Work, Energy and Power

- The scalar product or dot product of any two vectors A and B, denoted as A.B is defined as

  $A.B = A\ B \cos\theta$

  where θ is the angle between the two vectors. Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction.
- Work is done by a force on the body overa certain displacement.
- The work-energy theorem states that the change in kinetic energy of a body is the workdone by the net force on the body.

  $K_f - K_i = W_{net}$
- The change inkinetic energy of a particle is equal to thework done on it by the net force.

- The work done by the force is defined to bethe product of component of the force in thedirection of the displacement and themagnitude of this displacement. Thus

  W = (F cos θ)d = F.d

- If an object of mass m hasvelocity v, its kinetic energy K is

  K = (1/2) m v.v = (1/2) mv^2

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\Delta x$$

$$= \int_{x_i}^{x_f} F(x)\, dx$$

- Gravitational potential energy of an object, as a function of the height h, is denoted by V(h) and it is the negative of work done by the gravitational force in raising the object to that height.

  $V(h) = mgh$

- The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.
- If the total energy of the reactants is morethan the products of the reaction, heat is releasedand the reaction is said to be an exothermicreaction. If the reverse is true, heat is absorbed andthe reaction is endothermic.
- Massand energy are equivalent and are related bythe relation

  $E = m c^2$

  where c, the speed of light in vacuum isapproximately 3×10^8 m s^{-1}. Thus, a staggeringamount of energy is associated with a merekilogram of matter

  $E = 1\times (3 \times 10^8)^2$ J $= 9 \times 10^{16}$ J.

This is equivalent to the annual electrical outputof a large (3000 MW) power generating station.

- Energy may be transformed fromone form to another but the total energy of anisolated system remains constant.

  Energy canneither be created, nor destroyed.

- Power is defined as thetime rate at which work is done or energy istransferred.

  The average power of a force is defined as theratio of the work, W, to the total time t taken

  $P_{av} = W/t$

  The instantaneous power is defined as the limiting value of the average power as time interval approaches zero,

  $P = dW/dt$

  The work dWdone by a force F for a displacement dr is dW = F.dr. The instantaneous power can also be expressed as

  $P = F.(dr/dt) = F.v$

  where v is the instantaneous velocity when the force is F.

- In all collisions the total linear momentum isconserved; the initial momentum of the systemis equal to the final momentum of the system.
- Acollision in which the two particles move togetherafter the collision is called a completely inelasticcollision. The intermediate case where thedeformation is partly relieved and some of theinitial kinetic energy is lost is more common andis appropriately called an inelastic collision.

Sample Examples

- It is well known that araindrop falls under the influence of thedownward gravitational force and theopposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass1.00 g falling from a height 1.00 km. It hits the ground with a speed of 50.0 m s^{-1}. (a) What is the work done by the gravitational force? What is the work done by the unknown resistive force?

  Solution

  (a) The change in kinetic energy of the drop is $K= (1/2)mv^2$

  $= 1.25$ J

  where we have assumed that the drop is initially at rest.

  Assuming that g is a constant with a value 10 m/s^2, the work done by the gravitational forceis,

  $Wg = mgh$

  $= 10^{-3} \times 10 \times 10^3 = 10.0$ J

(b) From the work-energy theorem

$\Delta K = Wg + Wr$

where Wr is the work done by the resistive force on the raindrop. Thus

$Wr = \Delta K - Wg$

$= 1.25 - 10$

$= -8.75$ J

- An elevator can carry amaximum load of 1800 kg (elevator +passengers) is moving up with a constantspeed of 2 m s^{-1}. The frictional force opposingthe motion is 4000 N. Determine theminimum power delivered by the motor to the elevator in watts as well as in horsepower.

Solution

The downward force on the elevator is

$F = m g + F$

$= (1800.10) + 4000 = 22000$ N

The motor must supply enough power to balance this force. Hence, $P = F. v = 22000.2 = 44000$ W = 59 hp