SET I

DIRECTIONS : Let a, b, c be real and α and β be the roots of the equation $ax^2 + bx + c = 0$.

1. If $\alpha < -1$ and $\beta > 1$, then

   (1) $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$ (2) $1 + \frac{c}{a} + \left|\frac{b}{a}\right| > 0$ (3) $1 + \frac{c}{a} + \left|\frac{b}{a}\right| = 0$ (4) None of these

2. If $c < 0 < b$ and $\alpha < \beta$, then

   (1) $0 < \alpha < \beta$ (2) $\alpha < 0 < \beta < |\alpha|$ (3) $\alpha < \beta < 0$ (4) $\alpha < 0 < |\alpha| < \beta$

3. The roots of equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ are

   (1) $\alpha - 1, \beta - 1$ (2) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (3) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$ (4) None of these

4. If α, β are the roots of the equation $x^2 - a(x+1) - b = 0$, then $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + b} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + b}$ is equal to

   (1) 0 (2) 1 (3) 2 (4) 3

5. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if d/a, e/b, f/c are in

   (1) A.P. (2) G.P. (3) H.P. (4) None of these

6. If the roots α, β, γ of $x^3 - 3ax^2 + 3bx - c = 0$ are in H.P., then

   (1) $\beta = 1/\alpha$ (2) $\beta = b$ (3) $\beta = c/b$ (4) $\beta = b/c$

7. Let $f(x) = ax^2 + bx + c$, $a, b, c \in R$ and $a \neq 0$. Suppose $f(x) > 0$ for all $x \in R$. Let $g(x) = f(x) + f'(x) + f''(x)$. Then

   (1) $g(x) = 0 \ \forall\ x \in R$ (2) $g(x) < 0 \ \forall\ x \in R$
   (3) $g(x) = 0$ has non-real roots (4) $g(x) = 0$ has real roots

8. If p, q, r are positive and are in A.P., then the roots of the quadratic equation $px^2 + qx + r = 0$ are real for

   (1) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (2) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$ (3) all p and r (4) no p and r

9. If α and β be the roots of the equation $x^2 - ax + b = 0$ and $V_n = \alpha^n + \beta^n$, then

   (1) $V_{n+1} = aV_n - bV_{n-1}$ (2) $V_{n+1} = aV_n + bV_{n-1}$ (3) $V_{n+1} = bV_n - aV_{n-1}$ (4) $V_{n+1} = bV_n + aV_{n-1}$

10. In a triangle PQR. $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$. Then

    (1) $a + b = c$ (2) $b + c = a$ (3) $a + c = b$ (4) $b = c$

SET II

DIRECTIONS : *For the following questions, four options are given. Choose the correct option.*

1. If $x < 0$, $y < 0$, $x + y + x/y = 1/2$ and $(x + y)(x/y) = -1/2$, then the values of x and y are

(1) −1/4, −1/4 (2) 1/4, 1/4 (3) 1/4, −1/4 (4) −1, 2

2. Let p be a +ve integer. Then $\Delta = \begin{vmatrix} a^p - x & a^{p+1} - x & a^{p+2} - x \\ a^{p+3} - x & a^{p+4} - x & a^{p+5} - x \\ a^{p+6} - x & a^{p+7} - x & a^{p+8} - x \end{vmatrix} =$

(1) 0 (2) $(1 + a^2 + a^4)x$ (3) $a^p(1 + x)$ (4) $a^p(1 + x + x^2)$

3. Let p be a +ve integer. Then $\Delta = \begin{vmatrix} {}^{p+2}C_2 & {}^{p+3}C_2 & {}^{p+4}C_2 \\ {}^{p+3}C_2 & {}^{p+4}C_2 & {}^{p+5}C_2 \\ {}^{p+4}C_2 & {}^{p+5}C_2 & {}^{p+6}C_2 \end{vmatrix} =$

(1) 1 (2) −1 (3) p − 1 (4) $p^2 + p + 2$

4. Let p be a +ve integer. Then $\begin{vmatrix} 1 & {}^{p}C_1 & {}^{p}C_2 \\ 1 & {}^{p+1}C_1 & {}^{p+1}C_2 \\ 1 & {}^{p+2}C_1 & {}^{p+2}C_2 \end{vmatrix}$ is equal to

(1) 1 (2) −1 (3) p − 1 (4) $p^2 + p + 2$

5. Let a, b, c be three real numbers. Then $\begin{vmatrix} \sin 2a & \sin(a+b) & \sin(a+c) \\ \sin(b+a) & \sin 2b & \sin(b+c) \\ \sin(c+a) & \sin(c+b) & \sin 2c \end{vmatrix} =$

(1) 0 (2) cos (a + b + c) (3) sin (a + b + c) (4) None of these

6. Let a and b be two real numbers. Then $\begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix} =$

(1) 0 (2) $(a^2 + b^2)b$ (3) $\frac{3}{2}ab + \frac{a}{2} + \frac{b}{2}$ (4) a (a + b) (a + 2b)

7. Let x and y be real numbers. Then $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$ is equal to

(1) xy (2) $x^2 + y^2$ (3) $-2(x^3 + y^3)$ (4) $-x^2 - y^2$

8. Suppose p, q, r ≠ 0 and system of equation

$(p + a)x + by + cz = 0$

$ax + (q + b)y + cz = 0$

$ax + by + (r + c)z = 0$

has a non-trivial solution, then value of $\frac{p}{a} + \frac{q}{b} + \frac{r}{c}$ is

(1) −1 (2) 0 (3) 1 (4) 2

9. The determinant $\begin{vmatrix} xp+y & x & y \\ py+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

(1) x, y, z are in A.P. (2) x, y, z are in G.P. (3) x, y, z are in H.P. (4) xy, yz, zx are in A.P.

10. If α, β, γ are the roots of the equation $x^3 + px + 9 = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(1) $\alpha+\beta+\gamma$ (2) $\alpha\beta\gamma$ (3) $\alpha^{-1}\beta^{-1}\gamma^{-1}$ (4) 0

SET III

DIRECTIONS : *For the following questions, four options are given. Choose the correct option.*

1. If f(x) be a polynomial function satisfying $f(x).f\left(\frac{1}{2}\right) = f(x) + f\left(\frac{1}{x}\right)$ and f(4) = 65, then the value of f(6) is

(1) 216 (2) 217 (3) 37 (4) 28

2. If f(x) satisfies the relation f(x + y) = f(x) + f(y) for all x, y $\in$ R and f(1) = 5, then the value of $\sum_{n=1}^{m} f(n)$ is

(1) $\frac{5m(m+1)}{2}$ (2) $\frac{5m(m-1)}{2}$ (3) $\frac{7m(m-1)}{2}$ (4) $\frac{7m(m+1)}{2}$

3. Let f : R $\to$ R be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, f(0) = 3 and f'(0) = 3, then

(1) $\frac{f(x)}{x}$ is differentiable in R. (2) f(x) is continuous but not differentiable in R.

(3) f(x) is continuous in R. (4) f(x) is bounded in R.

4. Let f : N $\to$ N be a function such that

(i) $x - f(x) = 19\left\lfloor\frac{x}{19}\right\rfloor - 90\left\lfloor\frac{f(x)}{90}\right\rfloor \forall x \in N$ where [.] denotes the greatest integer function.

(ii) 1900 < f (1990) < 2000. Then the possible values of f(1990) are

(1) 1904, 1994 (2) 1908, 1994 (3) 1904, 1996 (4) 1904, 1992

5. Let g : R $\to$ R be given by g(x) = 3 + 4x. If $g^n(x)$ = gogo .... og (x), then $g^n(x)$ is

(1) $(4^n + 1) + 4^n x$ (2) $(4^n - 1) + 4^n x$ (3) $(4^n - 1) - 4^n x$ (4) $(4^n + 1) - 4^n x$

6. Consider a real valued function f(x) satisfying $2f(xy) = f(x)^y + (f(y))^x$ for all x, y $\in$ R and f(1) = a, where a $\neq$ 1, then the value of $(a-1)\sum_{i=1}^{n} f(i)$ is

(1) $a^{n+1} + a$ (2) $a^{n-1} - a$ (3) $a^{n+1} - a$ (4) $a^{n-1} + a$

7. If p and q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$, then

(1) $q = p^2$ (2) $q^2 = p$ (3) q = p (4) $q = p^4$

8. Let f(x, y) be a periodic function satisfying the condition f(x, y) = f(2x + 2y, 2y−2x) $\forall$ x, y $\in$ R. Now define a function g by g (x) = f(2^x, 0) and g(x) is a periodic function then the period of g(x) is

(1) 6 (2) 12 (3) 9 (4) 24

9. The domain of function $f(x) = \dfrac{1}{[|x-1|]+[|7-x|]-6}$ is (where [.] denotes greatest integral function)

(1) f(x) $\in$ R − (0, 1) $\cup$ {1, 2, 3, 4, 5, 6, 7} $\cup$ [7, 8] (2) f(x) $\in$ R − (0, 1) $\cap$ {1, 2, 3, 4, 5, 6, 7} $\cup$ [7, 8]

(3) f(x) $\in$ R − (0, 1) $\cup$ {1, 2, 3, 4, 5, 6, 7} $\cap$ [7, 8] (4) None of these

10. If f(x) satisfies the relation, f(x + y) = f(x) + f(y) for all x, y $\in$ R and f(1) = 5, then the value of $\sum_{n=1}^{101} f(n)$ is

(1) 25755 (2) 25750 (3) 25760 (4) None of these

SET IV

DIRECTIONS : *N, a set of natural numbers is partitioned in to subsets. S_1 = [1], S_2 = [2, 3], S_3 = [4, 5, 6] S_4 = [7, 8, 9, 10] and so on.*

1. The first term of the subset S_{25} is

(1) 201 (2) 301 (3) 402 (4) None of these

2. The sum of the element of the subset S_{10} is

(1) 405 (2) 505 (3) 435 (4) None of these

3. The sum of the element of the subset S_{30} is

(1) 12505 (2) 14115 (3) 13515 (4) None of these

4. The first term of the subset S_{18} is

(1) 155 (2) 153 (3) 154 (4) None of these

5. The difference between the first and last term of the subset S_{12} is

(1) 11 (2) 10 (3) 12 (4) None of these

6. The last term of the subset S_{15} is

(1) 120 (2) 125 (3) 130 (4) None of these

7. The sum of first and last two terms of subset S_8 is

(1) 68 (2) 65 (3) 75 (4) None of these

8. The total number of terms used up to S_{15} is

(1) 110 term (2) 90 term (3) 120 term (4) None of these

9. The 6th term of S_{22} is

(1) 237 (2) 238 (3) 239 (4) None of these

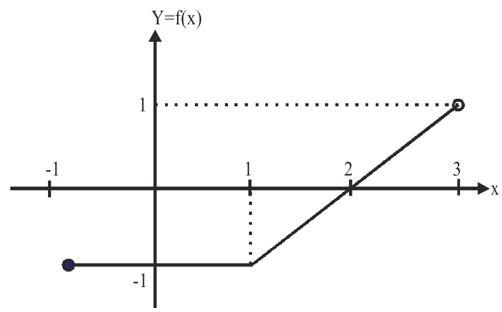
10. Sum of all values up to S_5 is

(1) 90 (2) 120 (3) 150 (4) None of these

SET V

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

Refer to the following graph to answer the questions that follow.



