Linear Inequalities

- Two real numbers or two algebraic expressions related by the symbol ‘$<$’, ‘$>$’, ‘$\leq$’ or ‘$\geq$’ form an inequality.
- $3 < 5$; $7 > 5$ are the examples of numerical inequalities
- $x < 5$; $y > 2$; $x \geq 3$, $y \leq 4$ are the examples of literal inequalities
- Rules for solving linear inequalities

  - Rule 1:-Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality.
  - Rule 2:-Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied or divided by a negative number, then the sign of inequality is reversed.

- Graphical solution
  - The region containing all the solutions of an inequality is called the solution region.
  - In order to identify the half plane represented by an inequality, it is just sufficient to take any point (a, b) (not online) and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane and shade the region which contains the point, otherwise, the inequality represents that half plane which does not contain the point within it. For convenience, the point (0, 0) is preferred.
  - If an inequality is of the type $ax + by \geq c$ or $ax + by \leq c$, then the points on the line $ax + by = c$ are also included in the solution region. So draw a dark line in the solution region.
  - If an inequality is of the form $ax + by > c$ or $ax + by < c$, then the points on the line $ax + by = c$ are not to be included in the solution region. So draw a broken or dotted line in the solution region.

Examples

- Solve $4x + 3 < 6x + 7$.

Solution:-

$4x + 3 < 6x + 7$

$4x - 6x < 6x + 4 - 6x$

$-2x < 4$ or $x > -2$

i.e., all the real numbers which are greater than -2, are the solutions of the given inequality. Hence, the solution set is $(-2, \infty)$.

- The marks obtained by a student of Class XI in first and second terminal examination are 62 and 48, respectively. Find the number of minimum marks he should get in the annual examination to have an average of at least 60 marks.

  Solution:-

  Let x be the marks obtained by student in the annual examination.

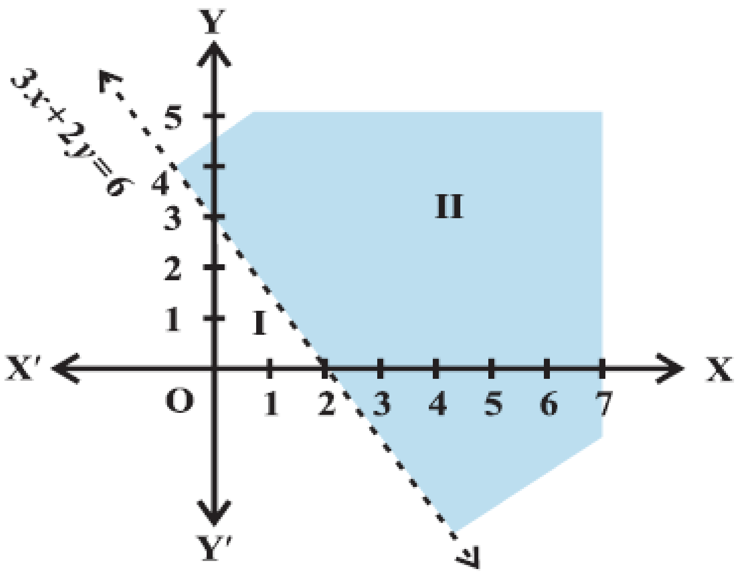
  $(62+48+x)/3 \geq 60$

  $110+x\geq180$

  $x\geq70$

  The student must obtain a minimum of 70 marks to get an average of at least 60 marks.

- Solve 3x + 2y > 6 graphically.



This line divides the xy-plane in two half planes I and II. We select a point (not on the line), say (0, 0), which lies in one of the half planes (Fig 6.8) and determine if this point satisfies the given inequality, we note that

3 (0) + 2 (0) > 6

$0 > 6$,

which is false.

Hence, half plane I is not the solution region of the given inequality. Clearly, any point on the line does not satisfy the given strict inequality. In other words, the shaded half plane II excluding the points on the line is the solution region of the inequality.