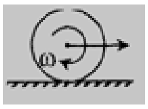
ROTATIONAL MOTION

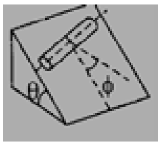
CHOOSE THE CORRECT ALTERNATIVE.

1. Identify the situation (s) in which the body is rotating about an axis which is fixed in direction only with respect to an inertial frame.

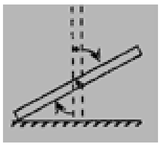
a) A cylinder rolling on a horizontal surface .



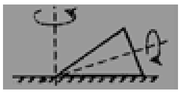
b) A cylinder rolling down on inclined surface at an angle θ with respect to the line of greatest slope.



c) A uniform rod slipping down a smooth horizontal surface.



d) A cone lying on a horizontal surface with its curved surface in contact. It rolls about its axis around a vertical axis passing through its apex.



2. Moment of inertia of a body

a) is always defined with respect to an axis.

b) depends on the mass of the body.

c) depends on the distribution of the mass with respect to the axis.

d) resists change in its rotational state of motion.

3. the perpendicular axes theorem

a) can be applied to lamina shaped bodies only.

b) can applied to any body.

c) can be applied on mutually perpendicular axes passing through its center of mass only.

d) can be applied on mutually perpendicular axes passing through any point on the body.

4. Mark the correct statement (s) about the instantaneous center of rotation.

a) It is a stationary point with respect to ground.

b) It may lie within or outside the body.

c) The angular velocity of the body about this pint is the same as that about its center of mass.

d) The body purely rotation abut this point.

5. The mathematical statement $\vec{v} = \vec{v}_c + \vec{v}'$, where $\vec{v}_c$ is the velocity of center of mass. $\vec{v}'$ is the velocity of the point with respect to the center of mass and $\vec{v}$ is the total velocity of the point with respect to ground.

a) is true for all the moving bodies.

b) is true only for rolling bodies.

c) is true only for rotating bodies.

d) is true only for bodies undergoing translatory motion.

6. When a body rolls on a horizontal surface

a) its point of contact does not slip with respect to the surface

b) its point of contact moves with the speed and acceleration of the surface.

c) its center of mass moves along a straight line.

d) its top most point always move faster than the lowest point of contact.

7. When a ring rolls on a horizontal rough surface., the point of contact of the ring moves along

a) a parabolic path

b) a circular arc whose center is the point of contact

b) an elliptical path.

d) none of these.

8. In one complete revolution of a rolling body of radius R

a) its center of mass travels a distance equal to $2\pi R$.

b) its point of contact travels a distance equal to 8R.

c) its point of contact undergoes a displacement equal to 2πR.

d) any point of its periphery always travel more distance than its center of mass.

9. When a uniform ring of mass m and radius R rolls on a horizontal surface

a) One-third mass of the ring moves with a velocity less than the velocity of its center of mass.

b) Half of the ring moves with a velocity more that the velocity of its center of mass.

c) There are two points on its periphery that moves with the velocity of center of mass.

d) the maximum velocity of any point on the ring is two times the velocity of its center of mass

Problem 10 to 13

The fig., 5.16 shown an isosceles triangular plate of mass M and base L. the angle at the apex is 90°. The apex lies at the origin and the base is parallel to y– axis.

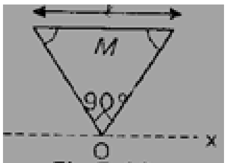


Fig.5.16

10. The moment of inertia of the plate about the z-axis is

a) $\frac{ML^2}{12}$

b) $\frac{ML^2}{24}$

c) $\frac{ML^2}{6}$

d) none of these

11. The moment of the plate about the x-axis is

a) $\frac{ML^2}{8}$

b) $\frac{ML^2}{32}$

c) $\frac{ML^2}{24}$

d) $\frac{ML^2}{6}$

12. The moment of inertia of the plate about its base parallel to the x-axis is

a) $\frac{ML^2}{18}$

b) $\frac{ML^2}{36}$

c) $\frac{ML^2}{24}$

d) none of these

13. the moment of inertia of the plate about the y-axis is

a) $\frac{ML^2}{6}$

b) $\frac{ML^2}{8}$

c) $\frac{ML^2}{24}$

d) None of these

Problem 14 to 18

A cylinder and a ring of same mass M and radius R are placed on the top of a rough inclined plane of inclination θ. Both are released simultaneously form the same height h.

