$= \frac{3}{4}\left[-\frac{1}{\sqrt{2}}+1\right] + \frac{1}{12}\left[-\frac{1}{\sqrt{2}}-1\right]$

$= \frac{9}{12}\left[-\frac{1}{\sqrt{2}}+1\right] + \frac{1}{12}\left[\frac{-1}{\sqrt{2}}-1\right]$

$= \frac{-10}{12\sqrt{2}} + \frac{8}{12} = \frac{2}{3} - \frac{5}{6\sqrt{2}}$

5. $\int_0^{\pi/4} \frac{dx}{1+\sin x}$

Sol. $\int_0^{\pi/4} \frac{(1-\sin x)dx}{1-\sin^2 x}$

$= \int_0^{\pi/4} \left[\sec^2 x - \tan x \cdot \sec x\right] dx$

$= \left[\tan x - \sec x\right]_0^{\pi/4} = (1-\sqrt{2}) - (0-1) = 2-\sqrt{2}$

6. $\int_1^2 \frac{\log x}{x^2} dx$

Sol. $\int_1^2 \frac{\log x}{x^2} dx = \int_1^2 \log x.\ \frac{1}{x^2} dx$

$\uparrow f \quad \uparrow g$

$\left[\log x \int \frac{dx}{x^2} - \int \frac{d}{dx}(\log x) \int \frac{dx}{x^2} dx\right]_1^2$

$$= \int_1^2 \left[-\frac{\log x}{x} - \int \frac{1}{x}\left(-\frac{1}{x}\right)\right] dx = \left[\frac{\log x}{x} - \frac{1}{x}\right]_1^2$$

$$= \left(\frac{-\log 2 - 1}{2}\right) + (1) = \frac{1}{2}(1 - \log 2)$$

7. $\int_{-1}^{1} |x|\, dx$

Sol. $\int_{-1}^{1} |x|\, dx$

We know that |x| = x, if x ≥ 0, |x| = –x, if x < 0

$$\int_{-1}^{1} |x|\, dx = \int_{-1}^{1} -x\, dx + \int_{0}^{1} x\, dx$$

$$= \left[\frac{-x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} + \frac{1}{2} = 1$$

8. $\int_1^2 \frac{(x+1)^2}{\sqrt{x}} dx$

Ans: $\sqrt{2}\frac{94}{15} - \frac{56}{15}$

9. $\int_{\pi/6}^{\pi/3} \cot^2 x dx$

Ans: $\frac{2}{\sqrt{3}} - \frac{\pi}{6}$

10. $\int_0^{\pi/4} \frac{1 + \sin 2x}{\cos x + \sin x} dx$

Sol. $\int_0^{\pi/4} \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos x + \sin x} dx$

$$= \int_0^{\pi/4} \frac{(\cos x + \sin x)^2}{\cos x + \sin x} dx = \int_0^{\pi/4} (\cos x + \sin x) dx$$

$$= [\sin x - \cos x]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) - (\sin 0 - \cos 0)$$

$$= \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - (0 - 1) = 1$$

III. Evaluate the following integrals.

1. $\int_0^1 (2x+3)\sqrt{3-2x}\,dx$

Sol. $\int_0^1 (2x+3)\sqrt{3-2x}\,dx$

Put 3 – 2x = t^2

–2 dx = 2t dt

dx = –t dt

U.L. x=1$\Rightarrow$ 1 = t^2 $\Rightarrow$ t=1

L.L.X=0, 3 = t^2 $\Rightarrow t = \sqrt{3}$

$$= \int_{\sqrt{3}}^{1} \left\{2\left[\frac{3-t^2}{2}\right]+3\right\} t(-t)dt$$

$$= \int_{1}^{\sqrt{3}} (3-t^2+3)t^2 dt = \int_{1}^{\sqrt{3}} (6t^2-t^4)dt$$

$$= \int\left[\frac{6t^3}{3}-\frac{t^5}{5}\right]_1^{\sqrt{3}} = \left[t^3\left(2-\frac{t^2}{5}\right)\right]_1^{\sqrt{3}}$$

$$= 3\sqrt{3}\left(2-\frac{3}{5}\right)-\left(2-\frac{1}{5}\right)$$

$$= \frac{21\sqrt{3}}{5}-\frac{9}{5} = \frac{-9+21\sqrt{3}}{5}$$

2. $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$

Sol. $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$

Put x = cos 2θ, dx = –2 sin 2θ dθ

U.L. x=1$\Rightarrow$ 1 = cos 2θ, $\Rightarrow$ 2θ = 0$\Rightarrow$ θ = 0

L.L.x=0$\Rightarrow$ 0 = cos 2θ$\Rightarrow$ 2θ = π/2$\Rightarrow$ θ = π/4

$$\int_{\pi/4}^{0} \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}}(-2\sin 2\theta)d\theta$$

$$= \int_{0}^{\pi/4} \frac{\sin\theta}{\cos\theta}\cdot 4\sin\theta\cos\theta\, d\theta$$

$$= \int_0^{\pi/4} 4\sin^2\theta d\theta = 2\int_0^{\pi/4} [1-\cos 2\theta] d\theta$$

