## Straight Lines

- Distance between the points P $(x_1, y_1)$ and Q $(x_2, y_2)$ is

  PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- The coordinates of a point dividing the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ internally, in the ratio m: n are

  $[(mx_2 + nx_1)/(m + n) , (my_2 + ny_1)/(m + n)]$.

- If m = n, the coordinates will be $[(x_1 + x_2)/2 , (y_1 + y_2)/2]$.
- Area of the triangle whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is

  $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

- If the area of the triangle ABC is zero, then three points A, B and C lie on a line, i.e., they are collinear.

- If θ is the inclination of a line l, then tan θ is called the slope or gradient of the line l. The slope of a line whose inclination is 90° is not defined. The slope of a line is denoted by m. Thus, m = tan θ, θ ≠ 90°.
- Slope $m = (y_2 - y_1)/(x_2 - x_1)$
- If the line $l_1$ is parallel to $l_2$ , then their inclinations are equal.
- If the lines $l_1$ and $l_2$ are perpendicular such that $l_1$ makes an angle β and $l_2$ makes an angle α with the x-axis, then β = α + 90°.
- Two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other $m_1m_2 = -1$.

- An acute angle (say θ) between lines L1 and L2 with slopes m1 and m2 is given by
  $\tan\theta = |(m2 - m1)/(1 + m1m2)|$ , $1 + m1m2 \neq 0$.

- Two lines are parallel if and only if their slopes are equal.
- Two lines are perpendicular if and only if product of their slopes is –1.
- Three points A, B and C are collinear, if and only if slope of AB = slope of BC.
- Equation of the horizontal line having distance a from the x-axis is either

  $y = a$ or $y = -a$.
- Equation of the vertical line having distance b from the y-axis is either

  $x = b$ or $x = -b$.
- The point (x, y) lies on the line with slope m and through the fixed point $(x_o, y_o)$, if and only if its coordinates satisfy the equation $y - y_o = m(x - x_o)$.
- Equation of the line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by

  $(y - y_1) = [(y_2 - y_1)/(x_2 - x_1)] (x - x_1)$

- The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$ .
- If a line with slope m makes x-intercept d. Then equation of the line is $y = m (x – d)$.
- Equation of a line making intercepts a and b on the x-and y-axis, respectively, is
  $x/a + y/b = 1$.
- The equation of the line having normal distance from origin p and angle between normal and the positive x-axis ω is given by $x \cos\omega + y \sin\omega = p$ .
- Any equation of the form $Ax + By + C = 0$, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.
- The perpendicular distance (d) of a line $Ax + By+ C = 0$ from a point (x1, y1) is given by
  $d = | (Ax1 + By1 + c)| / \sqrt{A^2 + B^2}$
- Distance between the parallel lines $Ax + By + C1 = 0$ and $Ax + By + C2 = 0$, is given by
  $d = |C1 – C2| / \sqrt{A^2 + B^2}$.

## Examples

- If the angle between two lines is $\pi/4$ and slope of one of the lines is 1/2, find the slope of the other line.

  Solution:-

  $\tan \theta = | (m2 – m1)/(1 + m1m2) |$

  $m1 = 1/2$

  $m2 = m$

  $\theta = \pi/4$

  Substituting values,

  $| (m – 1/2)/(1 + m/2) | = 1$

  $(m – 1/2)/(1 + m/2) = 1$ and $-(m – 1/2)/(1 + m/2) = 1$

  $m = 3$ and $m = -1/3$

- Three points P (h, k), Q ($x_1$, $y_1$) and R ($x_2$, $y_2$) lie on a line. Show that (h – $x_1$) ($y_2$ – $y_1$) = (k – $y_1$) ($x_2$ – $x_1$).

  Solution:-

  Since points P, Q and R are collinear, we have Slope of PQ = Slope of QR

  $(y_1 - k)/(x_1 - h) = (y_2 - y_1)/(x_2 - x_1)$

  $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$

- Find the equation of the line through (– 2, 3) with slope – 4.

  Solution:-

  Here m = – 4 and given point ($x_0$, $y_0$) is (– 2, 3). By slope-intercept form formula, equation of the given line is

  $y - 3 = -4(x + 2)$ or $4x + y + 5 = 0$, which is the required equation.

- Equation of a line is $3x – 4y + 10 = 0$. Find its (i) slope, (ii) x – and y-intercepts.

Solution:-

(1) Given equation $3x – 4y + 10 = 0$ can be written as

$y = 3/4\ x + 5/2$

Comparing with $y = mx + c$, we have slope of the given line as $m = 4/3$

(2)Given equation can be written as

$x/(-10/3) + y/(5/2) = 1$

y intercept is 5/2.

- Find the equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1, – 2).

Solution:-

Given line $x - 2y + 3 = 0$ can be written as

$y = x/2 + 3/2$

Slope of the line (1) is $m1 = 2$. Therefore, slope of the line perpendicular to line (1) is $m2 = -1/m1 = -1/2$

Equation of the line with slope – 2 and passing through the point (1, – 2) is $y - (-2) = -2(x - 1)$ or $y = -2x$.