Sets

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z^+ : the set of positive integers

Q^+ : the set of positive rational numbers, and

R^+ : the set of positive real numbers.

- If a is an element of a set A, we say that " a belongs to A" the Greek symbol ∈ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write a ∈ A. If 'b' is not an element of a set A, we write b ∉ A and read "b does not belong to A".
- There are two methods of representing a set:

(i) Roster or tabular form

(ii) Set-builder form.

- In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. Example - The set of all natural numbers which divide 42 is {1, 2, 3, 6, 7, 14, 21, 42}
- In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. Example - in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet.

- A set which does not contain any element is called the empty set or the null set or the void set.

  Example - D = {x: $x^2 = 4$, x is odd}. Then D is the empty set, because the equation $x^2 = 4$ is not satisfied by any odd value of x.

- A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

  Example - Let W be the set of the days of the week. Then W is finite.

  Example - Let G be the set of points on a line. Then G is infinite.

- Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write $A \neq B$.

  Example - Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

- A set A is said to be a subset of a set B if every element of A is also an element of B. In other words, $A \subset B$ if whenever $a \in A$, then $a \in B$. It is often convenient to use the symbol "$\Rightarrow$" which means implies. Using this symbol, we can write the definition of subset as follows:

  $A \subset B$ if $a \in A \Rightarrow a \in B$

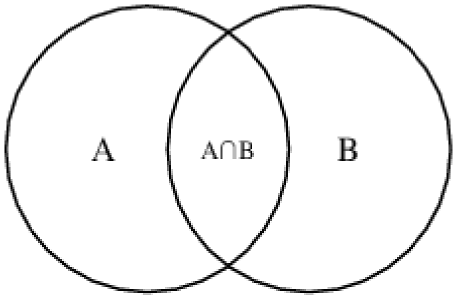
  Example - If A is the set of all divisors of 56 and B the set of all prime divisors of 56, then B is a subset of A and we write $B \subset A$.

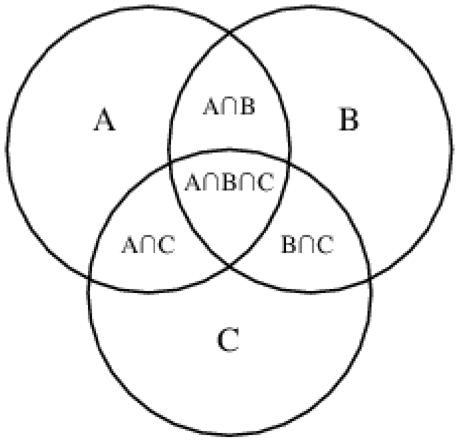
- Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

- Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{ y : a < y < b\}$ is called an open interval and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval.
  The interval which contains the end points also is called closed interval and is denoted by [a, b]. Thus $[a, b] = \{x : a \leq x \leq b\}$
  We can also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x : a \leq x < b\}$ is an open interval from a to b, including a but excluding b.$(a, b] = \{ x : a < x \leq b \}$ is an open interval from a to b including b but excluding a.

- The collection of all subsets of a set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.
  If $A = \{1, 2 \}$, then
  $P(A) = \{ \varphi,\{ 1 \}, \{ 2 \}, \{ 1,2 \}\}$
  Also, note that $n [P (A)] = 4 = 2^2$.

- Venn Diagram





- Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once. The symbol ‘∪’ is used to denote the union. Symbolically, we write $A \cup B$ and usually read as ‘A union B’.

- Properties of Union Operation

  (i) $A \cup B = B \cup A$ (Commutative law)

  (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

  (iii) $A \cup \varphi = A$ (Law of identity element, φ is the identity of $\cup$)

  (iv) $A \cup A = A$ (Idempotent law)

  (v) $U \cup A = U$ (Law of U)

- The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol '$\cap$' is used to denote the intersection. The intersection of two sets A and B is the set of all those elements which belong to both A and B.

- .Properties of Intersection operation.

  (i) $A \cap B = B \cap A$ (Commutative law).

  (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

  (iii) $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).

  (iv) $A \cap A = A$ (Idempotent law)

  (v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)

- The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write $A - B$ and read as "A minus B".
- Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U. Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$. Obviously $A' = U - A$.
- De Morgan's Laws
  $(A \cup B)' = A' \cap B'$
  $(A \cap B)' = A' \cup B'$

- Properties of Complement sets
  Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$
  De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
  Law of double complementation: $(A')' = A$
  Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

- If A and B are finite sets such that $A \cap B = \varphi$, then

  $n(A \cup B) = n(A) + n(B)$.
- If $A \cap B \neq \varphi$, then

  $(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

Examples

- Write the set A = {1, 4, 9, 16, 25, . . . }in set-builder form.

  Solution:-

  We may write the set A as

  A = {x:x is the square of a natural number}

  Alternatively, we can write

  A = {x:x = n2, where n ∈ N}

- State which of the following sets are finite or infinite :

  (1) {x:x ∈ N and (x – 1) (x –2) = 0}

  (2) {x:x ∈ N and x2 = 4}

  (3) {x:x ∈ N and x is prime}

  (4) {x:x ∈ N and x is odd}

Solution:-

(1) Given set = {1, 2}. Hence, it is finite.

(2) Given set = {2}. Hence, it is finite.

(3) The given set is the set of all prime numbers and since set of prime numbers is infinite. Hence the given set is infinite

(4) Since there are infinite numbers of odd numbers, hence, the given set is infinite.

- Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$?. If not, give an example.

Solution:-

No. Let A = {1}, B = {{1}, 2} and C = {{1}, 2, 3}. Here $A \in B$ as A = {1} and $B \subset C$. But $A \not\subset C$ as $1 \in A$ and $1 \notin C$.

Note that an element of a set can never be a subset of itself.

- Let A = {2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find $A \cup B$.

Solution:-

We have $A \cup B$ = {2, 4, 6, 8, 10, 12} Note that the common elements 6 and 8 have been taken only once while writing $A \cup B$.