## Conic Sections

- A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.
- The equation of a circle with centre (h, k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.
- The equation of the parabola with focus at (a, 0) $a > 0$ and directrix $x = -a$ is $y^2 = 4ax$.
- Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the hyperbola.
- Length of the latus rectum of the parabola $y^2 = 4ax$ is 4a.

- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.
- The equations of an ellipse with foci on the x-axis is $(x^2/a^2) + (y^2/b^2) = 1$.
- Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.
- Length of the latus rectum of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ is $2b^2/a$
- The eccentricity of an ellipse is the ratio between the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse.

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The equation of a hyperbola with foci on the x-axis is $(x^2/a^2) - (y^2/b^2) = 1$.
- Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.
- Length of the latus rectum of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$ is $2b^2/a$.
- The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

## Sample Examples

- Find the equation of the circle with centre (–3, 2) and radius 4.

  Solution:-

  Here h = –3, k = 2 and r = 4. Therefore, the equation of the required circle is $(x + 3)^2 + (y - 2)^2 = 16$

- Find the equation of the parabola with focus (2,0) and directrix x = – 2.

  Solution:-

  Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola. Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$. Since the directrix is x = – 2 and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with a = 2. Hence the required equation is $y^2 = 4(2)x = 8x$

- Find the equation of the ellipse whose vertices are (± 13, 0) and foci are (± 5, 0).

  Solution:-

  Since the vertices are on x-axis, the equation will be of the form $(x^2/a^2) + (y^2/b^2) = 1$.

  Given that a = 13, c = ± 5.

  Therefore, from the relation $c^2 = a^2 – b^2$, we get $25 = 169 – b^2$ , i.e., b = 12

  $(x^2/169) + (y^2/144) = 1$.

- Find the equation of the hyperbola where foci are (0, ±12) and the length of the latus rectum is 36.

  Solution:-

  Since foci are (0, ± 12), it follows that $c = 12$.

  Length of the latus rectum = $2b^2/a = 36$, $b^2 = 18a$

  $c^2 = a^2 + b^2$ gives

  $144 = a^2 + 18a$

  i.e., $a^2 + 18a – 144 = 0$,

  So $a = – 24, 6$.

  Since a cannot be negative, we take $a = 6$ and so $b^2 = 108$.

  Equation is $(x^2/36) - (y^2/108) = 1$