

Current Electricity

- Current through a given area of a conductor is the net charge passing per unit time through the area.
- To maintain a steady current, we must have a closed circuit in which an external agency moves electric charge from lower to higher potential energy. The work done per unit charge by the source in taking the charge from lower to higher potential energy (i.e., from one terminal of the source to the other) is called the electromotive force, or emf, of the source. Note that the emf is not a force; it is the voltage difference between the two terminals of a source in open circuit.
- Ohm's law: The electric current I flowing through a substance is proportional to the voltage V across its ends, i.e.,
 $V \propto I$ or $V = RI$,
where R is called the resistance of the substance. The unit of resistance is ohm: $1\Omega = 1 \text{ V A}^{-1}$.
- The resistance R of a conductor depends on its length l and constant cross-sectional area A through the relation,
 $R = \rho l/A$
where ρ , called resistivity is a property of the material and depends on temperature and pressure.

- Electrical resistivity of substances varies over a very wide range. Metals have low resistivity, in the range of $10^{-8} \Omega \text{ m}$ to $10^{-6} \Omega \text{ m}$. Insulators like glass and rubber have 10^{22} to 10^{24} times greater resistivity. Semiconductors like Si and Ge lie roughly in the middle range of resistivity on a logarithmic scale.
- In most substances, the carriers of current are electrons; in some cases, for example, ionic crystals and electrolytic liquids, positive and negative ions carry the electric current.
- Current density j gives the amount of charge flowing per second per unit area normal to the flow,

$$j = nq v_d$$
 where n is the number density (number per unit volume) of charge carriers each of charge q , and v_d is the drift velocity of the charge carriers. For electrons $q = -e$. If j is normal to a cross-sectional area A and is constant over the area, the magnitude of the current I through the area is $n_e v_d A$.
- Using $E = V/l$, $I = n_e v_d A$, and Ohm's law, one obtains

$$eE/m = \rho n_e^2 v_d/m$$

The proportionality between the force eE on the electrons in a metal due to the external field E and the drift velocity v_d (not acceleration) can be understood, if we assume that the electrons suffer collisions with ions in the metal, which deflect them randomly. If such collisions occur on an average at a time interval τ ,

$$v_d = a\tau = eE\tau/m$$

where a is the acceleration of the electron. This gives

$$\rho = m/(n_e^2\tau)$$

- In the temperature range in which resistivity increases linearly with temperature, the temperature coefficient of resistivity α is defined as the fractional increase in resistivity per unit increase in temperature.
- Ohm's law is obeyed by many substances, but it is not a fundamental law of nature. It fails if
 - (a) V depends on I non-linearly.
 - (b) the relation between V and I depends on the sign of V for the same absolute value of V .
 - (c) The relation between V and I is non-unique.

An example of (a) is when ρ increases with I (even if temperature is kept fixed). A rectifier combines features (a) and (b). GaAs shows the feature (c).

- When a source of emf ϵ is connected to an external resistance R , the voltage V_{ext} across R is given by

$$V_{\text{ext}} = IR = \epsilon R / (r + R)$$

where r is the internal resistance of the source.

- Total resistance R of n resistors connected in series is given by

$$R = R_1 + R_2 + \dots + R_n$$

Total resistance R of n resistors connected in parallel is given by

$$(1/R) = (1/R_1) + (1/R_2) + \dots + (1/R_n)$$

- Kirchhoff's Rules –

(a) Junction Rule: At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.

(b) Loop Rule: The algebraic sum of changes in potential around any closed loop must be zero.

- The Wheatstone bridge is an arrangement of four resistances – R1, R2, R3, R4 as shown in the text. The null-point condition is given by
$$R1/R2 = R3/R4$$
using which the value of one resistance can be determined, knowing the other three resistances.
- The potentiometer is a device to compare potential differences. Since the method involves a condition of no current flow, the device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources.

Sample Examples

- The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5Ω and at steam point is 5.23Ω . When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795Ω . Calculate the temperature of the bath.

Solution

$$R_0 = 5 \Omega, R_{100} = 5.23 \Omega \text{ and } R_t = 5.795 \Omega$$

$$T = (R_t - R_0)/(R_{100} - R_0)$$

$$= (5.795 - 5)/(5.23 - 5) * 100$$

$$= (0.795/0.23) * 100$$

$$= 345.65 \text{ } ^\circ\text{C}$$

- (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7} \text{ m}^2$ carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9.0 \times 10^3 \text{ kg/m}^3$, and its atomic mass is 63.5 u. (b) Compare the drift speed obtained above with, (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speed of propagation of electric field along the conductor which causes the drift motion.

Solution

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., electrons drift in the direction of increasing potential. The drift speed v_d is given by Eq. (3.18)

$$v_d = I / neA$$

Now, $e = 1.6 \times 10^{-19} \text{ C}$, $A = 1.0 \times 10^{-7} \text{ m}^2$, $I = 1.5 \text{ A}$. The density of conduction electrons, n is equal to the number of atoms per cubic metre (assuming one conduction electron per Cu atom as is reasonable from its valence electron count of one). A cubic metre of copper has a mass of $9.0 \times 10^3 \text{ kg}$. Since 6.0×10^{23} copper atoms have a mass of 63.5 g,

$$N = (6.0 \times 10^{23} * 9.0 \times 10^6) / 63.5 = 8.5 \times 10^{28} \text{ m}^{-3}$$

Which gives $v_d = 1.1 \text{ mm s}^{-1}$

(b) (i) At a temperature T , the thermal speed* of a copper atom of mass M is obtained from $[\langle (1/2) Mv^2 \rangle = (3/2) k_B T]$ and is thus typically of the order of $\sqrt{3 k_B T / M}$, where k_B is the Boltzmann constant. For copper at 300 K, this is about $2 \times 10^2 \text{ m/s}$. This figure indicates the random vibrational speeds of copper atoms in a conductor. Note that the drift speed of electrons is much smaller, about 10^{-5} times the typical thermal speed at ordinary temperatures.

(ii) An electric field travelling along the conductor has a speed of an electromagnetic wave, namely equal to $3.0 \times 10^8 \text{ m s}^{-1}$