WORK POWER AND ENERGY

Work

Work is said to be done by a force when the point of application of the force is displaced. The work done by the force is measured by the dot product of force (F) and the displacement (s) i..e. $W = \vec{F} \cdot \vec{s} = F(s\cos\theta)$. Here θ is angle between

If the applied force is variable and s_1 and s_2 be the magnitudes of initial and final displacements of a body with respect to some reference point, then work done under varying force $\vec{F}$ is: $W = \int_{s_1}^{s_2} \vec{F} \cdot \vec{ds}$ where $\vec{ds}$ is the small displacement of the body between s_1 and s_2.

1. The work done is zero, if θ = 90°. If θ < 90° then work done is positive. But, if θ > 90°, then the work done is negative. For example, when an object is pulled on a rough surface, the work done by the pulling force is positive, while work done by frictional force is negative.
2. If the force and displacement both are variable quantities, then the work done is represented by the area under force-displacement graph added with sign.
3. In a uniform circular motion of a body, the work done by centripetal force is zero. That is why the speed or KE remains constant.
4. The magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$ on a moving charged particle is always perpendicular to $\vec{v}$ or $\vec{dr}$ therefore, work done by magnetic force on a moving charge is zero.
5. The SI unit of work is joule (J). Other units are erg, eV, kWh. ; 1 joule = 10^7 erg,

Energy

The energy of a body is defined as the capacity of doing work. The unit of energy is same as that of work, i.e., joule. Energy can be classified further into various well defined forms such as (a) mechanical (b) heat (c) electrical (d) chemical (e) atomic energy etc.

In many processes that occur in nature energy may be transformed from one form to other. Mass can also be transformed into energy and vice-versa by relation $E = mc^2$. In dynamics, we are mainly concerned with purely mechanical energy, which may be kinetic or potential.

Kinetic energy

The energy possessed by a body by virtue of its motion is known as kinetic energy (KE). The KE of a moving body is equal to the amount of work that must be done to bring a body from rest into the state of motion.

Conversely, the amount of work that we must do in order to bring a moving body to rest is equal to the negative of the kinetic energy of the body, i.e.,

KE = work done to put the body into motion = - work done to bring the body to a stop = $\frac{1}{2}mv^2$

1. KE is always positive. The KE is a scalar quantity.
2. KE depends on the frame of reference, e.g., KE of a person of mass *m* sitting in a train moving with speed *v* is zero w.r.t. frame of train but $(1/2)mv^2$ w.r.t. frame of reference of earth.
3. In terms of momentum p (= *mv*), KE of a particle can be expressed as $K = p^2/2m$. This implies that a body cannot possess KE without having momentum and *vice-versa*.

Potential energy

The energy possessed by a body or a system by virtue of its configuration or its position in a field is called its potential energy. The potential energy of a particle at a point is defined as the amount of work done by an external force in moving the particle from infinity to that point.

1. Conservative force is the negative gradient of potential energy i.e. $-\frac{dU}{dr} = F$ (v) Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.
2. If the particle moves opposite to the conservative field, work done by the field will be negative and so change in potential energy will be positive, i.e., potential energy will increase.
3. If the particle moves in the direction of conservative field, work done will be positive and so change in potential energy will be negative, i.e., potential energy will decrease.
4. The PE does not have an absolute value. It may be positive, negative or zero. The value depends on the choice of reference level or where the zero of potential energy is taken. For example, if zero of U is taken at the surface of the earth, then PE of mass m at a height h (when $h << R_e$) is: $U = mgh$. But if the zero of PE is chosen when mass m is at an infinite separation from earth's centre, then $U = -\frac{GM_e m}{(R_e + h)}$

Work-Energy theorem

The net work done by the forces acting on a particle is equal to the change in the KE of the particle

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

1. When the magnitude of the velocity of a particle is constant, there is no change in KE. For example, for a particle moving in a circular path with uniform speed, the direction of velocity vector keeps on changing, but since the force is perpendicular to the displacement, no work is done. As a result, the magnitude of velocity does not change and KE remains constant.
2. If a body moving with velocity u and if by the action of a retarding force the body comes to rest after travelling a distance s, then $W = Fs = \frac{1}{2}mv^2$ and Stopping distance $= \frac{\text{initial KE}}{\text{retarding force}}$

Mechanical energy

1. Mechanical energy E of a particle, object or system is defined as the sum of kinetic energy K and potential energy K, i.e., E = K + U. It depends on frame of reference.
2. Mechanical energy of a body or a system can be negative, and negative mechanical energy implies that potential energy is negative and in magnitude it is more than KE. Such a state is called bound state.
3. In presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion. This is known as the law of conservation of mechanical energy.