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= 2\left[\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) - 0\right] = \frac{\pi}{2} - 1$$

3. $\int_1^{\sqrt{e}} x \cdot \log x \, dx$

g↑ f↑

Sol. $= \left[\log x \int x dx - \int\left(\frac{d}{dx}(\log x)\int x \cdot dx\right)dx\right]_1^{\sqrt{e}}$

$$= \left[\left(\log x \frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2} dx\right)\right]_1^{\sqrt{e}}$$

$$= \left[(\log x)\frac{x^2}{2} - \frac{x^2}{4}\right]_1^{\sqrt{e}} = \left(\frac{1}{2}\cdot\frac{e}{2} - \frac{e}{4}\right) + \frac{1}{4} = \frac{1}{4}$$

4. $\int_0^1 \frac{xe^x}{(1+x)^2} dx$

Sol. $\int_0^1 \frac{xe^x}{(1+x)^2} dx = \left[\int \frac{(x+1-1)e^x}{(1+x)^2} dx\right]_0^1$

$$= \int_0^1 e^x \left(\frac{1}{x+1} - \frac{1}{(1+x)^2}\right)$$

$$= \left(e^x \frac{1}{1+x}\right)_0^1 \qquad \left(\because \int e^x (f + f') = ef\right)$$

$$= \frac{e}{2} - 1$$

5. $\int_{-1}^{1}\log\left[\frac{2-x}{2+x}\right]dx$

Sol. let $f\left(x\right)=\log\left(\frac{2-x}{2+x}\right)$

$$f(-x)=\log\left(\frac{2+x}{2-x}\right)=-\log\left(\frac{2-x}{2+x}\right)$$

f(x) = –f(–x)

f(x) is odd function, therefore $\int_{-1}^{1}\log\left[\frac{2-x}{2+x}\right]dx$ **=0**

6. $\int_{0}^{\pi}\sin^3\theta(1+2\cos\theta)(1+\cos\theta)^2 d\theta$

Sol. $\int_{0}^{\pi}\sin^3\theta(1+2\cos\theta)(1+\cos\theta)^2 d\theta$

Put cos θ = t $\Rightarrow$ –sin θ dθ = dt

L.L. θ=0 $\Rightarrow$ t=1

U.L. θ=π $\Rightarrow$ t= -1

$$\int_{0}^{\pi}\sin^3\theta(1+2\cos\theta)(1+\cos\theta)^2 d\theta$$

$$=\int_{0}^{\pi}\left(1-\cos^2\theta\right)(1+2\cos\theta)(1+\cos\theta)^2\sin\theta d\theta$$

$$=\int_{1}^{-1}(1-t^2)(1+2t)(1+t)^2 dt$$

$$=\int_{1}^{-1}(1-t^2)(1+2t)(1+2t+t^2)dt$$

$$=\int_{-1}^{1}(1-t^2)(1+4t^2+4t+t^2+2t^3)dt$$

$$=\int_{-1}^{1}(1-t^2)(1+4t+5t^2+2t^3)dt$$

$$=\int_{-1}^{1}(1+4t+4t^2-2t^3-t^2-4t^3-5t^4-2t^5)dt$$

$$=\int_{-1}^{1}(1+4t+4t^2-2t^3-5t^4-2t^5)dt$$

$$=\left[t+2t^2+\frac{4}{3}t^3-\frac{t^4}{2}-t^5-\frac{t^6}{3}\right]_{-1}^{1}=\left(1+2+\frac{4}{3}-\frac{1}{2}-1-\frac{1}{3}\right)-\left(-1+2-\frac{4}{3}-\frac{1}{2}+1-\frac{1}{3}\right)$$

$$=\frac{15}{6}+\frac{1}{6}=\frac{16}{6}=\frac{8}{3}$$

PROBLEMS FOR PRACTICE

1. Evaluate $\int_1^2 x^5 dx$ **.**

Ans. 21/2

2. Evaluate $\int_0^{\pi} \sin x dx$

Ans. 2

3. Evaluate $\int_0^a \frac{dx}{x^2+a^2}$

Ans. $\frac{\pi}{4a}$

4. Evaluate $\int_1^4 x\sqrt{x^2-1}dx$

Ans. $\frac{1}{3}(15)^{3/2}$

5. Evaluate $\int_0^2 \sqrt{4-x^2}\, dx$

Ans. π

6. Evaluate $\int_0^{16} \frac{x^{1/4}}{1+x^{1/2}} dx$

Ans. $4\left[\frac{2}{3}+\tan^{-1}2\right]$

7. Evaluate $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx$

Ans. 2

8. Show that $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx$ **.**

9. Evaluate $\int_0^{\pi/2} \frac{\cos^{5/2} x}{\sin^{5/2} x + \cos^{5/2} x} dx$

Ans. π/4

10. Show that

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$$

Sol. Let I = $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right)}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} \left(\frac{x}{\sin x + \cos x} + \frac{(\pi/2 - x)}{\sin x + \cos x} \right) dx$$

