Differential Equations

- An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation.
- Order of a differential equation is the order of the highest order derivative occurring in the differential equation.
- Degree of a differential equation is defined if it is a polynomial equation in its derivatives.
- Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

- A function which satisfies the given differential equation is called its solution.
- The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called particular solution.
- To form a differential equation from a given function we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

- Variable separable method is used to solve such an equation in which variables can be separated completely i.e. terms containing y should remain with dy and terms containing x should remain with dx.
- A differential equation which can be expressed in the form dy/dx = f(x,y) where f(x,y) is a homogenous function of degree zero is called a homogeneous differential equation.
- A differential equation of the form (dy/dx)+Py = Q, where P and Q are constants or functions of x only is called a first order linear differential equation.

Examples

- Form the differential equation representing the family of curves y = a sin (x + b), where a, b are arbitrary constants.

  Solution:-

  We have y = a sin (x + b)

  dy/dx = a cos (x + b)…………………………..(1)

  d^2y/dx^2 = – a sin (x + b)……………………….(2)

  Eliminating a and b using equations 1 and 2 we get $(d^2y/dx^2) + y = 0$.

- Find the general solution of the differential equation $(dy/dx) = (1 + y^2)/ (1 + x^2)$

Solution:-

$(dy/(1 + y^2)) = (dx/(1 + x^2))$

Integrating both sides,

$\int (dy/(1 + y^2)) = \int (dx/(1 + x^2))$

$\tan^{-1} y = \tan^{-1} x + C$

- Find the general solution of the differential equation $y\,dx - (x + 2y^2)\,dy = 0$.

  Solution:-

  $(dx/dy) - (x/y) = 2y$

  This is of the type $(dx/dy) + Px = Q$ where $P = -(1/y)$ $Q = 2y$

  Hence IF $= e^{\int(-dy/y)} = e^{-\log y} = (1/y)$

  The solution of differential equation is

  $x(1/y) = \int(2y)(1/y)dy + C$

  $(x/y) = \int 2dy + C$

  $(x/y) = 2y + C$

  $x = 2y^2 + Cy$