Inverse Trigonometric Functions

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$\mathbf{R} - (-1,1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$\mathbf{R} - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1} x$	$\mathbf{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	$\mathbf{R}$	$(0, \pi)$

.

- The value of inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.
- $y = \sin^{-1} x \Rightarrow x = \sin y$
- $x = \sin y \Rightarrow y = \sin^{-1} x$
- $\sin (\sin^{-1} x) = x$
- $\sin^{-1}(\sin x) = x$
- $\sin^{-1} (1/x) = \text{cosec}^{-1} x$
- $\cos^{-1} (-x) = \pi - \cos^{-1} x$
- $\cos^{-1} (1/x) = \sec^{-1}x$
- $\cot^{-1} (-x) = \pi - \cot^{-1} x$

- $\tan^{-1}(1/x) = \cot^{-1}x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$
- $\sin^{-1}(-x) = -\sin^{-1} x$
- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \pi/2$
- $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1} x$
- $\sin^{-1} x + \cos^{-1} x = \pi/2$
- $\mathrm{cosec}^{-1} x + \sec^{-1} x = \pi/2$
- $\tan^{-1}x + \tan^{-1}y = (x+y)/(1 - xy)$
- $\tan^{-1}x - \tan^{-1}y = (x - y)/(1 + xy)$
- $2 \tan^{-1}x = \tan^{-1}(2x/(1-x^2))$
- $2 \tan^{-1}x = \sin^{-1}(2x/(1+x^2)) = \cos^{-1}((1 - x^2)/(1+x^2))$

Sample Examples

- Find the principal value of $\sin^{-1}(1/\sqrt{2})$

Solution:-

$\sin^{-1}(1/\sqrt{2}) = y$

$\sin y = (1/\sqrt{2})$

We know that the range of the principal value branch of sin–1 is, $(-\pi/2, \pi/2)$ and $\sin(\pi/4) = (1/\sqrt{2})$

Hence the value of $\sin^{-1}(1/\sqrt{2})$ is $\pi/4$.

- Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$

Solution:-

Let $x = \sin\theta$. Then $\sin^{-1} x = \theta$

$\sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\sin^{-1} x.$