$$= \frac{\pi}{2} \int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$$

Put $t = \tan\frac{x}{2} \Rightarrow dx = \frac{2dt}{1+t^2}$

$$I = \frac{\pi}{4} \int_0^1 \frac{2\frac{dt}{1+t^2}}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} = \frac{\pi}{2} \int_0^1 \frac{dt}{2t + 1 - t^2}$$

$$= \frac{\pi}{2} \int_0^1 \frac{dt}{(\sqrt{2})^2 + (t-1)^2} = \frac{\pi}{2} \left(\frac{1}{2\sqrt{2}} \log \frac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1} \right)_0^1$$

$$= \frac{-\pi}{4\sqrt{2}} \left(\log \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) = \frac{\pi}{4\sqrt{2}} \log(\sqrt{2} + 1)^2$$

$$= \frac{\pi}{4\sqrt{2}} 2 \log(\sqrt{2} + 1) = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$$

11. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Ans. π/12

12. Find $\int_{-a}^{a}(x^2+\sqrt{a^2-x^2})\,dx$.

Ans. $\frac{2a^3}{3}+a^2\frac{\pi}{2}$

13. Find $\int_0^{\pi}\frac{x\cdot\sin x}{1+\sin x}dx$

Sol. $I=\int_0^{\pi}\frac{x\cdot\sin x}{1+\sin x}dx=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}dx$

$$\left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\sin x}dx$$

$$I=\int_0^{\pi}\frac{\pi\sin x}{1+\sin x}dx-\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx$$

$$=\int_0^{\pi}\frac{\pi\sin x}{1+\sin x}dx-I$$

$$2I=\pi\int_0^{\pi}\frac{\sin x}{1+\sin x}dx$$

$$=\pi\int_0^{\pi}\left(1-\frac{1}{1+\sin x}\right)dx$$

$$=\pi\int_0^{\pi}dx-\pi\int_0^{\pi}\frac{1}{1+\sin x}dx \qquad \textbf{...(1)}$$

Consider $\int_0^{\pi}\frac{1}{1+\sin x}dx=2\int_0^{\pi/2}\frac{1}{1+\sin x}dx$

$$=2\int_0^{\pi/2}\frac{1}{1+\sin\left(\frac{\pi}{2}-x\right)}dx=2\int_0^{\pi/2}\frac{dx}{1+\cos x}$$

$$=2\int_0^{\pi/2}\frac{dx}{2\cos^2(x/2)}=\int_0^{\pi/2}\sec^2\frac{x}{2}dx$$

$$=\left(2\tan\frac{x}{2}\right)_0^{\pi/2}=2\cdot\tan\frac{\pi}{2}-2\cdot 0=2-0=2$$

FROM (i) $2I=\pi\left(x\right)_0^{\pi}-2\pi=\pi(\pi)-2=\pi^2-2\pi$

$$I=\frac{\pi^2}{2}-\pi$$

14. Evaluate $\int_0^{\pi/2} x\sin x dx$

Ans. 1

15. Let f : R → R be a continuous periodic function and T be the period of it. Then prove that for any positive integer n, $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$

16. Find (i) $\int_0^{\pi/2}\sin^4 x dx$ **, (ii)** $\int_0^{\pi/2}\sin^7 x dx$ **,**

(iii) $\int_0^{\pi/2}\cos^8 x dx$ **.**

Ans. (i) $\frac{3\pi}{16}$ **, (ii)** $\frac{16}{35}$ **, (iii)** $\frac{35\pi}{256}$

17. Evaluate $\int_0^{a}\sqrt{a^2-x^2}dx$

Ans. $\frac{\pi a^2}{4}$

18. Find (i) $\int_0^{\pi/2}\sin^4 x\cdot\cos^5 x\, dx$ **,**

(ii) $\int_0^{\pi/2}\sin^5 x\cdot\cos^4 x\, dx$ **,**

(iii) $\int_0^{\pi/2}\sin^6 x\cdot\cos^4 x\, dx$ **.**

Ans. (i) $\frac{8}{315}$ **, (ii)** $\frac{8}{315}$ **, (iii)** $\frac{3}{512}\pi$

19. Find $\int_0^{2\pi}\sin^4 x\cdot\cos^6 x\, dx$

Ans. $\frac{3}{128}\pi$

20. Find $\int_{-\pi/2}^{\pi/2} \sin^2 x \cdot \cos^4 x dx$

Ans. π/16

21. Find $\int_{0}^{\pi} x \sin^7 x \cdot \cos^6 x dx$

Ans. $\pi \frac{16}{3003}$

22. Find $\int_{-a}^{a} x^2 (a^2 - x^2)^{3/2} dx$

Ans. $\frac{\pi a^6}{16}$

23. Find $\int_{0}^{1} x^{3/2} \sqrt{1-x} dx$

Ans. π/16