1. $|f(x)|$ is an increasing function in the interval
   (1) (2, 3) (2) (1, 3) (3) (0, 2) (4) None of these

2. In the interval (–1, 3), $-|f(x)|$ is non differentiable at x equal to
   (1) 3 & 0 (2) 1 & 2 (3) 0 & 2 (4) None of these

3. $f(-|x|)$ is
   (1) an increasing function (2) a decreasing function (3) a constant function (4) positive $\forall\, x \in$ Domain

4. $g(x) = \frac{1}{2}(|f(x)| + f(x))$ is a constant function in the interval
   (1) [–1, 3] (2) [0, 3] (3) [–1, 2] (4) None of these

5. $g(x) = \frac{1}{2}(|f(x)| - f(x))$ is a constant function in the interval
   (1) [–1, 2] (2) $[-1, 1] \cup [2, 3]$ (3) [1, 3) (4) None of these

6. $g(x) = \frac{|f(x)|}{f(x)} = 1 \;\forall\, x \in$ is
   (1) (1, 3) (2) (2, 3) (3) (–1, 1) (4) (0, 2)

7. $g(x) = f\left(\frac{|x|}{x}\right)$ is
   (1) an increasing function (2) a decreasing function (3) a constant function (4) None of these

8. If $|f(|x|)| = x$, then
   (1) $-1 \le x \le 1$ (2) $x = 2$ (3) $1 < x < 2$ (4) None of these

9. If $|f(x) + 1| > 0$, then
   (1) $x > 0$ (2) $x < 0$ (3) $x > 1$ (4) None of these

10. If $f(|x|) < 0$, then
    (1) $x < 2$ (2) $x > 0$ (3) $1 < x < 2$ (4) None of these

SET VI

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. In a test of Mathematics, there are two types of questions to be answered – short answered and long answered. The relevant data are given below

	Time taken to solve	Marks	Number of questions
Short answered questions	5 minutes	3	10
Long answered questions	10 minutes	5	14

The total marks are 100. Student can solve all the questions. To secure maximum marks, student solve x short answered and y long answered questions in three hours, then the linear constraints except $x \geq 0$, $y \geq 0$, are

(1) $5x + 10y \leq 180$, $x \leq 10$, $y \leq 14$ (2) $x + 10y \geq 180$, $x \leq 10$, $y \leq 14$
(3) $5x + 10y \geq 180$, $x \geq 10$, $y \geq 14$ (4) $5x + 10y \leq 180$, $x \geq 10$, $y \geq 14$

2. The objective function for the above question is

(1) $10x + 14y$ (2) $5x + 10y$ (3) $3x + 5y$ (4) $5y + 3x$

3. The vertices of a feasible region of the above question are

(1) (0, 18), (36, 0) (2) (0, 18), (10, 13)
(3) (10, 13), (8, 14) (4) (10, 13), (8, 14), (12, 12)

4. The maximum value of objective function in the above question is

(1) 100 (2) 92 (3) 95 (4) 94

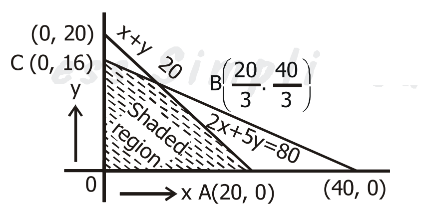
5. A firm produces two types of product A and B. The profit on both is Rs.2 per item. Every product processing on machines M_1 and M_2. For A, machines M_1 and M_2 takes 1 minute and 2 minutes respectively and that of for B, machines M_1 and M_2 takes the time 1 minute and 1 minute. The machines M_1, M_2 are not available more than 8 hours and 10 hours any of day respectively. If, the products made x of A and y of B, then the linear constraints for the L.P.P. except $x \geq 0$, $y \geq 0$, are

(1) $x + y \leq 480$, $2x + y \leq 600$ (2) $x + y \leq 8$, $2x + y \leq 10$
(3) $x + y \geq 480$, $2x + y \geq 600$ (4) $x + y \leq 8$, $2x + y \geq 10$

6. The objective function in the above question is

(1) $2x + y$ (2) $x + 2y$ (3) $2x + 2y$ (4) $8x + 10y$

7. Shaded region is represented by



(1) $2x + 5y \geq 80$, $x + y \leq 20$, $x \geq 0$, $y \leq 0$ (2) $2x + 5y \geq 80$, $x + y \geq 20$, $x \geq 0$, $y \geq 0$
(3) $2x + 5y \leq 80$, $x + y \leq 20$, $x \geq 0$, $y \geq 0$ (4) $2x + 5y \leq 80$, $x + y \leq 20$, $x \leq 0$, $y \leq 0$

8. What is the maximum value of $P = 3x + 2y$ when $x \geq 0$, $y \geq 0$, $2x + y \leq 12$ and $3x + 4y \leq 24$?

(1) 18 (2) 19.2 (3) 18.6 (4) None of these

9. If the equation of the lines of regression of y on x and that of x on y be $y = ax + b$ and $x = cy + d$ respectively, then $\bar{x}$ and $\bar{y}$ are equal to, respectively

(1) $\frac{ab+c}{1-ad}, \frac{cd+a}{1-ad}$ (2) $\frac{bc+d}{1-ac}, \frac{ad+b}{1-ac}$ (3) $\frac{ad+c}{1-bc}, \frac{cd+d}{1-bc}$ (4) None of these

10. If the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$, then the means of x and y are

(1) $-\frac{4}{7}, -\frac{11}{7}$ (2) $-\frac{4}{7}, \frac{11}{7}$ (3) $\frac{4}{7}, -\frac{11}{7}$ (4) 4, 7

SET VII

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. The value of $\frac{1}{1+x^{a-b}+x^{a-c}}+\frac{1}{1+x^{b-c}+x^{b-a}}+\frac{1}{1+x^{c-a}+x^{c-b}}$ is

(1) 1 (2) 2 (3) 4 (4) 0

2. The value of $\left(\frac{a^p}{a^q}\right)^{p+q}\cdot\left(\frac{a^q}{a^r}\right)^{q+r}\cdot\left(\frac{a^r}{a^p}\right)^{r+p}=$

(1) 0 (2) 1 (3) 2 (4) 3

3. If $a^x = b^y = c^z$ and abc = 1, then xy + yz + zx is equal to

(1) 0 (2) 1 (3) xyz (4) None of these

4. If $(2.381)^x = (0.2381)^y = 10^z$, then $z\left(\frac{1}{x}-\frac{1}{y}\right)=$

(1) 1 (2) 0 (3) 3 (4) $\frac{1}{2}$

5. Given log 2 = 0.30103; then the position of the first significant figure in 2^{20} is

(1) 4 (2) 3 (3) 6 (4) 7

6. If $\log_l x$, $\log_m x$ and $\log_n x$ are in arithmetic progression and $x \neq 1$, then n^2 is

(1) $\log_l m$ (2) (ln) log l (3) (ln) log l^m (4) None of these

7. $\frac{\sqrt{7}}{\sqrt{(16+6\sqrt{7})}-\sqrt{16-6\sqrt{7}}}$ is

(1) Irrational number (2) Complex number (3) Prime integer (4) Rational number

8. If $\frac{4+\sqrt{18}}{4\sqrt{48}-\sqrt{128}+\sqrt{200}-8\sqrt{12}+5\sqrt{8}} = a + b\sqrt{2}$, then a and b are

(1) a = 14, b = –9 (2) $a=\frac{1}{-14}, b=\frac{1}{9}$ (3) $a=\frac{1}{9}; b=\frac{1}{4}$ (4) $a=\frac{1}{4}; b=\frac{1}{6}$

9. The value of $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}=$

(1) 0 (2) 1 (3) 2 (4) 3

10. The values of l and m so that $lx^4 + mx^3 + 2x^2 + 4$ is exactly divisible by $x^2 - x - 2$ is

(1) $\frac{3}{2},\frac{5}{2}$ (2) $\frac{-5}{2},\frac{7}{2}$ (3) $\frac{5}{2},\frac{7}{2}$ (4) None of these

SET VIII

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. If $U = [2 \quad -3 \quad 4]$, $X = [0 \quad 2 \quad 3]$, $V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then UV + XY is equal to

   (1) 20 (2) [–20] (3) –20 (4) [20]

2. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 1 & 9 \end{bmatrix}$, then

   (1) AB and BA both are defined. (2) AB exist but BA does not exist.
   (3) BA exists but AB does not. (4) AB and BA both do not exist.

3. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$, then $[F(\alpha)]^{-1}$ is equal to

   (1) $F(-\alpha)$ (2) $F(\alpha^{-1})$ (3) $F(2\alpha)$ (4) None of these

4. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then A^{-1} is equal to

   (1) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ (2) $\begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -c \end{bmatrix}$ (3) $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$ (4) None of these

5. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible if

   (1) $\lambda \neq -15$ (2) $\lambda \neq -17$ (3) $\lambda \neq -16$ (4) $\lambda \neq -18$

6. The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ is

   (1) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 0 & 0 \\ b & -c & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac & b & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$

7. The values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have no solution are

   (1) $\lambda = 2$ and $\mu \neq 9$ (2) $\lambda = 5$ and $\mu \neq 9$ (3) $\lambda = 5$ and $\mu \neq 8$ (4) None of these

8. Given $A = \begin{bmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$ A is

(1) orthogonal (2) unitary (3) involutory (4) None of these

9. The matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is

(1) a Hermitian Matrix (2) skew hermitian (3) can't say (4) None of these

10. The rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ is

(1) 1 (2) 3 (3) 2 (4) None of these

SET IX

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. Sum of the series $1+\frac{3}{2^3}+\frac{1.3(3^3)}{1.2(2)^6}+\frac{1.3.5(3^3)}{1.2.3(2^9)}+\ldots\ldots\ldots\ldots \infty$ is

(1) 2 (2) 3 (3) 4 (4) 8

2. If $y = 2x + 3x^2 + 4x^3 + \ldots\ldots\infty$, then x is equal to

(1) $1-\frac{1}{\sqrt{1-y}}$ (2) $1+\frac{1}{\sqrt{1-y}}$ (3) $1-\frac{1}{\sqrt{1+y}}$ (4) $1+\frac{1}{\sqrt{1+y}}$

3. The value of the expression $1-n\frac{(1+x)}{(1+nx)}+\frac{n(n-1)}{1.2}\frac{(1+2x)}{(1+nx)^2}-\frac{n(n-1)(n-2)}{1.2.3}\frac{(1+3x)}{(1+nx)^3}+\ldots.$ is

(1) 2 (2) 1 (3) 3 (4) 0

4. The sum of $1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+\ldots\infty$ terms, is

(1) $(1 + x)^{1/3}$ (2) $(1 + x)^{-1/3}$ (3) $(1 - x)^{-1/3}$ (4) $(1 + x)^{1/6}$

5. The sum to infinity of $1+\frac{1}{2}.\frac{1}{2}+\frac{1.3}{2.4}.\frac{1}{2^2}+\ldots$ is

(1) $\sqrt{\left(\frac{2}{3}\right)}$ (2) $\sqrt{\left(\frac{1}{13}\right)}$ (3) $\sqrt{\frac{1}{2}}$ (4) $\sqrt{2}$

6. The sum of the series $\frac{4}{1!}+\frac{11}{2!}+\frac{22}{3!}+\frac{37}{4!}+\frac{56}{5!}$ is

(1) 6e (2) 6e – 1 (3) 5e (4) 5e + 1

7. If $y + \frac{y^3}{3} + \frac{y^5}{5} + \ldots\infty = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right)$, then

(1) $y = 2x$ (2) $\log y = 2 \log x$ (3) $x^2y = 2x - y$ (4) None of these

8. The sum to infinity of the series $1 - 3x + 5x^2 - 7x^3 + \ldots\ldots \infty$, when $|x| < 1$ is

(1) $\frac{x}{(1+x)^2}$ (2) $\frac{-x}{(1+x)^2}$ (3) $\frac{1-x}{(1+x)^2}$ (4) None of these

9. The sum to infinity of the series $1^2 + 5^2x + 9^2x^2 + 13^2 x^3 + \ldots\infty$, where $|x| < 1$ is

(1) x (2) x^3 (3) x^4 (4) None of these

10. If $|a| < 1$ and $|b| < 1$, then the sum of the series $1 + (1 + a) b + (1 + a + a^2) b^2 + (1 + a + a^2 + a^3) b^3 + \ldots$is

(1) $\frac{1}{(1-a)(1-b)}$ (2) $\frac{1}{(1-a)(1-ab)}$ (3) $\frac{1}{(1-b)(1-ab)}$ (4) $\frac{1}{(1-a)(1-b)(1-ab)}$

SET X

DIRECTIONS : For each of the following questions, four options are given. Choose the correct option.

1. The equation of the circle concentric with the circle $x + y + 8x + 10y - 7 = 0$ and passing through the centre of the circle $x^2 + y^2 - 4x - 6y = 0$ is

(1) $x^2 + y^2 + 8x + 10y + 59 = 0$ (2) $x^2 + y^2 + 8x + 10y - 59 = 0$

(3) $x^2 + y^2 - 4x - 6y + 87 = 0$ (4) $x^2 + y^2 - 4x - 6y - 87 = 0$

2. If the lengths of the chords intercepted by the circle $x^2 + y^2 + 2gx + 2fy = 0$ from the coordinates axes be 10 and 24 respectively, then the radius of the circle is

(1) 17 (2) 9 (3) 14 (4) 13

3. A circle has radius 3 units and its centre lies on the line $y = x - 1$. The equation of this circle if it passes through point (7, 3), is

(1) $x^2 + y^2 - 8x - 6y + 16 = 0$ (2) $x^2 + y^2 + 8x + 6y + 16 = 0$

(3) $x^2 + y^2 - 8x - 6y - 16 = 0$ (4) None of these

4. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with its sides parallel to the coordinate axes. The coordinates of its vertices are

(1) (–6, –9), (–6, 5), (8, –9) and (8, 5) (2) (–6, 9), (–6, –5), (8, –9) and (8, 5)

(3) (–6, 9), (–6, 5), (8, 9) and (8, 5) (4) (–6, –9), (–6, 5), (8, –9) and (8, –5)

5. If the equation $\frac{K\,(x+1)^2}{3} + \frac{(y+2)^2}{4} = 1$ represents a circle, then K is equal to

(1) 3/4 (2) 1 (3) 4/3 (4) 12

6. The equation of the circle which passes through the origin and cuts off intercepts of 2 units length from negative coordinates axis is

(1) $x^2 + y^2 - 2x + 2y = 0$ (2) $x^2 + y^2 + 2x - 2y = 0$

(3) $x^2 + y^2 + 2x + 2y = 0$ (4) $x^2 + y^2 - 2x - 2y = 0$

7. The equation of the circle which touches x-axis at (3, 0) and passes through (1, 4) is given by

(1) $x^2 + y^2 - 6x - 5y + 9 = 0$ (2) $x^2 + y^2 + 6x + 5y - 9 = 0$

(3) $x^2 + y^2 - 6x + 5y - 9 = 0$ (4) $x^2 + y^2 + 6x - 5y + 9 = 0$

8. The equation of the circle which passes through the points (2, 3) and (4, 5) and whose centre lies on the straight line $y - 4x + 3 = 0$, is

(1) $x^2 + y^2 + 4x - 10y + 25 = 0$ (2) $x^2 + y^2 - 4x - 10y + 25 = 0$

(3) $x^2 + y^2 - 4x - 10y + 16 = 0$ (4) $x^2 + y^2 - 14y + 8 = 0$

9. If the vertices of a triangle be (2, – 2), (– 1, – 1) and (5, 2), then the equation of its circumcircle is

(1) $x^2 + y^2 + 3x + 3y + 8 = 0$ (2) $x^2 + y^2 - 3x - 3y - 8 = 0$

(3) $x^2 + y^2 - 3x + 3y + 8 = 0$ (4) None of these

10. The number of circles touching the line $y - x = 0$ and the y-axis is/are

(1) Zero (2) One (3) Two (4) Infinite

SET XI

DIRECTIONS : *The hundred cells in the square below have been filled with letters. The columns and the rows are identified by the numbers 0 to 9. A letter in a cell is represented first by its column number and then by its row number e.g., G in column 3 and 1 is represented by 31. In each of the following questions, a word has been given which is represented by one of the four alternatives given under it. Find the correct alternative.*

	0	1	2	3	4	5	6	7	8	9
0	I	L	B	P	K	N	H	S	A	E
1	M	A	Q	G	T	V	I	O	N	U
2	H	R	W	J	A	X	B	E	C	I
3	T	Y	A	I	U	U	O	N	J	F
4	F	O	B	M	E	G	U	K	W	R
5	A	C	L	J	X	R	A	A	X	T
6	P	S	U	E	Z	K	V	W	D	L
7	Z	D	Y	V	F	O	H	Y	I	O
8	M	I	Z	Q	E	A	U	E	I	S
9	P	E	O	D	E	U	Q	O	C	G

1. MIND

(1) 01, 61, 73, 36 (2) 08, 61, 55, 44 (3) 34, 33, 50, 17 (4) 73, 33, 61, 17

2. JAIL

(1) 32, 05, 25, 44 (2) 32, 05, 87, 96 (3) 35, 23, 26, 33 (4) 83, 65, 25, 44

3. BLOT

(1) 20, 10, 71, 22 (2) 24, 10, 26, 48 (3) 34, 35, 63, 03 (4) 62, 25, 57, 95

4. JOKE

(1) 32, 14, 56, 44 (2) 35, 14, 37, 78 (3) 83, 63, 40, 59 (4) 83, 71, 25, 36

5. OMIT

(1) 14, 34, 88, 95 (2) 63, 44,, 88, 03 (3) 79, 09, 61, 41 (4) 97, 34, 62, 95

DIRECTIONS : *A painter is given a task to paint a cubical box with six different colours for different faces of the cube. The detailed account of it was given as*

(a) Red face should lie between Yellow and Brown faces.
(b) Green face should be adjacent to the Silver face.
(c) Pink face should lie adjacent to the Green face.
(d) Yellow face should lie opposite to the Brown one.
(e) Brown face should face down.
(f) Silver and Pink faces should lie opposite to each other.

6. The face opposite to Red is
(1) Yellow (2) Green (3) Pink (4) Silver

7. The upper face is
(1) Red (2) Pink (3) Yellow (4) Silver

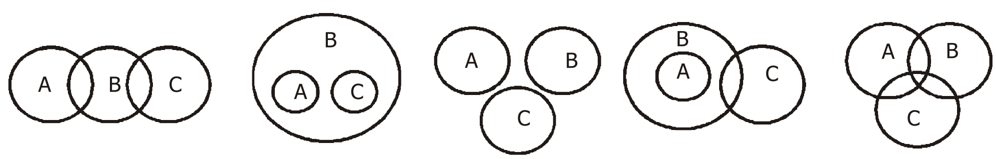
8. The faces adjacent to Green are
(1) Yellow, Pink, Red, Silver (2) Brown, Pink, Red, Silver
(3) Red, Silver, Yellow, Brown (4) Pink, Silver, Yellow, Brown

9. The face opposite to Silver is
(1) Pink (2) Brown (3) Red (4) Green

10. Three of the faces adjacent to Red face are
(1) Silver, Green, Brown (2) Silver, Brown, Pink (3) Silver, Pink, Green (4) Yellow, Pink, Green

SET XII

DIRECTIONS : *Given below arefive patterns represented by circles A, B and C which indicate the logical relationship between and among the respective descriptions. On the basis of description given for A, B and C respectively in the quesitons, decide which of the gien patterns (1), (2), (3), (4) or (5) best indicates the logical relationship.*



(1) (2) (3) (4) (5)

1. (A) Doctor (B) Male (C) Actor

2. (A) Rose (B) Flower (C) Lotus

3. (A) Father (B) Mother (C) Child

4. (A) Gold (B) Ornament (C) Silver

DIRECTIONS *: In each of the questions given below, use the following notations :*

A"B means 'add B to A'; **A'B means 'subtract B from A';**
A @ B means 'divide A by B'; **A * B means 'multiply A by B'.**
Now, answer the following questions.