14. Choose the correct statement (s) related to the motion of each body.

a) The friction force acting on each body opposite the motion of its center of mass.

b) The friction force provides the necessary torque to rotate the body about its center of mass.

c) Without friction none of the two bodies can roll.

d) The friction force keeps the point of contact stationary.

15. Identify the correct statement (s).

a) The friction force acting on the cylinder is more than that acting of the ring.

b) The friction force acing on the ring is more than that acting on the cylinder.

c) If the friction is sufficient to roll the cylinder then that ring will also roll.

d) If the friction is sufficient to roll the ring then the cylinder will also roll.

16. when these bodies roll down the inclined surface, then

a) The frictional force does rotational work only.

b) The frictional force does translational work only.

c) The frictional force does both translational and rotational work.

d) The net work done by friction is zero.

17. When these bodies roll down to the foot of the inclined plane, then

a) the mechanical energy of each body is conserved.

b) the velocity of center of mass of the cylinder is $2\sqrt{\frac{gh}{3}}$.

c) the velocity of center of mass of ring is $\sqrt{gh}$.

d) the velocity of center of mass of each body is $\sqrt{2gh}$.

Problem 18 to 19

A ring, cylinder, solid sphere and hollow sphere each of same mass and radius are placed at the same height on a rough inclined surface where the friction is just sufficient to roll the cylinder.

18. Identify the correct statement related to the friction force acting on each body.

a) Maximum friction force acts on the ring.

b) Minimum friction force acts on the solid sphere.

c) Same friction force acts on the ring and the hollow sphere only.

d) Same friction force acts on the ring, hollow sphere and cylinder,

19. Choose the correct statement (s) related to the time taken in reaching the foot of the inclined plane.

a) The solid sphere is the first to reach the foot of the incline.

b) The ring and the hollow sphere reach the foot of the incline, simultaneously.

c) The ring, hollow sphere and the cylinder reach the foot of the incline. simultaneously.

d) The order of reaching the foot of the incline, will be as follows first the solid sphere, second the cylinder, third the hollow sphere and fourth the ring.

20. Identify the correct statement (s) related to angular momentum

a) It is always defined with respect to an axis.

b) It is a vector quantity.

c) It is always zero for a body whose center of mass is stationary.

d) For a rotating a is never equal to zero.

21. Choose the correct statement (s) related to the applicability of the expression of angular momentum $\vec{L} = \vec{r}_c \times M\vec{v}_c + I_c\vec{\omega}$

a) It is applicable to a particle with respect to a fixed point.

b) It s applicable to all the particle of a body or system.

c) In general. The given expression can be modified as $\vec{L} = \vec{r}_C \times M\vec{v}_C + I_C\vec{\omega}$

d) None of these

22. A projectile of mass m is from the origin O with a velocity v_0 at an angle θ. Identify the correct statement(s)

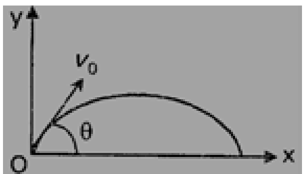


Fig.5.17

a) Angular momentum of the particle about O is always zero.
b) Angular momentum of the particle about O increases with time.
c) A constant torque acts on the particle about O in the clockwise sense.
d) Angular momentum of the particle is least when it is at th highest position.

Problem 23 to 27

A ring of mass M and radius R sliding with a velocity v_0 suddenly enter into rough surface where the coefficient of friction is μ, as shown in fig. 5.18.

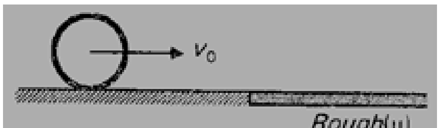


Fig.5.18

23. Chooser the correct statement (s)
a) As the ring enter on the rough surface m the limiting friction force acts on it
b) The direction of friction is opposite to the direction of motion.
c) The friction force accelerates the ring in the clockwise sense about its center of mass.
d) As the ring enters on the rough surface it starts rolling.

24. Choose the correct statement (s)
a) The friction does negative translational work.
b) The friction does positive rotational work.
c) The net work done by friction is zero.
d) Friction force converts translational kinetic energy into rotational kinetic energy.

25. Choose the correct statement(s).
a) The momentum of the ring is conserved.
b) The angular momentum of the ring is conserved about its center of mass
c) The angular momentum of the ring is conserved about any pint on the horizontal surface
d) The mechanical energy of the ring is conserved.

26. Choose the correct statement (s).
a) The ring starts its rolling motion when the center of mass becomes stationary,
c) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$.
d) The rolling velocity is $\frac{v_0}{2}$.

