Matrices and Determinants

- A matrix is an ordered rectangular array of numbers or functions.
- A matrix having m rows and n columns is called a matrix of order m × n.
- $[a_{ij}]_{m \times 1}$ is a column matrix.
- $[a_{ij}]_{1 \times n}$ is a row matrix.
- An m × n matrix is a square matrix if m = n.
- $A = [a_{ij}]_{m \times m}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$.
- $A = [a_{ij}]_{n \times n}$ is a scalar matrix if $a_{ij} = 0$, when $i \neq j$, $a_{ij} = k$, (k is some constant), when $i = j$.
- $A = [a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = 1$, when $i = j$, $a_{ij} = 0$, when $i \neq j$.

- A zero matrix has all its elements as zero.
- $A = [a_{ij}] = [b_{ij}] = B$ if (i) A and B are of same order, (ii) $a_{ij} = b_{ij}$ for all possible values of i and j.
- $kA = k[a_{ij}]m \times n = [k(a_{ij})]m \times n$
- $-A = (-1)A$
- $A - B = A + (-1) B$
- $A + B = B + A$
- $(A + B) + C = A + (B + C)$, where A, B and C are of same order.
- $k(A + B) = kA + kB$, where A and B are of same order, k is constant.
- $(k + l) A = kA + lA$, where k and l are constant.
- If $A = [a_{ij}]m \times n$ and $B = [b_{jk}]n \times p$, then $AB = C = [c_{ik}]m \times p$, where $c_{ik} = \Sigma a_{ij} b_{jk}$ (i) $A(BC) = (AB)C$, (ii) $A(B + C) = AB + AC$, (iii) $(A + B)C = AC + BC$

- If A = [aij]m × n, then A′ or AT = $[a_{ji}]$n × m (i) (A′)′ = A, (ii) (kA)′ = kA′, (iii) (A + B)′ = A′ + B′, (iv) (AB)′ = B′A′
- A is a symmetric matrix if A′ = A.
- A is a skew symmetric matrix if A′ = –A.
- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix.
- Elementary operations of a matrix are as follows: (i) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$ (ii) $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$ (iii) $R_i \rightarrow$ Ri + kR_j or $C_i \rightarrow C_i + kC_j$
- If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A–1 and A is the inverse of B.
- Inverse of a square matrix, if it exists, is unique.

Determinants

- Determinant of a matrix $A = [a_{11}]_{1\times1}$ is given by $|a_{11}| = a_{11}$
- For any square matrix A, the |A| satisfy following properties.
- $|A'| = |A|$, where A' = transpose of A.
- If we interchange any two rows (or columns), then sign of determinant changes.
- If any two rows or any two columns are identical or proportional, then value of determinant is zero.
- If we multiply each element of a row or a column of a determinant by constant k, then value of determinant is multiplied by k.
- Multiplying a determinant by k means multiply elements of only one row (or one column) by k.
- If $A =[a_{ij}]_{3\times3}$, then $k .A =k^3 A$
- If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.

- If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.
- Minor of an element aij of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and denoted by M_{ij}.
- Cofactor of a_{ij} of given by A_{ij} = (– 1)i+ j M_{ij}
- Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example,
- $A = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$.
- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For example, $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$
- A (adj A) = (adj A) A = |A| I, where A is square matrix of order n.

- A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$.
- If AB = BA = I, where B is square matrix, then B is called inverse of A.
- Also A–1 = B or B–1 = A and hence (A–1)–1 = A.
- A square matrix A has inverse if and only if A is non-singular.
- $A^{-1} = 1/|A|$
- Unique solution of equation AX = B is given by X = A–1 B, where $A \neq 0$.
- A system of equation is consistent or inconsistent according as its solution exists or not.

- For a square matrix A in matrix equation AX = B

  (i) $|A| \neq 0$, there exists unique solution

  (ii) $|A| = 0$ and (adj A) B $\neq 0$, then there exists no solution

  (iii) $|A| = 0$ and (adj A) B = 0, then system may or may not be consistent.

Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$