Probability

- Sample space: The set of all possible outcomes
- Outcomes: A possible result of a random experiment is called its outcome.
- Sample points: Elements of sample space
- Event: A subset of the sample space
- Impossible event : The empty set
- Sure event: The whole sample space
- Simple event: If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

- Compound Event If an event has more than one sample point, it is called a Compound event.
- Complementary event or ‘not event’ : The set A' or $S – A$
- Event A or B: The set $A \cup B$
- Event A and B: The set $A \cap B$
- Event A and not B: The set $A – B$
- Mutually exclusive event: A and B are mutually exclusive if $A \cap B = \varphi$
- Exhaustive and mutually exclusive events: Events E1, E2,..., En are mutually exclusive and exhaustive if $E1 \cup E2 \cup ...\cup En = S$ and $Ei \cap Ej = \varphi$ V $i \neq j$

- Probability: Number P (ωi) associated with sample point ω i such that

  (i) $0 \leq P(\omega i) \leq 1$ (ii) P(ωi) Σ for all $\omega i \in S = 1$ (iii) P(A) = P(ωi) Σ for all $\omega i \in A$. The number P (ωi) is called probability.

- Equally likely outcomes: All outcomes with equal probability
- Probability of an event: For a finite sample space with equally likely outcomes, Probability of an event P(A) = n(A)/n(S) where n(A) = number of elements in the set A, n(S) = number of elements in the set S.
- If A and B are any two events, then P(A or B) = P(A) + P(B) – P(A and B). Equivalently, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive, then P(A or B) = P(A) + P(B)
- If A is any event, then P(not A) = 1 – P(A)

- Two coins (a one rupee coin and a two rupee coin) are tossed once. Find a sample space.

  Solution:-

  Heads on both coins = (H,H) = HH

  Head on first coin and Tail on the other = (H,T) = HT

  Tail on first coin and Head on the other = (T,H) = TH

  Tail on both coins = (T,T) = TT

  Thus, the sample space is S = {HH, HT, TH, TT}

- Consider the experiment of rolling a die. Let A be the event ‘getting a prime number’, B be the event ‘getting an odd number’. Write the sets representing the events (i) Aor B (ii) A and B (iii) A but not B (iv) ‘not A’.

  Solution:-

  Here S = {1, 2, 3, 4, 5, 6}, A = {2, 3, 5} and B = {1, 3, 5}

  (i) ‘A or B’ = $A \cup B$ = {1, 2, 3, 5}

  (ii) ‘A and B’ = $A \cap B$ = {3,5}

  (iii) ‘A but not B’ = A – B = {2}

  (iv) ‘not A’ = A′ = {1,4,6}

- A coin is tossed three times, consider the following events. A: ‘No head appears’, B: ‘Exactly one head appears’ and C: ‘At least two heads appear’. Do they form a set of mutually exclusive and exhaustive events?

Solution:-

The sample space of the experiment is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

and A = {TTT}, B = {HTT, THT, TTH}, C = {HHT, HTH, THH, HHH}

Now $A \cup B \cup C$ = {TTT, HTT, THT, TTH, HHT, HTH, THH, HHH} = S

Therefore, A, B and C are exhaustive events.

Also, $A \cap B = \varphi$, $A \cap C = \varphi$ and $B \cap C = \varphi$

Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive.

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

- One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be (i) a diamond (ii) not an ace.

  Solution:-

  (1)Let A be the event 'the card drawn is a diamond'

  Clearly the number of elements in set A is 13.

  Therefore, $P(A) = 13/52 = 1/4$.

  (2) We assume that the event ‘Card drawn is an ace’ is B

  Therefore ‘Card drawn is not an ace’ should be B′.

  We know that $P(B') = 1 – P(B) = 1-(4/52) = 12/13$

- Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

  (a) Both Anil and Ashima will not qualify the examination.

  (b) At least one of them will not qualify the examination.

Solution:-

Let E and F denote the events that Anil and Ashima will qualify the examination, respectively. Given that $P(E) = 0.05$, $P(F)$ 0.10 and $P(E \cap F) = 0.02$.

Then

(a) The event 'both Anil and Ashima will not qualify the examination' may be expressed as $E' \cap F'$. Since, E' is 'not E', i.e., Anil will not qualify the examination and F' is 'not F', i.e., Ashima will not qualify the examination.

Also $E' \cap F' = (E \cup F)'$ (by Demorgan's Law)

Now $P(E \cup F) = P(E) + P(F) – P(E \cap F)$

or $P(E \cup F) = 0.05 + 0.10 – 0.02 = 0.13$

Therefore $P(E' \cap F') = P(E \cup F)' = 1 – P(E \cup F) = 1 – 0.13 = 0.87$

(b) P (at least one of them will not qualify)

= 1 – P(both of them will qualify)

= 1 – 0.02 = 0.98