



Permutations and Combinations.

- Fundamental principle of counting

If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$.

- A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.
- The number of permutations of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is $n (n - 1) (n - 2) \dots (n - r + 1)$, which is denoted by ${}^n P_r$.
- Factorial Notation : The notation $n!$ represents the product of first n natural numbers, i.e., the product $1 \times 2 \times 3 \times \dots$
- $1 \times (n - 1) \times n$ is denoted as $n!$

- ${}^n P_r = n!/(n-r)! \quad 0 \leq r \leq n.$
- The number of permutations of n objects, where p objects are of the same kind and rest are all different is $n!/p!$.
- The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, p_k are of k^{th} kind and the rest, if any, are of different kind is $n!(p_1!p_2!\dots p_k!).$

- Combinations

$${}^n P_r = {}^n C_r r! \quad 0 < r \leq n.$$

- ${}^n C_r = n!/(n-r)!r!$
- ${}^n C_r = {}^n C_{n-r}$
- If ${}^n C_a = {}^n C_b,$
 $a=b$ or $a=n-b.$

Sample Examples

- How many 2 digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

Solution:-

There will be as many ways as there are ways of filling 2 vacant places in succession by the five given digits. Here, in this case, we start filling in unit's place, because the options for this place are 2 and 4 only and this can be done in 2 ways; following which the ten's place can be filled by any of the 5 digits in 5 different ways as the digits can be repeated. Therefore, by the multiplication principle, the required number of two digits even numbers is 2×5 , i.e., 10.

- Find the number of permutations of the letters of the word ALLAHABAD.

Solution:-

Here, there are 9 objects (letters) of which there are 4A's, 2 L's and rest are all different. Therefore, the required number of arrangements = $\frac{9!}{4!2!} = \frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}{2} = 7560$.

- In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

Solution:-

Total numbers of discs are $4 + 3 + 2 = 9$. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).

Therefore, the number of arrangements = $9!/4!3!2! = 1260$.

- A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Solution:- Here, order does not matter. Therefore, we need to count combinations. There will be as many committees as there are combinations of 5 different persons taken 3 at a time. Hence, the required number of ways = ${}^5C_3 = 10$.

Now, 1 man can be selected from 2 men in 2C_1 ways and 2 women can be selected from 3 women in 3C_2 ways.

Therefore, the required number of committees = ${}^2C_1 * {}^3C_2 = 6$.