Complex Numbers and Quadratic Equations

- For the complex number $z = a + ib$, a is called the real part, denoted by Re z and b is called the imaginary part denoted by Im z of the complex number z.
- Let us denote $\sqrt{-1}$ by the symbol i. Then, we have $i^2 = -1$. This means that i is a solution of the equation $x^2 + 1 = 0$.
- Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i\,(b + d)$, which is again a complex number.
- Given any two complex numbers z_1 and z_2, the difference $z_1 – z_2$ is defined as follows: $z_1 – z_2 = z_1 + (– z_2)$.
- Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac – bd) + i(ad + bc)$.

- Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient z_1/z_2 is defined by

  z1/z2= $z_1 * 1/z_2$.
- $(z_1+z_2)^2=z_1^2+z_2^2+2z_1z_2$
- $(z_1-z_2)^2=z_1^2+z2^2-2z_1z_2$
- $z_1^2-z_2^2=(z1+z2)(z_1-z_2)$
- Let z = a + ib be a complex number. Then, the modulus of z, denoted by | z |, is defined to be the non-negative real number $\sqrt{a^2+b^2}$, i.e., $| z | = \sqrt{a^2 + b^2}$ and the conjugate of z, denoted as z, is the complex number a – ib, i.e., z = a – ib.
- $z z' = |z|^2$
- Polar representation the nonzero complex number z = x + iy -- z = r (cosθ + i sinθ)

  Where $r=\sqrt{x^2+y^2}$.

Quadratic Equations

Roots of equation ax^2+bx+c

- $x=(-b+\sqrt{b^2-4ac})/2a$ and $(-b-\sqrt{b^2-4ac})/2a$

or

- $(-b+\sqrt{4ac-b^2}\,i)/2a$ and $(-b-\sqrt{4ac-b^2}\,i)/2a$

Sample Examples

- Find the multiplicative inverse of 2 – 3i.

  Solution:-

  $z = 2 - 3i$

  $z' = 2 + 3i$

  $|z|^2 = (2^2 + (-3)^2) = 13$

  $z^{-1} = z'/|z|^2 = (2+3i)/13 = (2/13)+(3/13)i$.

- Represent the complex number $z = 1 + i\sqrt{3}$ in the polar form.

  $1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$

  $r^2(\cos^2\theta + \sin^2\theta) = 4$

  $r = 2$

  $\cos\theta = (1/2)$, $\sin\theta = (\sqrt{3}/2)$

  $\theta = (\pi/3)$

  $z = 2(\cos\pi/3 + i\sin\pi/3)$

- Solve $x^2 + 2 = 0$

  $x^2 + 2 = 0$

  or $x^2 = -2$ $x = \pm\sqrt{-2} = \pm\sqrt{2}\,i$.