Power

Rate of doing work is called power. It is a scalar quantity . Mathematically $P = \frac{dW}{dt}$

1. If velocity vector makes an angle θ with the force vector, then $P = \vec{F}.\vec{v} = Fv\cos\theta$
2. Unit of power is J/sec or watt in SI and erg/sec in CGS. Other units are horse power (= 746 watt)

Conservation of momentum

The law of conservation of linear momentum states that if the total external force acting on a system is equal to zero, then the final value of the total momentum of the system is equal to the initial value of the total momentum of the system. Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

Collisions

Perfectly elastic collision: If in a collision, alongwith momentum KE is also conserved, the collision is said to be perfectly elastic.

Inelastic collision: If in a collision, some kinetic energy is lost, the collision is said to be inelastic. All real collisions belong to this category. Here KE appears in other forms.

Perfectly inelastic collision: If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic, e.g., the collision between a bullet and a block of wood into which it is fired is completely inelastic, when the bullet remains embedded in the block.

1. The law of conservation of momentum holds good for any type of collision (elastic, inelastic or perfectly inelastic). The total momentum of the system after the collision must be equal to the total momentum of the system before collision, although the momentum of individual particles within the system may be changed, but the total momentum remains constant.
2. In collision, it is not necessary that the colliding particles come in contact physically.
3. Coefficient of restitution (e) : The coefficient of restitution in a collision of two bodies is defined as:

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{|v_2 - v_1|}{|u_1 - u_2|}$$

   For a perfectly elastic collision, e = 1

   For a perfectly inelastic collision, e = 0

   For an inelastic collision, 0 < e < 1

4. Collision in one dimension Consider collision of two bodies of unequal masses m_1 and m_2 moving with initial velocities u_1 and u_2. Let these bodies move with velocities v_1 and v_2 after the collision. Then

$$v_1 = \frac{(m_1 - em_2)u_1 + (1 + e)m_2u_2}{m_1 + m_2} \quad \text{and} \quad v_2 = \frac{(m_2 - em_1)u_2 + (1 + e)m_1u_1}{m_1 + m_2}$$

These formulae are valid for any type of one dimensional collision.

Special cases: (For perfectly elastic collision in one dimension)

(a) If masses are equal: $v_1 = u_2$ and $v_2 = u_1$, i.e., their velocities are mutually interchanged. Further if $u_2 = 0$, then $v_1 = 0$ and $v_2 = u_2$, i.e., the moving body stops and the body at rest starts moving with same velocity.

(b) If target particle is massive: If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$ and $v_1 = -u_1$ i.e. the light particle recoils with almost same speed while heavy target remains practically at rest.

(c) If projectile particle is massive i.e. $m_1 >> m_2$ and $u_2 = 0$, then $v_1 \approx u_1$ and $v_2 \approx 2u_1$. i.e. the motion of heavy projectile is almost unaffected while the light body flies away at speed twice that of heavier.

Inelastic collision of a ball with the earth: Let h_o be the initial height of the ball w.r.t. earth and ball strike first time with velocity v_o. Since the earth is massive, the initial and final velocities of the earth can be assumed to be zero. Because of inelastic collision the ball loose energy at every strike. It can be easily calculated that for coefficient of restitution 'e' the

Velocity of ball after n_{th} strike $v_n = e^n v_o$

Height of ball after n_{th} strike $h_n = e^{2n} h_o$

1. The height h_1 attained after first impact should be $h_1 = \frac{v_1^2}{2g} = e^2 h_o$ or $e = \sqrt{\frac{h_1}{h_o}}$
2. The velocity just after second impact, $v_2 = ev_1 = e^2u_1$ Hence velocity after n impacts $v_n = e^n u_1$
3. Height attained after second impact, $h_2 = \frac{v_2^2}{2g} = e^4 h_o$. Hence the height attained after n impacts, $h_n = e^{2n} h_o$