5. The time taken by two running trains in crossing each other is calculated by dividing the sum of the lengths of two trains by the total speed of the two trains. If the length of the first train is L_1, the length of the second train is L_2; the speed of the first train is V_1 and the speed of the second train is V_2, which of the following expressions would represent the time taken?
(1) $(L_1" L_2) * (V_1" V_2)$ (2) $(L_1" L_2)$ @ $(V_1" V_2)$
(3) $[(L_1" L_2)$ @ $(V_1" V_2)] * 60$ (4) $(L_1' L_2)$ @ $(V_1' V_2)$
(5) None of these

6. The total airfare is calculated by adding 15% of basic fare as fuel surcharge, 2% of the basic fare as IATA charges and Rs.200 as airport tax to the basic fare. If the basic fare of a sector is B, which of the following will represent the total fare?

(1) B″ (B * 15) @ 100″ (B * 2) @ 200″ 100
(2) B″ (B * 15) @ 100″ (B * 2) @ 100″ 200
(3) B″ (B * 15) @ 100′ (B * 2) @ 100″ 200
(4) B′ (B * 15) @ 100″ (B * 2) @ 100″ 200
(5) None of these

7. The profit percentage of a commodity is worked out by multiplying the quotient of the difference between the amount of sale price and the total expenses and divided by the amount of total expenses by 100. If the sale price of an article is S, the total expenses are equal to the sum of the cost price (C), transportation costs (T), labour charges (L), which of the following expressions would indicate the profit percentage?

(1) [{S – (C + L + T)} ÷ (C + L + T) × 100]
(2) [{S′ (C″ L″ T)} @ (C″L″ T) @ 100]
(3) [{S′ (C″ L″ T)} @ (C″L″ T) * 100]
(4) [{S″ (C′L′ T)} * (C″L″ T) @ 100]
(5) None of these

8. While considering employees for promotion, an organisation gives 2 marks for every year of service beyond the first two years, four-thirds of the marks obtained in an examination out of 90 marks, five marks for each level of education-matriculation, graduation and post-graduation. Which of the following represents the total marks a candidate gets if he has put in T years of service, obtained K marks in the examination and passed Xth, XIIth and Graduation level examinations?

(1) (T′2) * 3″ 5 * 2″ 4 * T @ 3
(2) (K′ 2) * 2″ 5 * 3″ 4 * T @ 3
(3) (T″ 2) * 2″ 5 * 3″ 4 * K @ 3
(4) (T′2) * 2″ 5 * 3 ‶ 4 * K @ 3
(5) None of these

9. In a semester system of examination, the total marks obtained is arrived at by adding 10% of the marks obtained in first periodical, 15% of the marks obtained in the second periodical and 75% of the marks obtained in the final examination. If a student secures P marks out of 150 in first periodical, T marks out of 180 in second periodical and M marks out of 400 in the final examination, which of the following will represent the total marks obtained by him?

(1) (P @ 150 * 10)″ (T @ 400 * 15)″ (M @ 180 * 75)
(2) (P @ 150 * 10)″ (T @ 180 * 15)″ (M @ 400 * 75)
(3) (P * 150 * 10)″ (T * 180 @ 15)″ (M * 400 @ 75)
(4) (P @ 10 * 10)″ (T @ 180 * 15)″ (M @ 400 * 75)
(5) None of these

10. Every ten years, the Indian government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Mota Hazri

Chota Hazri has 4,522 fewer males than Mota Hazri.

Mota Hazri has 4,020 more females than males.

Chota Hazri has twice as many females as males.

Chota Hazri has 2,910 fewer females than Mota Hazri.

What is the total number of males in Chota Hazri?

(1) 11264 (2) 14174 (3) 5632 (4) 10154

SET XIII

DIRECTIONS : In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchange of number of marked '1' and the number marked '4' is required to establish the series, your answer is '1'. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

1.	17	16	15	13	7	–17
		(1)	(2)	(3)	(4)	(5)
2.	2	1	195	9	40	4
		(1)	(2)	(3)	(4)	(5)
3.	16	15	29	343	86	1714
		(1)	(2)	(3)	(4)	(5)

4. 1728 1452 1526 1477 1607 1443
(1) (2) (3) (4) (5)

5. 1 1 1 2 8 4
(1) (2) (3) (4) (5)

DIRECTIONS : In each of the following questions a number series is given. After the series, below it, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the given number following the sequence for the given series. Then answer the questions given below it.

6. 18 22 38 74
121 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
(1) 141 (2) 125 (3) 341 (4) 177
(5) 241

7. 4 7 24 93
2 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (d)?
(1) 12 (2) 230 (3) 3 (4) 51
(5) 1205

8. 4 2 2 3
12 (a) (b) (c) (d) (e)
Which of the following number will come in place of (e)?
(1) 45 (2) 6 (3) 9 (4) 18
(5) None of these

9. 264 136 72 40
488 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (a)?
(1) 128 (2) 248 (3) 38 (4) 23
(5) 68

10. 2 17 121 729
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
(1) 289 (2) 41 (3) 17393 (4) 1448
(5) 5796

SET XIV

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. Three students appear at an examination of mathematics. The probability of their success are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability of success of at least two.

(1) $\frac{1}{6}$ (2) $\frac{1}{30}$ (3) $\frac{5}{6}$ (4) $\frac{1}{15}$

2. A problem of mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the chance that the problem will be solved?

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) $\frac{1}{3}$

3. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{4}{9}$ (4) $\frac{1}{4}$

4. A man throws two dice, one the common cube and the other a regular tetrahedron, the number on the lowest face being taken in the case of the tetrahedron. What is the chance that the sum of the numbers thrown is not less than 5?

(1) $\frac{1}{3}$ (2) $\frac{3}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

5. Find the minimum number of tosses of a pair of dice so that the probability of getting the sum of the digits equal to 7 at least one toss, is greater than 0.95. (Given $\log_{10} 2 = 0.3010$, $\log_{10}3 = 0.4771$).

(1) 15 (2) 16 (3) 17 (4) 18

6. It is known that each of four people A, B, C and D tells the truth in a given instance with probability $\frac{1}{3}$. Suppose that A makes a statement and then D says that C says that B says that A was telling the truth. What is the probability that A was actually telling the truth?

(1) $\frac{13}{41}$ (2) $\frac{14}{27}$ (3) $\frac{13}{27}$ (4) None of these

7. The mean and the variance of a binomial variable X are 2 and 1, respectively. Find the probability that X takes values greater than 1.

(1) 17 (2) 18 (3) 19 (4) 20

8. A fair coin is tossed four times. Let X denotes the number of times a head is followed immediately by a tail. Find the mean and variance of X.

(1) $\frac{5}{16}$ (2) $\frac{3}{4}$ (3) 7 8 (4) None of these

9. A tosses 2 fair coins and B tosses 3 fair coins. The game is won by the person who throws greater number of heads. In case of a tie, the game is continued under the identical rules until someone wins the game. Find the probability of A winning the game.

(1) $\frac{5}{16}$ (2) $\frac{3}{11}$ (3) $\frac{3}{16}$ (4) None of these

10. The digits 1, 2, 3, ..., 9 are written in random order to form a nine-digit number. Find the probability that this number is divisible by 11.

(1) $\frac{11}{126}$ (2) $\frac{9}{126}$ (3) $\frac{17}{126}$ (4) None of these

SET XV

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. The numbers of the sequence 121, 12321, 1234321, ... are.

(1) each a perfect square of odd integer (2) each a perfect square of even integer
(3) both (1) and (2) (4) None of these

2. If $x_1, x_2, x_3, \ldots, x_n$ are in H.P., then $x_1x_2 + x_2x_3 + x_3x_4 + \ldots + x_{n-1} x_n$ is

(1) $(n-1) x_1 x_n$ (2) $(n + 1) x_1 x_n$ (3) $(n - 2) x_1 x_n$ (4) $(n + 2) x_1 x_n$

3. Two consecutive numbers from 1, 2, 3, ..., n are removed Arithmetic mean of the remaining numbers is $\frac{105}{4}$. Find the removed numbers.

(1) 6, 7 (2) 7, 8 (3) 8, 9 (4) 7, 9

4. The natural numbers are arranged in groups as given below such that the rth group contains 2^{r-1} numbers. The sum of the numbers in the nth group is

$$\begin{array}{cccccccc} & & & 1 & & & & \\ & & 2 & & 3 & & & \\ & 4 & 5 & & 6 & 7 & & \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{array}$$

....

....

(1) $2^{n-2}[2^n + 2^{n-1} + 1]$ (2) $2^{n-2}[2^n + 2^{n-1} - 2]$ (3) $2^{n-2}[2^n + 2^{n-1} + 2]$ (4) $2^{n-2}[2^n + 2^{n-1} - 1]$

5. Find out the largest term of the sequence $\frac{1}{503}, \frac{4}{524}, \frac{9}{581}, \frac{16}{692}, ...$

(1) $\frac{39}{1529}$ (2) $\frac{98}{1529}$ (3) $\frac{49}{1529}$ (4) $\frac{24}{1529}$

6. Given that a, b, c, α, β, γ, are all positive quantities and $a\alpha$, $b\beta$, $c\gamma$ are all distinct if a, b, c are in A.P. , α, β, γ, are in H.P. and $a\alpha$, $b\beta$, $c\gamma$ are in G.P., then a : b: c = $\frac{1}{\gamma}:\frac{1}{\beta}:\frac{1}{\alpha}$.

(1) $\frac{1}{\beta}:\frac{1}{\gamma}:\frac{1}{\alpha}$ (2) $\frac{1}{\gamma}:\frac{1}{\beta}:\frac{1}{\alpha}$ (3) $\frac{1}{\alpha}:\frac{1}{\beta}:\frac{1}{\gamma}$ (4) $\frac{1}{\beta}:\frac{1}{\alpha}:\frac{1}{\gamma}$

7 If the sum of the terms of an infinitely decreasing G.P. is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ on the interval [–5, 3] and the difference between the first and second terms is f '(0), then the common ratio of the progression is

(1) 1/3 (2) 3/2 (3) 4/3 (4) 2/3

8. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} (ab)^n$, where a, b < 1, then

(1) xy + z = z (x + y) (2) xy – z = z (x + y) (3) xy + z = z (x – y) (4) None of these

9. 25 trees are planted in a straight line at interval of 5 metres. To water them the gardener must bring water for each tree separately from a well 10 metres from the first tree in line with the trees. How far he will have to cover in order to water all the trees beginning with the first if he starts from the well.

