

Electromagnetic Induction

- The magnetic flux through a surface of area **A** placed in a uniform magnetic field **B** is defined as,

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$

where θ is the angle between **B** and **A**.

- Faraday's laws of induction imply that the emf induced in a coil of N turns is directly related to the rate of change of flux through it,

$$\varepsilon = -N(d\Phi_B/dt)$$

Here Φ_B is the flux linked with one turn of the coil. If the circuit is closed, a current $I = \varepsilon/R$ is set up in it, where R is the resistance of the circuit

- Lenz's law states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it. The negative sign in the expression for Faraday's law indicates this fact.
- When a metal rod of length l is placed normal to a uniform magnetic field B and moved with a velocity v perpendicular to the field, the induced emf (called motional emf) across its ends is

$$\varepsilon = Blv$$

- Changing magnetic fields can set up current loops in nearby metal (any conductor) bodies. They dissipate electrical energy as heat. Such currents are called eddy currents.
- Inductance is the ratio of the flux-linkage to current. It is equal to $N\Phi/I$.

- A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1). This relation is given by,

$$\epsilon_1 = M_{12}(di_2/dt)$$

The quantity M_{12} is called mutual inductance of coil 1 with respect to coil 2. One can similarly define M_{21} . There exists a general equality,

$$M_{12} = M_{21}$$

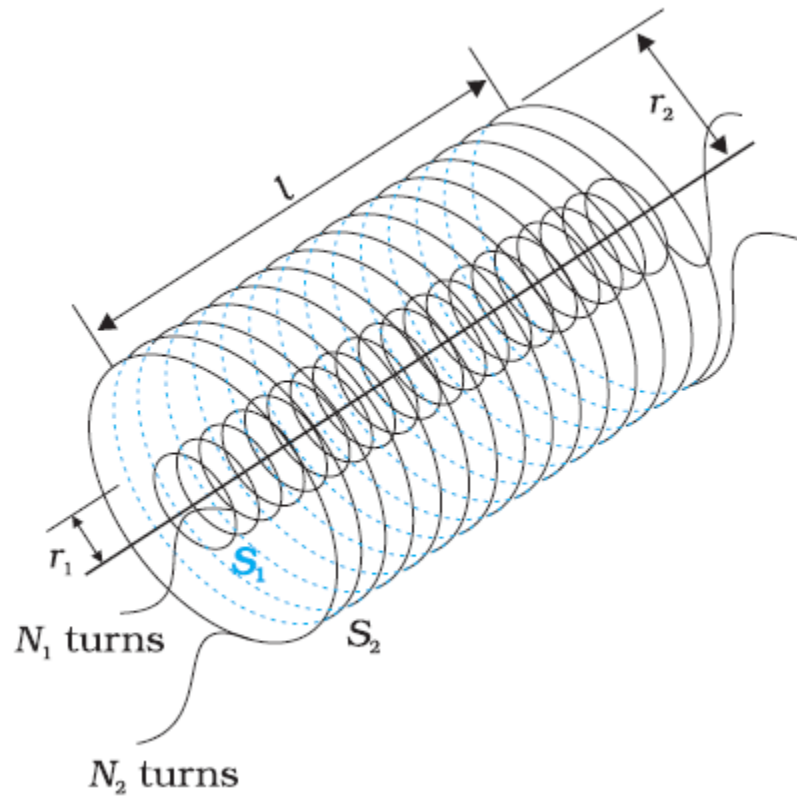
- When a current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by,

$$\epsilon = L(di/dt)$$

L is the self-inductance of the coil. It is a measure of the inertia of the coil against the change of current through it.

- The self-inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r , is given by

$$L = \mu_r \mu_0 n^2 Al$$



where A is the area of cross-section of the solenoid, l its length and n the number of turns per unit length.

- In an ac generator, mechanical energy is converted to electrical energy by virtue of electromagnetic induction. If coil of N turn and area A is rotated at ν revolutions per second in a uniform magnetic field B , then the motional emf produced is

$$\varepsilon = NBA (2\pi\nu) \sin (2\pi\nu t)$$

where we have assumed that at time $t = 0$ s, the coil is perpendicular to the field.

Sample Examples

- A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field H_E at a place. If $H_E = 0.4$ G at the place, what is the induced emf between the axle and the rim of the wheel?

Note that $1 \text{ G} = 10^{-4} \text{ T}$.

Solution

$$\begin{aligned}\text{Induced emf} &= (1/2) \omega B R^2 \\ &= (1/2) \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2 \\ &= 6.28 \times 10^{-5} \text{ V}\end{aligned}$$

The number of spokes is immaterial because the emf's across the spokes are in parallel.

- A circular coil of radius 10 cm, 500 turns and resistance 2Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is $3.0 \times 10^{-5} \text{ T}$.

Solution

Initial flux through the coil,

$$\begin{aligned}\Phi_B (\text{initial}) &= BA \cos \theta \\ &= 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 0^\circ \\ &= 3\pi \times 10^{-7} \text{ Wb}\end{aligned}$$

Final flux after the rotation,

$$\begin{aligned}\Phi_B (\text{final}) &= 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) \times \cos 180^\circ \\ &= -3\pi \times 10^{-7} \text{ Wb}\end{aligned}$$

Therefore, estimated value of the induced emf is,

$$\varepsilon = N (\Delta\Phi / \Delta t)$$

$$= 500 \times (6\pi \times 10^{-7}) / 0.25$$

$$= 3.8 \times 10^{-3} \text{ V}$$

$$I = \varepsilon / R = 1.9 \times 10^{-3} \text{ A}$$

Note that the magnitudes of ε and I are the estimated values. Their instantaneous values are different and depend upon the speed of rotation at the particular instant.