WORK POWER AND ENERGY Assignment

1. A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. The work required to pull the hanging part on the table is
(a) MgL (b) MgL/3 (c) MgL/9 (d) MgL/18

2. The work done in holding a mass of 50 kg at a height of 2 m above the ground is
(a) 0 (b) 25 J (c) 100J (d)980J

3. A car weighing 1 ton is moving twice as fast as another car weighing 2 tons. The K.E of the one ton car is
(a) less than that of the two-ton car.
(b) same as that of the two-ton car
(c) more than that of the two-ton car
(d) impossible to compare with that of the two - ton car unless the height of each car above the sea level is known.

4. Energy required to accelerate a car from 10 to 20 m/s compared with that required to accelerate it from 0 to 10 m/s is
(a) twice (b) three times
(c) four times (d) same

5. A force of (5 + 3x) N, acting on a body of mass 20 kg along the x -axis, displaces it from x = 2m to x = 6m. The work done by the force is
(a) 20 J (b) 48 J (c) 68 J (d) 86 J

6. An electric motor creates a tension of 4500 N in a hoisting cable and reels it in at the rate of 2 m/s. The power of the motor is
(a) 15 kW (b) 9 kW (c) 225 W (d) 9000 H.P.

7. A cord is used to lower vertically a block of mass M a distance d at a constant downward acceleration of g/4. Then the work done by the cord on block is
(a) Mgd/4 (b) -Mgd/4 (c)3Mgd/4 (d) -3Mgd/4

8. A body of mass 5 kg, initially at rest, is moved by a horizontal force of 2 N on a smooth horizontal surface. The work done by the force in 10 s is
(a) 20 J (b) 30 J (c) 40 J (d) 60 J

9. If the kinetic energy of a body is increased by 300%, its momentum will increase by
(a) 100% (b) 150% (c) 200% (d) 400%

10. Two bodies of masses m and 4m are dropped from the top of a tower. When they reach the ground their kinetic energies will be in the ratio
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

11. A pump can take out 36000 kg of water per hour from a 100 m deep well. If the efficiency of the pump is 50%, its power is (g = 10 m/s^2)
(a) 5 kW (b) 10 kW (c) 15 kW (d) 20 kW

12. A truck of mass 30000 kg moves up an inclined plane rising 1 in 100 at a speed of 30 km/h. The power of the engine of the truck is (g = 10 m/s^2)
(a) 25 kW (b) 90 kW (c) 2.5 kW (d) 9.0 kW

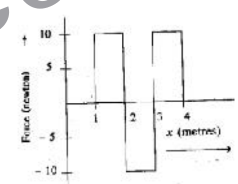
13. A body of mass 0.5 kg is taken up an inclined plane of length 10 m and height 5 m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.1. The work done by the frictional force over the round trip is (g = 10 m/s^2)
(a) 5 J (b) 5 3 J (c) - 5 J (d) - 5 3 J

14. A uniform rod of length 1 m and mass 100 g is pivoted at one end and is hanging vertically. It is displaced through 60° from the vertical. The increase in its potential energy is (g = 10 m/s^2)
(a) 0.25 J (b) 0.5 J (c) 0.75J (d) 1.0 J

15. A bullet, moving with a speed of 150 m/s, strikes a wooden plank. After passing through the plank, its speed becomes 125 m/s. Another bullet of the same mass and size strikes the plank with a speed of 90 m/s. Its speed after passing through the plank would be
(a) 25 m/s (b) 35 m/s (c) 50 m/s (d) 70 m/s

16. The kinetic energy acquired by a body of mass m in travelling a certain distance starting from rest, under a constant force is
(a) directly proportional to m
(b) directly proportional to m
(c) inversely proportional to m
(d) independent of m

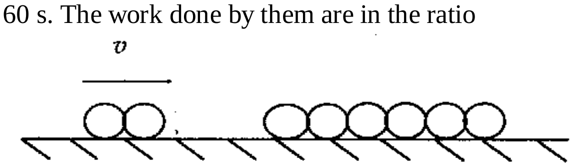
17. The figure shows the force-distance curve of a body moving along a straight line. The work done by the force is