(1) 3070 metre (2) 3520 metre (3) 3370 metre (4) 3660 metre

10. Find a three digit number whose consecutive numbers from a G.P. if we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now if we increase the second digit of the required number by 2, the resulting number will form an A.P.

(1) 931 (2) 1030 (3) 732 (4) 638

SET XVI

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. 6 balls marked as 1, 2, 3, 4, 5 and 6 are kept in a box. Two players A and B start to take out 1 ball at a time from the box one after another without replacing the ball till the game is over. The number marked on the ball is added each time to the previous sum to get the sum of numbers marked on the balls taken out. If this sum is even then 1 point is given to the player. The first player to get 2 points is declared winner. At the start of the game the sum is 0. If A starts to take out the ball then find the number of ways in which the game can be won.

   (1) 72 (2) 90 (3) 96 (4) 78

2. A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. Find the number of ways of choosing P and Q so that $P \cap Q$ contains exactly two elements.

   (1) ${}_nC^2 \times 3^n$ (2) $3^{n-2} \times {}^nC_2$ (3) $3^{n-1} \times {}^nC_2$ (4) ${}^nC_1 \times 3^{n-2}$

3. How many natural numbers are there lying between 20,000 and 60,000, the sum of digits being even ?

   (1) 20,000 (2) 10,000 (3) 5,000 (4) None of these

4. In how many different ways can a set A of 3n elements be partitioned into 3 subsets of equal number of elements? (The subsets P, Q, R form a partition if $P \cup Q \cup R = A$, $P \cap R = \phi$, $Q \cap R = \phi$, $R \cap P = \phi$.)

   (1) 3n! (2) $\frac{3n!}{(n!)^3}$ (3) $\frac{3n!}{2.(n!)^3}$ (4) None of these

5. Find the sum of all the 4-digit nos. which can be formed from the digits 1, 2, 3, 4. If each digit may be repeated upto 4 times.

   (1) 711040 (2) 640000 (3) 64000 (4) None of these

6. In an examination, the maximum marks for each of the three papers are 50 each. Maximum marks for the fourth paper are 100. Find the number of ways in which the candidate can score 60% marks in the aggregate.

   (1) ${}^{153}C_{150}$ (2) ${}^{102}C_{99}$ (3) 110556 (4) 110550

7. m equi-spaced horizontal lines are intersected by n equi-spaced vertical lines. If m < n and the distance between two successive horizontal lines is the same as that between two successive vertical lines. The number of squares formed by the lines is

   (1) 1/3 (m – 1) (3n – m – 1) (2) 1/6 (m – 1) (3n – m – 1)
   (3) 1/12 (m – 1) (3n – m – 1) (4) 1/24 (m – 1) (3n – m – 1)

8. There are 5 letters and 5 directed envelopes. In how many ways can 2 letters be rightly places and 3 letters wrongly placed?

   (1) 44 (2) 24 (3) 10 (4) 20

9. A family consists of a grand father, 5 sons and daughters and 8 grand children. They are to be seated in a row for dinner. The grand children wish to occupy the 4 seats at each end and the grandfather refuses to have a grand child on either side of him. In how many ways can the family be made to sit?

   (1) 120 (2) 480 (3) 11520 (4) None of these

10. How many three digit numbers are of the form xyz with $x < y$, $z < y$ and $x \neq 0$.

   (1) 240 (2) 285 (3) 45 (4) None of these

SET XVII

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. A is a set containing n elements. A subset P_1 of A is chosen at random and the set A is then reconstructed by replacing the elements of P_1. A subset P_2 of A is now chosen at random and again the set A is reconstructed by replacing the elements of P_2. This process is continued by choosing subsets $P_2, P_3, \ldots, P_m$, with $m \geq 2$. Find the probability that $P_i \cap P_j = \phi$ for $i \neq j$ and i, j = 1, 2, ..., m.

   (1) $\frac{(m+1)^n}{2^{mn}}$ (2) $\frac{(m-1)^n}{2^{mn}}$ (3) $\frac{(m-2)^n}{2^{mn}}$ (4) $\frac{(m+2)^n}{2^{mn}}$

2. For three independent events A, B and C the probability for A to occur is a, the probability that A, B and C will not occur is b and the probability that at least one of A, B, C will not occur is c. If p denotes the probability that C occurs but neither A nor B occur, then p satisfies the quadratic equation.

   (1) $ap^2 + [ab + (1 - a)(a + c - 1)]\, p + b\,(1 - a)\,(1 - c) = 0$ (2) $ap^2 + [ab - (1 - a)(a + c - 1)]\, p + b\,(1 - a)\,(1 - c) = 0$
   (3) $ap^2 + [ab - (1 + a)(a + c - 1)]\, p + b\,(1 - a)\,(1 - c) = 0$ (4) $ap^2 + [ab - (1 - a)(a + c - 1)]\, p + b\,(1 - a)\,(1 + c) = 0$

3. Out of 3n consecutive integers, three are selected at random. Find the chance that their sum is divisible by 3.

   (1) $\frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$ (2) $\frac{3n^2 + 3n + 2}{(3n-1)(3n-2)}$ (3) $\frac{3n^2 + 3n + 2}{(3n-1)(3n+2)}$ (4) $\frac{3n^2 + 3n + 2}{(3n+1)(3n+2)}$

4. A, B, C and D cut a pack of 52 cards successively in the order given. If the person who cuts a spade first receives Rs.350, what are their respective expectations?

   (1) Rs.128 (2) Rs.97 (3) Rs.54 (4) All of these

5. Suppose a sample consists of the integers 1, 2, 3, ..., 2n. The probability of choosing an integer k is proportional to log k. Find the conditional probability of choosing the integer 2, given that an even integer is chosen, is

   (1) $\frac{\log 2}{[n \log 2 + \log(n!)]}$ (2) $\frac{\log 2}{[n \log 2 - \log(n!)]}$ (3) $\frac{\log 2}{[n \log 2 + 2\log(n!)]}$ (4) $\frac{\log 2}{[n \log 2 - 2\log(n!)]}$

6. An electric component manufactured by 'RASU electronics' is tested for its defectiveness by a sophisticated testing device. Let A denote the event "the device is defective" and B the event "the testing device reveals the component to be defective." Suppose $P(A) = \alpha$ and $P(B/A) = P(B'/A') = 1 - \alpha$, where $0 < \alpha < 1$. Find the probability that the component is not defective.

   (1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) None of these

7. Seven digits from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order. Find the probability that this seven–digit number is divisible by 9.

   (1) $\frac{1}{45}$ (2) $\frac{1}{50}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

8. Of three independent events, the chance that only the first occurs is a, the chance that only the second occurs is b and the chance of only third is c. Find the chances of three events are respectively a/(a + x), b/(b + x), c/(c + x), where x is a root of the equation

   (1) $(a + x)(b + x)(c + x) = x^2$ (2) $(a - x)(b + x)(c + x) = x^2$
   (3) $(a - x)(b - x)(c + x) = x^2$ (4) $(a + x)(b + x)(c - x) = x^2$

9. Two point P, Q are taken at random on a straight line OA of length a, find the chance that PQ > b, where b < a is

   (1) $\left(\frac{a-b}{b}\right)^2$ (2) $\left(\frac{a-b}{a}\right)^2$ (3) $\left(\frac{a-b}{2b}\right)^2$ (4) None of these

10. If x + y = 2a where a is constant and that all values of x between 0 and 2a are equally likely, then the chance that $xy > \frac{3}{4}a^2$, is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) None of these

SET XVIII

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. Out of the set S = {0, 1, 2 ..., 189} two numbers x and y are chosen at random without any replacement(s). What is the probability the $x^2 + y^2$ is a perfect square?

(1) $\frac{278}{17955}$ (2) $\frac{236}{17955}$ (3) $\frac{231}{17955}$ (4) None of these

2. The altitude through A of ΔABC meets BC at D and the circumscribed circle at E. If D = (2, 3), E = (5, 5), the ordinate of the orthocentre being a natural number. Find the probability that the orthocentre lies on the lines

y = l
y = 2
y = 3
......
......
......
y = 10.

(1) $\frac{3}{5}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$

3. There are 6 red and 8 green balls in a bag. 5 balls are drawn at random and placed in a red box. The remaining balls are placed in a green box. What is the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number?

(1) $\frac{213}{2002}$ (2) $\frac{213}{1001}$ (3) $\frac{214}{2002}$ (4) $\frac{214}{1001}$

4. An artillery target may be either at point I with probability $\frac{8}{9}$ or at point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each shell may hit the target independently of the other shell with probability $\frac{1}{2}$. How many shells must be fired at point I to hit the target with maximum probability?

(1) 6 (2) 8 (3) 24 (4) 12

5. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

(1) 0.83 (2) 0.38 (3) 0.62 (4) None of these

6. Two integers x and y are chosen (without random) at random from the set $\{x : 0 \le x \le 10, x \text{ is an integer}\}$ then find the probability for $|x - y| \le 5$.

(1) $\frac{81}{121}$ (2) $\frac{71}{121}$ (3) $\frac{85}{121}$ (4) $\frac{91}{121}$

7. A sum of money is rounded off to the nearest rupee; find the probability that the round off error is at least ten paise.

(1) $\frac{19}{100}$ (2) $\frac{19}{50}$ (3) $\frac{81}{100}$ (4) None of these

8. A die is rolled three times, find the probability of getting a large number than the previous number.

(1) $\frac{5}{18}$ (2) $\frac{45}{216}$ (3) $\frac{5}{54}$ (4) None of these

9. A natural number x is chosen at random from the first 100 natural numbers. Then find the probability for the question $x + \frac{100}{x} > 50$.

(1) $\frac{11}{20}$ (2) $\frac{1}{2}$ (3) $\frac{53}{100}$ (4) None of these

10. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8 ?

(1) $\frac{2}{11}$ (2) $\frac{193}{792}$ (3) $\frac{11}{36}$ (4) None of these

SET XIX

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. A code word is to consist of two distinct English alphabets followed by two distinct numbers from 1 to 9. For example, CA 23 is a code word. How many of them end with an even integer?

(1) 46800 (2) 5200 (3) 10400 (4) 20800

2. The Principal wants to arrange 5 students on the platform such that the boy 'SALIM' occupies the second position and such that the girl, 'SITA' is always adjacent to the girl 'RITA'. How many such arrangements are possible?

(1) 4 (2) 6 (3) 8 (4) 3

3. In an examination hall there are four rows of chairs. Each row has 8 chairs one behind the other. There are two classes sitting for the examination with 16 students in each class. It is desired that in each row, all students belong to the same class and that no two adjacent rows are allotted to the same class. In how many ways can these 32 students be seated?