27. Choose the correct alternatives(s)

a) The linear distance moved by the center of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$.

b) The net work done by friction force is $-\frac{3}{8}mv_0^2$

c) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$

d) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

ANSWERS

1abc	2abcd	3ad	4abcd	5a	6abc	7d	8abcd	9acd	10c
11a	12c	13c	14abcd	15bd	16cd	17abc	18bd	19ac	20abd
21abc	22bc	23abc	24abd	25c	26bcd	27acd			

SOLUTIONS

1.

- In situation (a), (b) and (c),the axis passing through the center of mass of the body is moving but remains constant in direction.
- In situation (d) and inclined axis about which the cone is rolling does not remain constant in direction.

2. a, b, c & d

All the statements confirm to the definition of moment of inertia.

3. a & d

The perpendicular axis theorem is applicable lamina shaped bodies only, because in that case only one can distinguish between the axis lying in the plane of the body an perpendicular to the plane of the body.

4. a, b, c & d

All the statement confirm to the definition and properties of the instantaneous axis of rotation.

5. a

The relations $\vec{r} = \vec{r}_c + \vec{r}\,'$

$\vec{v} = \vec{v}_c + \vec{v}\,'$

$\vec{a} = \vec{a}_c + \vec{a}\,'$

$K = K_c + K'$

$$L = L_c + L'$$

are all general in nature. These are applicable to a body undergoing any kind of motion.

6. a, b & c

- Although the center of mass is rotating about the instantaneous axis of rotation but is trajectory is a straight line.

- The velocity of the top most point depends on the linear velocity of center as well as on the angular velocity of the body rotating about the center of mass. If the horizontal surface is stationary, then the velocity of the topmost velocity is double the velocity of center of mass.

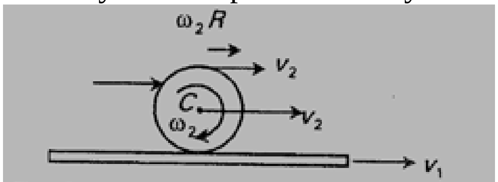


Fig. S 5.14 A The velocity of the *top* most point is *more* than the velocity of centre of mass

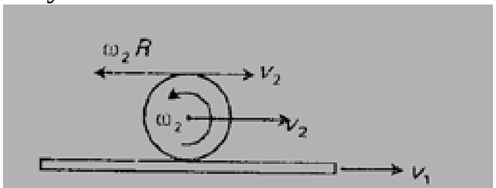


Fig. S 5.14 B. The velocity of the *top* most po is *less* than the velocity of centre of mass.

7. d

Any point on the periphery of a rolling ring follow a cycloid.

8. a, b, c & d

- According to the definition of rolling motion, the linear distance moved by the center of mass in one revolution in 2πR.
- The point of contact moves along a cycliod, the length of the cycloid in one revolution is 8R.
- Note that the distance move by the point of contact is 8R but the net displacement of the point of contact is 2πR.
- Since of speed of any point on the periphery of a ring is more than that of the center of mass, therefore, this point always travel a larger distance.

9. a, c & d

- From the fig. S 5.15, it is obvious that the points lying in the range AOB have velocities less than or equal to the velocity of center of mass. The segment AOB constitutes one–third mass of the ring.

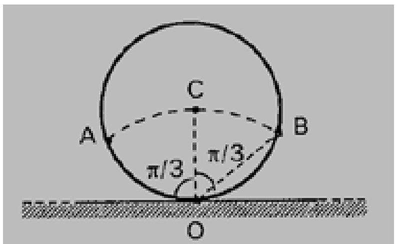


Fig S 5.15

- Two points A and B on the periphery of the ring which moves with the velocity of center of mass.
- The top most point of the ring moves with a velocity two times the velocity of center of mass.

Problems 10 to 13

10. c

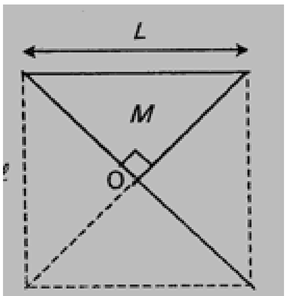


Fig S 5.16

The given plate is one–fourth of a square plates of side l .