(a) 10 J (b) 20 J
(c) 30 J (d) 40 J

18. The displacement of a body of mass 2 kg as a function of time is given by $x = 2t^2 + 5$, where x is in metres and t in seconds. The increase in its kinetic energy one second after the start of motion is
(a) 8 J (b) 16 J (c) 32 J (d) 64 J

19. A body is lifted by a man to a height of 1 m in 30 s. Another man lifts the same mass to the same height in 60 s. The work done by them are in the ratio



(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 4 : 1

20. A 4.0 kg body, moving with a speed of 2.0 m/s. collides with a spring bumper of negligible mass and force constant 100 N/m. The maximum compression of the spring is
(a) 0.4 m (b) 0.8 m (c) 1.6m (d) 2.0m

21. A simple pendulum of length 1 m has a bob of mass 100 g. It is displaced through an angle of 60° from the vertical and then released. The kinetic energy of the bob when it passes through the mean position is (g = 10 m/s^2)
(a) 0.25 J (b) 0.5 J (c) 1.0 J (d) 1.4 J

22. A bomb of mass 12 kg, initially at rest, explodes into two pieces of masses 4 kg and 8 kg. The speed of the 8 kg mass is 6 m/s. The kinetic energy of the 4 kg mass is
(a) 32 J (b) 48 J (c) 144 J (d) 288 J.

23. A moving particle of mass m makes a head on elastic collision with a particle of mass 2m which is initially at rest. The fraction of the initial kinetic energy lost by the colliding particle is
(a) 1/9 (b) 2/9 (c) 4/9 (d) 8/9.

24. A shell is fired from a canon with a velocity v at an angle with the horizontal. At the highest point it explodes into two pieces of equal masses. One of the pieces retraces its path to the canon. The speed of the other piece immediately after the explosion is
(a) 3 v cos (b) 2 v cos
(c) (3/2) v cos (d) v cos .

25. Three particles A, B and C of equal masses, moving with the same speed v along the medians of an equilateral triangle, collide at the centroid of the triangle. After collision, A comes to rest and B retraces its path with speed v. The speed of C after the collision is
(a) zero (b) v/2 (c) v (d) 4v.

26. Two skaters A and B, having masses 50 kg and 70 kg respectively, stand facing each other 6m apart on a horizontal smooth surface. They pull on a rope stretched between them. 'How far does each move before they meet ?
(a) both move 3 m
(b) A moves 2.5 m and B moves 3.5 m
(c) A moves 3.5 m and B moves 2.5 m
(d) none of the above

27. A body of mass 2.9 kg is suspended from a string of length 2.5 m and is at rest. A bullet of mass 100 g, moving horizontally with a speed of 150 m/s, strikes and sticks to it. What is the maximum angle made by the string with the vertical after the impact? ($g = 10\ m/s^2$)
(a) 30° (b) 45° (c) 60° (d) 90°.

28. A ball A, moving with a speed u, collides directly with another similar ball B moving with a speed v in the opposite direction, A comes to rest after the collision. If the coefficient of restitution is e then u/v is
(a) $\frac{1 \quad e}{1 \quad e}$ (b) $\frac{1 \quad e}{1 \quad e}$ (c) $\frac{e}{1 \quad e}$ (d) $\frac{e}{1 \quad e}$

29. A uniform wooden plank of mass 200 kg and length 5 m is floating on still water with a man of 50 kg at one end of it. The man walks to the other end of the plank and stops. The distance moved by the man relative to water is
(a) 2.5 m (b) 4 m (c) 5m (d) zero.

30. A ball is dropped from a height of 1 m. If the coefficient of restitution between the surface and the ball is 0.6, the ball rebounds to a height of
(a) 0.6 m (b) 0.4 m (c) 0.16 m (d) 0.36 m.

Answers: 1-d, 2-a, 3-c, 4-b, 5-c, 6-d, 7-d, 8-c, 9-a, 10- c, 11-d, 12-a, 13-d, 14-a, 15-b, 16-d, 17-a, 18-d, 19-b, 20-a, 21-b, 22-d, 23-d, 24-a, 25-c, 26-c, 27-c, 28-a, 29-b, 30-d.