(1) $2 \times (16!)^2$ (2) $(16!)^2$ (3) $2 \times (16!)$ (4) $2 \times (8!)^2$

4. A question paper which is divided into two groups containing three and four questions respectively, carries the note that it is not necessary to answer all the questions. One question must be answered from each group. In how many ways can an examinee select the questions?

(1) 105 (2) 104 (3) 22 (4) 21

5. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Chemistry Part II, unless Chemistry Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

(1) 21 (2) 20 (3) 41 (4) 420

6. A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are at least two balls of each colour?

(1) 150 (2) 200 (3) 425 (4) 75

7. In a village, there are 87 families of which 52 families have at most 2 children. In a rural development programme, 20 families are to be helped chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

(1) ${}^{52}C_{18} \times {}^{35}C_2$ (2) ${}^{52}C_{19} \times {}^{35}C_1$ (3) ${}^{52}C_{20} \times {}^{35}C_0$ (4) None of these

8. There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. Find the number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected.

(1) 2 (2) 3 (3) 4 (4) 10

9. How many integral solutions are there to $x + y + z + t = 29$, when $x \geq 1$, $y \geq 2$, $z \geq 3$ and $t \geq 0$?

(1) 1300 (2) 2600 (3) 3900 (4) None of these

10. Find the number of triangles whose vertices are at the vertices of an octagon but none of whose sides happen to come from the sides of the octagon.

(1) $^8C_3$ (2) 24 (3) 16 (4) 32

SET XX

DIRECTIONS : *For each of the following questions, four options are given. Choose the correct option.*

1. A question paper is split into two parts-Part A and Part B. Part A contains 5 questions and Part B has 4. Each question in Part A has an alternative. A student has to attempt at least one question from each part. Find the number of ways in which the student can attempt the question paper.

(1) 243 (2) 258 (3) 257 (4) 3630

2. What can be the maximum population of a country in which no two persons have an identical set of teeth. (Disregard the shape and size of the teeth. Take only the positioning of the teeth in consideration. Also, assume that , there is no person without a tooth and no person has more than 32 teeth.)

(1) $2^{32} - 1$ (2) 2^{32} (3) 64 (4) 63

3. How many different car licence plates can be constructed if the licences contain three letters of the English alphabet followed by a three digit number if repetitions are allowed?

(1) 26×999 (2) $(261) \times 999$ (3) $(26)^3 \times 999$ (4) None of these

4. A tea party is arranged for 2m people along two sides of a long table with m chairs on each side. r men wish to sit on one particular side and s on the other. In how many ways can they be seated? (Assume that $r, s \leq m$.)

(1) $^mp_r + {}^mp_s$ (2) $2(^mp_r) \times (^mp_s)$ (3) $^mp_r \times {}^mp_{s-r}$ (4) None of these

5. A family consists of a grandfather, m sons and daughters and 2n grandchildren. They are to be seated in a row for dinner. The grandchildren wish to occupy the n seats at each end and the grandfather refuses to have a grandchild on either side of him. In how many ways can the family be made to sit?

(1) (2n!) (m!) (m – 1) (2) (2n!) (m!) (m + 1) (3) (2n!) (m!) (4) None of these

6. Find the sum of all the 4-digit nos. which can be formed from the digits 1, 2, 3, 4. If each may be used only once?

(1) 6666 (2) 66666 (3) 66660 (4) None of these

7. There are 5 letters and 5 directed envelopes. In how many ways can all the letters be put into wrong envelopes?

(1) 44 (2) 20 (3) 24 (4) None of these

8. Bhawna has 4 different toys and Quincy has 7 different toys. Find the number of ways in which they can exchange their toys so that each keeps her initial number of toys.

(1) 330 (2) 328 (3) 329 (4) 331

9. There are 5 mangoes and 4 apples. In how many different ways can selection of fruits be made if fruit of the same kind are different

(1) $2^9 - 2$ (2) $2^9 - 1$ (3) $2^9 - 3$ (4) None of these

10. Find the number of whole numbers formed on the screen of a calculator which can be recognised as numbers with (unique) correct digits when they are read inverted. The greatest number that can be formed on the screen of the calculator is 999999.

(1) 10843 (2) 100843 (3) 10043 (4) 100943

Detailed Solutions

SET I

1. Taking a > 0, we have
$ax^2 + bx + c = f(x) = a(x - \alpha)(x - \beta)$, $\alpha < \beta$.
f(x) is +ive for all value of x which are such that either $x < \alpha$ or $> \beta$,
and f(x) is –ive for all values of x which are such that $\alpha < x < \beta$.
Now under given condition both –1 and 1 lie between α and β.
$\therefore$ f(1) and f(–1) both are –ive.
or $a + b + c < 0$ and $a - b + c < 0$
Dividing by a which is +ive

$1+\frac{c}{a}+\frac{b}{a}<0$ and $1+\frac{c}{a}-\frac{b}{a}<0$

$\therefore 1+\frac{c}{a}+\left|\frac{b}{a}\right|<0$. **Ans.(1)**

2. Given $\alpha < \beta$, c is –ive and b = +ive.
$\alpha + \beta = -b$ = –ive, $\alpha\beta = c$ = –ive
$\alpha < \beta \Rightarrow \alpha$ must be a –ive root and β a +ive root as $\alpha\beta$ is –ive.
Again $\alpha + \beta < 0 \Rightarrow \beta < -\alpha \Rightarrow \beta < |\alpha|$. **Ans.(2)**

3. Let $D = b^2 - 4ac$. Let $\alpha=\frac{-b+\sqrt{D}}{2a}$ and $\beta=\frac{-b-\sqrt{D}}{2a}$.
We can write the equation $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$,
as $(a - b + c)x^2 + x(b - 2c) + c = 0$.
Discriminant of this equation is $D = (b - 2c)^2 - 4(a - b + c)c$
$= b^2 - 4bc + 4c^2 - 4ac + 4bc - 4c^2 = b^2 - 4ac = D$

If A, B are roots of (1), then $A, B=\frac{-(b-2c)\pm\sqrt{D}}{2(a-b+c)}=\frac{-\frac{b}{2a}+\frac{c}{a}\pm\frac{\sqrt{D}}{2a}}{1-\frac{b}{a}+\frac{c}{a}}$.

Let $A=\frac{\alpha+\alpha\beta}{1+\alpha+\beta+\alpha\beta}=\frac{\alpha(1+\beta)}{(1+\alpha)(1+\beta)}=\frac{\alpha}{1+\alpha}$

and $B=\frac{\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}=\frac{\beta(1+\alpha)}{(1+\alpha)(1+\beta)}=\frac{\beta}{1+\beta}$. **Ans.(2)**

4. We have $\alpha + \beta = a$, $\alpha\beta = -a - b$. Also $(\alpha + 1).(\beta + 1)$
$= \alpha\beta + \alpha + \beta + 1 = -a - b + a + 1 = 1 - b$.

Now $E=\frac{(\alpha+1)^2}{(\alpha+1)^2+b-1}+\frac{(\beta+1)^2}{(\beta+1)^2+b-1}$

$=\frac{(\alpha+1)^2}{(\alpha+1)^2-(\alpha+1)(\beta+1)}+\frac{(\beta+1)^2}{(\beta+1)^2-(\alpha+1)(\beta+1)}$

$=\frac{\alpha+1}{(\alpha+1)-(\beta+1)}+\frac{\beta+1}{(\beta+1)-(\alpha+1)}$

$=\frac{\alpha+1}{\alpha-\beta}+\frac{\beta+1}{\beta-\alpha}=\frac{(\alpha+1)-(\beta+1)}{\alpha-\beta}=1$. **Ans.(2)**

5. As a, b, c are in G.P. $b^2 = ac$. Now, the equation $ax^2 + 2bx + c = 0$ can be written as, $ax^2+2\sqrt{ac}\,x+c=0 \Rightarrow \left(\sqrt{a}x+\sqrt{c}\right)^2=0$

$\Rightarrow x=-\sqrt{\frac{c}{a}},\ -\sqrt{\frac{c}{a}}$

If the two given equations have a common root, then this root must be $-\sqrt{(c/a)}$. Thus $d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f=0$

$\Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{c}\sqrt{\frac{c}{a}}=\frac{2e}{\sqrt{ac}}=\frac{2e}{b} \Rightarrow \frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P. **Ans.(1)**

6. Since α, β, γ are in H.P., we take $\alpha=\frac{1}{A-D}$, $\beta=\frac{1}{A}$ and $\gamma=\frac{1}{A+D}$. We have

$\frac{1}{A-D}+\frac{1}{A}+\frac{1}{A+D}=3a$ .... (1)

$\Rightarrow \frac{1}{(A-D)A}+\frac{1}{(A+D)A}+\frac{1}{(A-D)(A+D)}=3b$ .....(2)

$\Rightarrow \frac{1}{(A-D)\,A\,(A+D)}=c$ .... (3)

We can write (2) as $\frac{(A+D)+(A-D)+A}{(A-D)\,A\,(A+D)}=3b \Rightarrow c(3A) = 3b$

$\Rightarrow A = b/c$ or $\beta=\frac{1}{A}=\frac{c}{b}$. **Ans.(3)**

7. Since $f(x) > 0\ \forall x \in R$, $a > 0$ and $b^2 - 4ac < 0$, we have $f'(x) = 2ax + b$ and $f''(x) = 2a$. Thus $g(x) = ax^2 + bx + c + 2ax + b + 2a$
$= ax^2 + (2a + b)x + (2a + b + c)$.
We have $a > 0$ and $D = (2a + b)^2 - 4a(2a + b + c)$
$= 4a^2 + 4ab + b^2 - (8a^2 + 4ab + 4ac) = b^2 - 4ac - 4a^2 < 0$.
Thus $g(x) > 0\ \forall x \in R$.
Therefore g (x) = 0 has non-real complex roots. **Ans.(3)**

8. Since p, q, r are A.P. therefore 2 q = p + r.
The roots of $px^2 + qx + r = 0$ are real if $q^2 - 4pr \ge 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2-4pr\ge 0 \Rightarrow p^2+r^2-14pr\ge 0$

$\Rightarrow \left(\frac{r}{p}\right)^2-14\left(\frac{r}{p}\right)+1\ge 0$.