Therefore, its moment of inertia is

$$I_0 = \frac{1}{4}\left[\frac{4ML^2}{6}\right] = \frac{ML^2}{6}$$

11. a

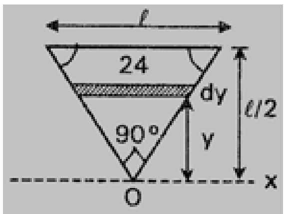


Fig S 5.17

Image a horizontal strip at a distance y form the apex O. Since the apex angel is 90°, therefore, length of the strip is 2y.

area of the strip is dA = 2y dy

Mass of the strip is dm = $\frac{M}{L^2/4} 2ydy = \frac{8M}{L^2} ydy$

Now, $I_x = \int y^2 dm = \frac{8M}{L^2}\int y^3 dy = \frac{8M}{L^2}\left[\frac{y^4}{4}\right]_0^{1/2}$

$$I_x = \frac{ML^2}{8}$$

12. c

Using parallel – axes theorem, the moment of inertia about the base of the plate is

$$I = I_C + M\left(\frac{1}{3}\frac{L}{2}\right)^2 = I_C + \frac{ML^2}{36}$$

Also, $I_C = I_x - M\left(\frac{2}{3}\frac{L^2}{2}\right)^2 = \frac{ML^2}{8} - \frac{ML^2}{9} = \frac{ML^2}{72}$

$$\therefore I = \frac{ML^2}{72} + \frac{ML^2}{36} = \frac{ML^2}{24}$$

13. c

Using perpendicular – axes theorem, we know that $I_x + I_y = I_z$

$$\therefore I_y = I_z - I_x = \frac{ML^2}{6} - \frac{ML^2}{8} = \frac{ML^2}{24}$$

14. a, b, c & d

From the free body diagram of the cylinder one can easily conclude that

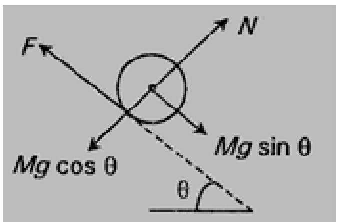


Fig S 5.18

- Friction force opposes the motion of the center of mass.
- Friction force provides the clockwise torque about its center of mass.
- Without friction the bodies can only slide down without any rotation.
- contact stationary, otherwise, it will slip downward.

15. b & d

- Since the moment of inertia of the ring is more than that of the disc, therefore., more friction force is required to roll a ring as compared to a disc.
- Hence, if the friction is sufficient to roll the ring, then the cylinder can also roll but the reverse is not true.
- The minimum friction force required in case of ring and cylinder are:

Ring : $f_{\min} = \frac{Mg \sin\theta}{2}$

Cylinder : $f_{\min} = \frac{mg \sin\theta}{3}$

16. c & d

The frictional force does both the work:

- the rotational work is positive.
- the translational work is negative.

Since the point of application of friction force is stationary, therefore, the net work done by friction force is zero.

17. a, b & c

- Since $W_{NC} = 0$, $W_{PS} = 0$ and $W_{other} = 0$, therefore, mechanical energy of the system is conserved with respect to ground. E = K + U= constant.
- For the cylinder :

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \frac{3}{4}Mv^2 \qquad (\because v = \omega R)$$

$$\therefore \quad v = \sqrt{\frac{4}{3}gh} = 2\sqrt{\frac{gh}{3}}$$

- For the ring :

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(MR^2\right)\omega^2 = Mv^2 \qquad (\because v = \omega R)$$

$$\therefore \quad v = \sqrt{gh}.$$

Problems 18 to 19

18. b & d

- The friction force required to roll a cylinder is $\frac{Mg\sin\theta}{3}$. The friction force required to roll a hollow sphere and a ring is more than this. Therefore, the same friction force $\left(=\frac{Mg\sin\theta}{3}\right)$ acts on the cylinder, ring and the hollow sphere.
- the friction required to roll a solid sphere is $\frac{2}{7}$Mg sin θ which is les than $\frac{1}{3}$Mg sin θ. Therefore, the least friction force acts on the solid sphere.

19. a & c

- The acceleration of ring, cylinder and hollow sphere are equal and it is given by
$$a = \frac{Mg\sin\theta - f}{M} = \frac{2}{3}g\sin\theta$$
- The acceleration of the solid sphere is
$$a = \frac{Mg\sin\theta - \frac{2}{7}M\sin\theta}{M} = \frac{5}{7}g\sin\theta$$
- Hence, the solid sphere is the first to reach the foot of the incline and all the other three bodies reach simultaneously at some later time.

20. a, b & d

- A body may possess infinite values of angular momentum depending upon the axis under consideration.
- When the center of mass of a body is stationary even then the body may possess angular momentum if it is rotating about its center of mass.

