Alternating Current

- An alternating voltage $v = v_m \sin \omega t$ applied to a resistor R drives a current $i = i_m \sin\omega t$ in the resistor, $i_m = v_m/R$. The current is in phase withthe applied voltage.
- For an alternating current $i = i_m \sin\omega t$ passing through a resistor R, the average power loss P (averaged over a cycle) due to joule heating is $(1/2)i_m^2R$. To express it in the same form as the dc power ($P = I\ 2R$), a special value of current is used. It is called root mean square (rms) current and is denoted by I:

  $I = i_m/\sqrt{2}$

  Similarly, the rms voltage is defined by

  $V = v_m/\sqrt{2}$

  We have $P = IV = I^2R$

- An ac voltage $v = v_m \sin\omega t$ applied to a pure inductor L, drives a current in the inductor $i = i_m \sin(\omega t - \pi/2)$, where im $= v_m/X_L$.
- $X_L = \omega L$ is called inductive reactance. The current in the inductor lags the voltage by $\pi/2$. The average power supplied to an inductor over one complete cycle is zero.
- An ac voltage $v = v_m \sin\omega t$ applied to a capacitor drives a current in the capacitor: $i = i_m \sin(\omega t + \pi/2)$. Here,

  $I_m = v_m/X_c$, $X_c = 1/\omega C$ is called capacitive reactance.

  The current through the capacitor is $\pi/2$ ahead of the applied voltage.As in the case of inductor, the average power supplied to a capacitorover one complete cycle is zero.
- For a series RLC circuit driven by voltage $v = v_m \sin\omega t$, the current isgiven by $i = i_m \sin(\omega t + \phi)$

  $i_m = v_m/(\sqrt{R^2 + (X_c - X_L)^2}$

  $\phi = \tan^{-1}[(X_c - X_L)/R]$

  $Z = \sqrt{R^2 + (X_c - X_L)^2}$ is called the impedance of the circuit.

  The average power loss over a complete cycle is given by

  $P = V I \cos\phi$

  The term $\cos\phi$ is called the power factor.

- In a purely inductive or capacitive circuit, $\cos\phi = 0$ and no power is dissipated even though a current is flowing in the circuit. In such cases, current is referred to as a wattless current.
- The phase relationship between current and voltage in an ac circuit can be shown conveniently by representing voltage and current by rotating vectors called phasors. A phasor is a vector which rotates about the origin with angular speed ω. The magnitude of a phasor represents the amplitude or peak value of the quantity (voltage or current) represented by the phasor.

  The analysis of an ac circuit is facilitated by the use of a phasor diagram.

- An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant frequency,

  $\omega_0 = (1/\sqrt{LC})$

  The quality factor Q defined by

  $Q = (\omega_0 L/R) = (1/\ \omega_0 CR)$ is an indicator of the sharpness of the resonance,

  the higher value of Q indicating sharper peak in the current.

- A circuit containing an inductor L and a capacitor C (initially charged) with no ac source and no resistors exhibits free oscillations. The charge q of the capacitor satisfies the equation of simple harmonic motion:

  $(d^2q/dt^2) + (1/LC)q = 0$

  and therefore, the frequency ω of free oscillation is $\omega_0 = (1/\sqrt{LC})$

  The energy in the system oscillates between the capacitor and the inductor but their sum or the total energy is constant in time.

- A transformer consists of an iron core on which are bound a primary coil of N_p turns and a secondary coil of Ns turns. If the primary coil is connected to an ac source, the primary and secondary voltages are related by

  $V_s = (N_s/N_p)V_p$

  and the currents are related by

  $I_s = (N_p/N_s)I_p$

If the secondary coil has a greater number of turns than the primary, the voltage is stepped-up ($V_s > V_p$). This type of arrangement is called a stepup transformer. If the secondary coil has turns less than the primary, we have a step-down transformer.

Sample Examples

- A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which R = 3 Ω, L = 25.48 mH, and C = 796 μF. Find (a) the impedance of the circuit; (b) the phase difference between the voltage across the source and the current; (c) the power dissipated in the circuit; and (d) the power factor.

  Solution

  (a) To find the impedance of the circuit, we first calculate XL and XC.

  $X_L = 2\ \pi \nu L$

  $= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3}\ \Omega = 8\ \Omega$

  $X_C = (1/2\ \pi \nu C)$

  $= [1/(3.14*2*50*796*10^{-6})]$

  $= 4\ \Omega$

  $Z = \surd R^2 + (X_c - X_L)^2 = \surd 9+(8-4)^2 = 5\ \Omega$

$\phi = \tan^{-1}[(X_c - X_L)/R]$

$\tan^{-1}[(4 - 8)/3]$

$= -53.1°$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) The power dissipated in the circuit is

$P = I^2R$

$I = i_m/\sqrt{2} = 283/5\sqrt{2} = 40$ A

Therefore, P = (40A)2 × 3Ω = 4800W

(d) Power factor = cosϕ = cos 53.1° = 0.6