Mathematical Reasoning

- A mathematically acceptable statement is a sentence which is either true or false. Whenever we mention a statement here, it is a “mathematically acceptable” statement. Example - Two plus two equals four. The sum of two positive numbers is positive.
- Negation of a statement p: If p denotes a statement, then the negation of p is denoted by ~p.
- Compound statements and their related component statements:
- A statement is a compound statement if it is made up of two or more smaller statements. The smaller statements are called component statements of the compound statement.
- The role of “And”, “Or”, “There exists” and “For every” in compound statements.
- The meaning of implications “If ”, “only if ”, “ if and only if ”.

- A sentence with if p, then q can be written in the following ways.
  - p implies q (denoted by $p \Rightarrow q$)
  - p is a sufficient condition for q
  - q is a necessary condition for p
  - p only if q
  - $\sim q$ implies $\sim p$
- The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$. The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$. $p \Rightarrow q$ together with its converse, gives p if and only if q.

- The following methods are used to check the validity of statements:
  - direct method
  - contrapositive method
  - method of contradiction
  - using a counter example.

Examples

- Check whether the following sentences are statements. Give reasons for your answer.

  (i) 8 is less than 6. (ii) Every set is a finite set.(iii) The sun is a star. (iv) Mathematics is fun.

  Solution:-

  (i) This sentence is false because 8 is greater than 6. Hence it is a statement.

  (ii) This sentence is also false since there are sets which are not finite. Hence it is a statement.

  (iii) It is a scientifically established fact that sun is a star and, therefore, this sentence is always true. Hence it is a statement.

  (iv) This sentence is subjective in the sense that for those who like mathematics, it may be fun but for others it may not be. This means that this sentence is not always true. Hence it is not a statement.

- Write the negation of the following statements and check whether the resulting statements are true,

  (i) Australia is a continent.

  (ii) There does not exist a quadrilateral which has all its sides equal.

  Solution :-

  (i) The negation of the statement is:- It is false that Australia is a continent. This can also be rewritten as

  Australia is not a continent.

  We know that this statement is false.

  (ii) The negation of the statement is:- It is not the case that there does not exist a quadrilateral which has all its sides equal.

  This also means the following:

  There exists a quadrilateral which has all its sides equal.

  This statement is true because we know that square is a quadrilateral such that its four sides are equal.

- Write the component statements for the following statement

  A line is straight and extends indefinitely in both directions.

  Solution:-

  p: A line is straight.

  q: A line extends indefinitely in both directions.

- Determine whether inclusive OR or exclusive Or is used for the following sentences.

  (1) The school is closed if it is a holiday or a Sunday.
  (2) Two lines intersect at a point or are parallel.

  Solution:-

  (1) Here “Or” is inclusive since school is closed on holiday as well as on Sunday.
  (2) Here “Or” is exclusive because it is not possible for two lines to intersect and parallel together.

- Write the contrapositive of the following statement:

  (1) If a number is divisible by 9, then it is divisible by 3.

  (2) If you are born in India, then you are a citizen of India.

  Solution:-

  (1)If a number is not divisible by 3, it is not divisible by 9.

  (2) If you are not a citizen of India, then you were not born in India.