Principle of Mathematical Induction.

Suppose there is a given statement P(n) involving the natural number n such that

(i) The statement is true for n = 1, i.e., P(1) is true, and

(ii) If the statement is true for n = k (where k is some positive integer), then the statement is also true for n = k + 1, i.e., truth of P(k) implies the truth of P (k + 1).Then, P(n) is true for all natural numbers n.

Sample Examples

- Prove that 2n > n for all positive integers n.

Solution:-

Let P(n): $2^n > n$

When n =1, $2^1 > 1$. Hence P(1) is true.

Assume that P(k) is true for any positive integers k, i.e.,

$2^k > k$ ... (1)

We shall now prove that P(k +1) is true whenever P(k) is true.

Multiplying both sides of (1) by 2, we get $2 \cdot 2^k > 2k$ i.e., $2^{k+1} > 2k = k + k > k + 1$

Therefore, P(k + 1) is true when P(k) is true. Hence, by principle of mathematical induction, P(n) is true for every positive integer n.

- For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

Solution:-

P(n) : $7^n - 3^n$ is divisible by 4.

We note that

P(1): 71 – 31 = 4 which is divisible by 4. Thus P(n) is true for n = 1

Let P(k) be true for some natural number k,

i.e., P(k) : $7^k - 3^k$ is divisible by 4.

We can write $7^k - 3^k = 4d$, where $d \in N$.

Now, we wish to prove that P(k + 1) is true whenever P(k) is true.

Now $7^{(k+1)} - 3^{(k+1)} = 7^{(k+1)} - 7.3^k + 7.3^k - 3^{(k+1)}$

$= 7(7^k - 3^k) + (7 - 3)3^k = 7(4d) + (7 - 3)3^k$

$= 7(4d) + 4.3^k = 4(7d + 3^k)$

From the last line, we see that $7^{(k+1)} - 3^{(k+1)}$ is divisible by 4. Thus, P(k + 1) is true when P(k) is true. Therefore, by principle of mathematical induction the statement is true for every positive integer n.