21. a, b, c

- The expression $\vec{L} = \vec{r} \times \vec{p}$ is general and can be applied to a single particle or a system of particle.
- Since $\vec{r} = \vec{r}_c + \vec{r}\,'$ and $\vec{v} = \vec{v}_c \times \vec{v}\,'$, therefore, the given expression may be modified as for a system of particles as

$$\vec{L} = \sum(\vec{r}_c + \vec{r}\,') \times m(\vec{v}_c + \vec{v}\,')$$

or $$\vec{L} = \vec{r} \times (\sum m)\vec{v}_c + \vec{r}_c \times \sum m\vec{v}\,' + (\sum m\vec{r}\,') \times \vec{v}_c + \sum(\vec{r}\,' \times m\vec{v}\,')$$

here $$\sum m\vec{v}\,' = 0 \text{ and } \sum m\vec{r}\,' = 0$$

∴ $$\vec{L} = \vec{r} \times M\vec{v}_c + \sum(\vec{r}\,' \times m\vec{v}\,') \text{ where } M = \sum m$$

If $\vec{r}\,' \perp \vec{v}\,'$, then $v' = r'\omega$ where ω is angular velocity with respect to center of mass, then.

$$\sum(\vec{r}\,'\times m\vec{v}\,') = \left(\sum m r'^2\right)\vec{\omega} = I_c\vec{\omega}$$

Thus, $\vec{L} = \vec{r}_c \times M\vec{v}_c + I_c\vec{\omega}$

22. b & c

- Initially, the angular momentum of the projectile about the origin is zero.
- Since a clockwise torque always acts on the projectile about O, Therefore, its angular momentum always increases with time.

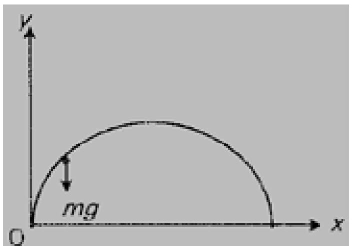


Fig S 5.19

Problems 23 to 27

23. a, b & c

- Since the relative motion exists, therefore, the limiting friction force acts on the ring, i.e. $f = \mu mg$.

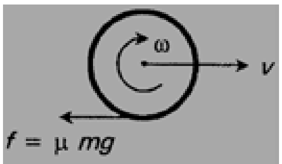


Fig S 5.20

- Friction oppose relative motion, therefore, it acts backward on the ring.
- Friction produced clockwise torque on the ring in the clockwise sense.

24. a, b & d

- The friction force oppose translatory motion of the ring, therefore, $W_{trans} < 0$.
- The friction favours rotation about center of mass, therefore, $W_{rot} > 0$.
- The friction force converts translational kinetic energy into rotational kinetic energy.
- $W_{net} = W_{trans} + W_{rot} < o$

25. c

- Momentum is not conserved because a net force ($= \mu mg$) is acting in the horizontal direction.
- Angular momentum about c.m. is not conserved because a net torque ($= \mu mgR$) acts about the c.m.
- Angular momentum about any point on the surface is conserved because the net force passes through this point.
- The mechanical energy is not conserved because friction does a negative work.

26. b, c & d

- Rolling condition, the point of contact should be stationary, i.e. $v = \omega R$.
- Conserving Angular Momentum about the point of contact $L_i = L_f$.

  $mv_0 R = mvR + mvR^2\omega = 2mRv$

or v = $\frac{v_0}{2}$

- Applying the equation of kinematics

  v – u = at

$\therefore \quad \frac{v_0}{2} - v_0 = (-\mu g)t$

or $t = \frac{v_0}{2\mu g}$.

27. a, c & d

- Using the equation of kinematics

$$v^2 - v_0^2 = 2ax$$

or $\left(\frac{v_2}{2}\right)^2 - v_0^2 = 2(-\mu g)x$

or $x = \frac{3}{8}\frac{v_0^2}{\mu g}$

- The loss in kinetic energy is

$$\Delta K_{loss} = k_i - k_f = \frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 = \frac{mv_0^2}{4}$$

- The translational work done by friction is

$$W_{trans} = -(\mu mg)x = -\frac{3}{8}mv_0^2$$

- the rotational work done by friction is

$$W_{rot} = (K_f - K_i) - W_{trans}$$

or $W_{rot} = -\frac{mv_0^2}{4} - \left(-\frac{3}{8}mv_0^2\right)$

or $W_{rot} = +\frac{mv_0^2}{8}$