Now, $\left(\frac{r}{p}\right)^2-14\left(\frac{r}{p}\right)+1=0 \Rightarrow \frac{r}{p}=7\pm 4\sqrt{3}$ $\therefore \left(\frac{r}{p}\right)^2-14\left(\frac{r}{p}\right)+1\ge 0$

$\Rightarrow \frac{r}{p}\le 7-4\sqrt{3}$ or $\frac{r}{p}\ge 7+4\sqrt{3} \Rightarrow \left|\frac{r}{p}-7\right|\ge 4\sqrt{3}$. **Ans.(1)**

9. We have $x^2 - ax - b$
Multiplying by x^{n-1}, we get
$x^{n+1} = ax^n - bx^{n-1}$.
In this putting $x = \alpha, \beta$ and adding, we get
$\alpha^{n+1} + \beta^{n+1} = a(\alpha^n + \beta^n) - b(\alpha^{n-1} + \beta^{n-1})$
$\therefore V_{n+1} = aV_n - bV_{n-1}$. **Ans.(1)**

10. $t_1 + t_2 = -\frac{b}{a}, t_1t_2 = \frac{c}{a}$

$P + Q = \pi - R = \frac{\pi}{2}$

$\therefore \frac{P}{2}+\frac{Q}{2}=\frac{\pi}{4}$

$\frac{t_1+t_2}{1-t_1t_2}=\tan\frac{\pi}{4}=1 \qquad \because \left[\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A.\tan B}\right]$

or $-\frac{b}{a}=1-\frac{c}{a} \quad \Rightarrow a+b=c$. **Ans.(1)**

SET II

1. Put x + y = u, x/y = v

u + v = 1/2, uv = −1/2 then u and v are roots of $t^2 - \frac{1}{2}t - \frac{1}{2} = 0$

or $2t^2 - t - 1 = 0$

Hence u = 1. v = −1/2 or u = −1/2, v = 1.

∴ x + y = 1, x/y = −1/2

or x + y = −1/2, x/y = 1.

Solving these, we shall get

x = −1, y = 2 or x = y = −1/4.

Since x < 0, y < 0 we get the answer

x = y = −1/4. **Ans.(1)**

2. Operate $C_3 - C_2$ and $C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} a^p - x & a^p(a-1) & a^{p+1}(a-1) \\ a^{p+3}-x & a^{p+3}(a-1) & a^{p+4}(a-1) \\ a^{p+6}-x & a^{p+6}(a-1) & a^{p+7}(a-1) \end{vmatrix}$$

$$= a^p.a^{p+1}.(a-1)^2 \begin{vmatrix} a^p - x & 1 & 1 \\ a^{p+3}-x & a^3 & a^3 \\ a^{p+6}-x & a^6 & a^6 \end{vmatrix} = 0.$$

[∵ C_1, C_2 are identical]. **Ans.(1)**

3. Operate $C_3 - C_2$ and $C_2 - C_1$ and using $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$

$$\Delta = \begin{vmatrix} ^{p+2}C_2 & ^{p+2}C_1 & ^{p+3}C_1 \\ ^{p+3}C_2 & ^{p+3}C_1 & ^{p+4}C_1 \\ ^{p+4}C_2 & ^{p+4}C_1 & ^{p+5}C_1 \end{vmatrix}$$

Operate $R_3 - R_2$, $R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} ^{p+2}C_2 & ^{p+2}C_1 & ^{p+3}C_1 \\ ^{p+2}C_1 & 1 & 1 \\ ^{p+3}C_1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} ^{p+2}C_2 & p+2 & p+3 \\ p+2 & 1 & 1 \\ p+3 & 1 & 1 \end{vmatrix}$$

Operate $C_3 - C_2$,

$$\Delta = \begin{vmatrix} ^{p+2}C_2 & p+2 & 1 \\ p+2 & 1 & 0 \\ p+3 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} p+2 & 1 \\ p+3 & 1 \end{vmatrix} = -1.$$ **Ans.(2)**

4. $$\begin{vmatrix} 1 & ^pC_1 & ^pC_2 \\ 1 & ^{p+1}C_1 & ^{p+1}C_2 \\ 1 & ^{p+2}C_1 & ^{p+2}C_2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & p & \frac{p(p-1)}{2} \\ 1 & p+1 & \frac{(p+1)p}{2} \\ 1 & p+2 & \frac{(p+2)(p+1)}{2} \end{vmatrix}$$

Operate $R_3 - R_2$, $R_2 - R_1$, we get

$$= \begin{vmatrix} 1 & p & \frac{p(p-1)}{2} \\ 0 & 1 & \frac{p}{2}(2) \\ 0 & 1 & \frac{p+1}{2}(2) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & p \\ 1 & p+1 \end{vmatrix} = p + 1 - p = 1.$$ **Ans.(1)**

5. $$\begin{vmatrix} \sin 2a & \sin(a+b) & \sin(a+c) \\ \sin(b+a) & \sin 2b & \sin(b+c) \\ \sin(c+a) & \sin(c+b) & \sin 2c \end{vmatrix} = \begin{vmatrix} \sin a & \cos a & 0 \\ \sin b & \cos b & 0 \\ \sin c & \cos c & 0 \end{vmatrix} \begin{vmatrix} \cos a & \sin a & 0 \\ \cos b & \sin b & 0 \\ \cos c & \sin c & 0 \end{vmatrix} = 0.$$

Ans.(1)

6. $$\begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$$

$= a[(2a + b)(2a + 3b) - (a + b)^2] - a^2(2a + 3b)]$

$= a[4a^2 + 8ab + 3b^2 - a^2 - b^2 - 2ab] - (2a^3 + 3a^2b)$

$= a[3a^2 + 2b^2 + 6ab] - (2a^3 + 3a^2b)$

$= a^3 + 2ab^2 + 3a^2b$

$= a(a^2 + 2b^2 + 3ab) = a(a + b)(a + 2b)$. **Ans.(4)**

7. $$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$ [Operate $R_1 + R_2 + R_3$]

$$= \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Operate $C_3 - C_1$, $C_2 - C_1$

we get

$$= 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix}$$

$= 2(x + y)(-x^2 + xy - y^2)$

$= -2(x + y)(x^2 - xy + y^2)$

$= -2(x^3 + y^3)$. **Ans.(3)**

8. Since given system of equation has a non-trivial solution.

$$\therefore \begin{vmatrix} p+a & b & c \\ a & q+b & c \\ a & b & r+c \end{vmatrix} = 0$$

Operate $R_1 - R_2$ and $R_2 - R_3$, we get

$$\begin{vmatrix} p & -q & 0 \\ 0 & q & -r \\ a & b & r+c \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} q & -r \\ b & r+c \end{vmatrix} + a\begin{vmatrix} -q & 0 \\ q & -r \end{vmatrix} = 0$$

$\Rightarrow$ p (qr + qc + br) + a (qr) = 0

$$\Rightarrow 1+\frac{c}{r}+\frac{b}{q}+\frac{a}{p}=0$$

$$\Rightarrow \frac{a}{p}+\frac{b}{q}+\frac{c}{r}=-1.$$ **Ans.(1)**

9. Operate $C_1 - pC_2 - C_3$, given equation become

$$\begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -p(xp+y)-(yp+z) & xp+y & yp+z \end{vmatrix} = 0$$

$\Rightarrow (zx - y^2)(-p(xp + y) - (yp + z)) = 0$

$\Rightarrow zx - y^2 = 0 \Rightarrow y^2 = zx$

$\Rightarrow$ x, y, z are in G.P. **Ans.(2)**

10. Given α, β, γ are the roots of the equation, therefore $\alpha + \beta + \gamma = 0$. Now

$$\begin{bmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{bmatrix} = \begin{bmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{bmatrix} \Rightarrow (\alpha+\beta+\gamma)\begin{bmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{bmatrix}$$

$$\Rightarrow 0\begin{bmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{bmatrix} = 0$$. **Ans.(4)**

SET III

1. Here, $f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

$$\Rightarrow f(x).f\left(\frac{1}{x}\right) - f(x) = f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x) = \frac{f(1/x)}{f(1/x)-1} \quad ....(1)$$

Also $f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

$$\Rightarrow f(x).f\left(\frac{1}{x}\right) - f\left(\frac{1}{x}\right) = f(x)$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{f(x)}{f(x)-1} \quad ....(2)$$

On multiplying (1) and (2); we get

$$f(x).f\left(\frac{1}{x}\right) = \frac{f(1/x).f(x)}{\{f(1/x)-1\}\{f(x)-1\}}$$

$$\Rightarrow f\left(\frac{1}{x}-1\right)(f(x)-1) = 1 \quad ....(3)$$

Since, f(x) is polynomial function,

so, {f(x)−1} and $\left\{f\left(\frac{1}{x}\right)-1\right\}$ are reciprocal of each other, also x and $\frac{1}{x}$ are reciprocal of each other.

Thus, (3) can hold only when

$f(x)-1 = \pm x^n$, where $n \in R$.

$\therefore f(x) = \pm x^n + 1$; but f(4) = 65,

$\Rightarrow \pm 4^n + 1 = 65$

$\Rightarrow 4^n = 64$

$\Rightarrow 4^n = 4^3$ [$\because 4^n > 0$]

$\Rightarrow$ n = 3

So, $f(x) = x^3 + 1$

Hence, $f(6) = 6^3 + 1 = 217$. **Ans.(2)**

2. Here, f(r) = f((r − 1) + 1)

f(r) = f(r−1) + f(1) ....(1)

[using definition]

$\therefore$ f(r) = f(r − 1) + 5

$\Rightarrow$ f(r) = f(r−2) + 5} + 5

$\Rightarrow$ f(r) = f(r−2) + 2.(5)

$\Rightarrow$ f(r) = f(r−3) + 3}.5

....

....

....

$\Rightarrow$ f(r) = f(r−(r−1)) + (r − 1).5

$\Rightarrow$ f(r) = f(1) + (r − 1).5

$\Rightarrow$ f(r) = 5+(r−1).5

$\Rightarrow$ f(r) = 5r

$$\therefore \sum_{n=1}^{m} f(n) = \sum_{n=1}^{m}(5n) = 5[1+2+3+.....+m]$$

$$= \frac{5m(m+1)}{2}$$

Hence, $\sum_{n=1}^{m} f(n) = \frac{5m(m+1)}{2}$. **Ans.(1)**

3. Given $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$. Replacing x by 3x and y by zero, then

$$f(x) = \frac{f(3x)+f(0)}{3} \Rightarrow f(3x) - 3f(x) = -f(0) \quad ....(1)$$

and $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f\left(\frac{3x+3h}{3}\right)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\frac{f(3x)+f(3h)}{3}-f(x)}{h} = \lim_{h\to 0}\frac{f(3h)-f(0)}{3h}$$ [from (1)] = f'(0) = 3

$\therefore$ f(x) = 3x + c, $\because$ f(0) = 0 + c = 3, $\therefore$ c = 3, then f(x) = 3x + 3

Hence f(x) is continuous and differentiable every where. **Ans.(3)**

4. Since, 1900 < f (1990) < 2000

$$\Rightarrow \frac{1900}{90} < \frac{f(1990)}{90} < \frac{2000}{90}$$

$$\Rightarrow 21.1 < \frac{f(1990)}{90} < 22.2$$

$\therefore \left[\frac{f(1990)}{90}\right] = 21, 22$ .....(1)

Given $x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right]$

$\therefore$ Taking Case I : $\left[\frac{f(1990)}{90}\right] = 21$

We have $1990 - f(1990) = 19\left[\frac{1990}{19}\right] - 90\left[\frac{f(1990)}{90}\right]$

1990 − f(1990) = 19.(104) − 90.(21)

$\Rightarrow$ f(1990) = 1904 ....(2)

Again taking case II :

$\left[\frac{f(1990)}{90}\right] = 22$

we have, $1990 - f(1990) = 19\left[\frac{1990}{19}\right] - 90\left[\frac{f(1990)}{90}\right]$

$\Rightarrow$ f(1990) = 1994 ....(3)

From (2) and (3); we have

f(1990) = 1904 or 1994. **Ans.(1)**

5. Here,

$g^2(x) = (gog)(x) = g(g(x)) = g(3 + 4x)$

$g^2(x) = 3 + 4(3 + 4x)$

$\Rightarrow g^2(x) = 15 + 4^2x$

$\Rightarrow g^2(x) = (4^2 - 1) + (4^2)x$

On generalizing we have

$g^n(x) = (4^n - 1) + (4^n)x$. **Ans.(2)**

6. We have

$2f(xy) = f(x)^y + (f(y))^x$

Replacing y by 1, we get $2f(x) = f(x) + (f(1))^x$

$\Rightarrow f(x) = a^x$ {as f(1) = a}

$\therefore \sum_{i=1}^{n} f(i) = f(1) + f(2) + f(3) + \ldots\ldots + f(n)$

$= a^1 + a^2 + a^3 + \ldots\ldots\, a^n$

$= \frac{a(a^n - 1)}{(a-1)}$

$= \frac{a^{n+1} - a}{(a-1)}$

$\Rightarrow (a-1)\sum_{i=1}^{n} f(i) = (a^{n+1} - a)$. **Ans.(3)**

7. For $x = \frac{1}{f(y)}$, we have

$f\left(x.\frac{1}{x}\right) = \frac{1}{f(y)^p}.y^q$

$\Rightarrow f(1) = \frac{y^q}{(f(y))^p}$

$\Rightarrow f(y) = \frac{y^{q/p}}{(f(1))^{1/p}}$

for y = 1, we have f(1) = 1

$\therefore f(y) = y^{q/p}$ or $f(x) = x^{q/p}$ .....(1)

Hence, $f(x.y^{q/p}) = x^p.y^q$

Let $y^{q/p} = z \Rightarrow y = z^{p/q}$

$\Rightarrow f(x.z) = x^p.z^p$

or $f(x) = x^p$ ....(2)

From (1) and (2) we have

$x^{q/p} = x^p$

$\Rightarrow \frac{q}{p} = p$

or $q = p^2$. **Ans.(1)**

8. f(x, y) = f(2x + 2y, 2y − 2x) .....(1)

$\Rightarrow$ f(2x + 2y, 2y − 2x) = f{2(2x + 2y) + 2(2y − 2x),
2(2y − 2x) − 2 (2x + 2y)} [using (1)]

$\Rightarrow$ f(x, y) = f(2x + 2y, 2y − 2x) = f(8y, −8x)

$\Rightarrow$ f(x, y) = f(8y, −8x) ....(2)

$\Rightarrow$ f(8y, −8x) = f{8(−8x), −8(8y)} [using (2)]

$\Rightarrow$ f(x, y) = f(2x + 2y, 2y − 2x) = f(8y, −8x)

$\Rightarrow$ f(−64, −64y)

$\Rightarrow$ f(x, y) = f(−64x −64y) ....(3)

$\Rightarrow$ f(−64x, −64y) = f(64 × 64x, 64 × 64y)

$= f(2^{12}x, 2^{12}y)$

$\Rightarrow f(x, y) = f(2^{12}x, 2^{12}y)$ [using (3)]

$\Rightarrow f(2^x, 0) = f(2^{12}.2^x, 0) = f(2^{12+x}, 0)$ ....(4)

given $g(x, 0) = f(2^x, 0)$

$\Rightarrow g(x, 0) = f(2^x, 0) = f(2^{12+x}, 0)$ [using (4)]

$\Rightarrow$ g(x, 0) = g(x + 12, 0)

Hence, g(x) is periodic with period 12. **Ans.(2)**

9. f(x) is defined when

[|x − 1|] + [|7 − x|] − 6 ≠ 0

$\begin{cases} [1-x]+[7-x] \neq 6; & \text{when} \quad x \leq 1 \quad ....(1) \\ [1-x]+[7-x] \neq 6; & \text{when} \quad 1 \leq x \leq 7 \quad ....(2) \\ [1-x]+[7-x] \neq 6; & \text{when} \quad x \geq 7 \quad ....(3) \end{cases}$

Taking (1), we have

[1 − x] + [7 − x] ≠ 6

1 + [−x] + 7 + [−x] ≠ 6

$\Rightarrow$ 2[−x] ≠ −2

$\Rightarrow$ [−x] ≠ −1

$\Rightarrow x \notin (0, 1)$ ....(A)

From (2), we have

[x − 1] + [7 − x] ≠ 6

$\Rightarrow$ [x] − 1 + 7 + [−x] ≠ 6

$\Rightarrow$ [x] + [−x] ≠ 0

$\Rightarrow x \notin$ integer

$\Rightarrow x \notin \{1, 2, 3, 4, 5, 6, 7\}$ ....(B)

From (3), we have

[x − 1] + [x − 7] ≠ 6

$\Rightarrow$ [x] − 1 + [x] − 7 ≠ 6

$\Rightarrow$ 2[x] ≠ 14

$\Rightarrow$ [x] ≠ 7

$\Rightarrow x \notin [7, 8]$ ....(C)

Hence, from (A), (B) and (C), we have

Domain $f(x) \in R - (0, 1) \cup \{1, 2, 3, 4, 5, 6, 7\} \cup [7, 8]$. **Ans.(1)**

10. Here, f(r) = f((r − 1) + 1)

f(r) = f(r − 1) + f(1) ....(1)

[using definition]

$\therefore$ f(r) = f(r − 1) + 5

$\Rightarrow$ f(r) = f(r − 2) + 5} + 5

$\Rightarrow$ f(r) = f(r − 2) + 2.(5)

$\Rightarrow$ f(r) = f(r − 3) + 3}.5

....

....

....

$\Rightarrow$ f(r) = f(r – (r – 1)) + (r – 1).5

$\Rightarrow$ f(r) = f(1) + (r – 1).5

$\Rightarrow$ f(r) = 5 + (r – 1).5

$\Rightarrow$ f(r) = 5r

$$\therefore \sum_{n=1}^{101} f(n) = \sum_{n=1}^{101}(5n) = 5[1+2+3+.....+101]$$

$$= \frac{5101(101+1)}{2}$$

Hence, $\sum_{n=1}^{101} f(n) = \frac{5101(101+1)}{2}$ = 25755. **Ans.(1)**

SET IV

1. S_n will have n terms T_1 of S_n will be T_n of 1, 2, 4, 7 ......by inspection

$$T_n = \frac{(n-1)n}{2} + 1$$

In S_{25}, the first term = [(25 – 1) 25/2] + 1 = 301. **Ans.(2)**

2. First term of S_{10} = $\frac{(10-1)10}{2} + 1 = 46$

$\Rightarrow$ S_{10} = [46, 47, ...55] Sum of all terms = 505. **Ans.(2)**

3. First term of S_{30} = 436 $\Rightarrow$ S_{30} = {436, 437, ...30 term}
= 13515 term (A.P.) **Ans.(3)**

4. First term of S_{18} = (18 – 1) $\frac{18}{2} + 1 = 154$. **Ans.(4)**

5. S_{12} {67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78)
$\therefore$ difference between first and last term = 78 – 67 = 11. **Ans.(1)**

6. T_1 of S_{15} = 106.
T_{15} of S_{15} = 106 + (15 – 1) × 1 = 120. **Ans.(1)**

7. T_1 of S_8 = 29 last term = 36
Sum = 29 + 36 = 65. **Ans.(2)**

8. Total no. of terms up to S_{15}
= 1 + 2 + ...up to 15
$\Rightarrow$ Sum of first 15 natural no.

$$= \frac{15(15+1)}{2} = 120$$. **Ans.(3)**

9. T_1 of S_{22} = 232
6th term = 237. **Ans.(1)**

10. Sum of All Values up to S_5 = 1 + 2 + 3 + 4 + 5 + 6 + ...15
$\Rightarrow$ Sum of first 15 natural no.

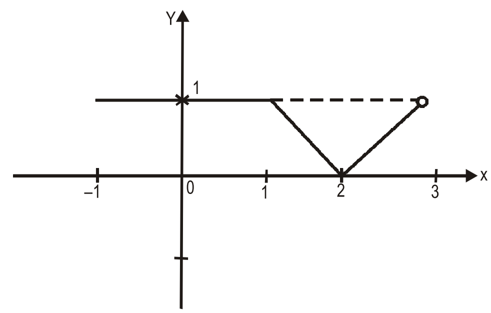
By formula $\frac{n(n+1)}{2}$

Where n is natural number

$$= 15 \times \frac{15+1}{2} = \frac{15 \times 8}{2} = 120.$$ **Ans.(2)**

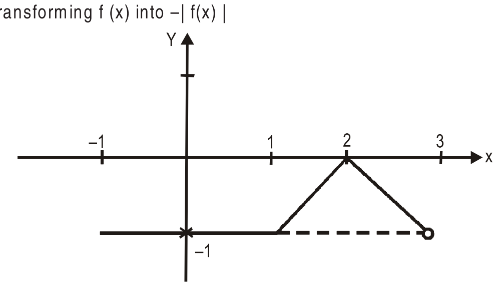
SET V

1. Transforming the graph of f(x) into |f(x)| we get



It is clearly evident that for x $\in$ (2, 3) the function is increasing. **Ans.(1)**

2. Transforming f (x) into –| f(x) |

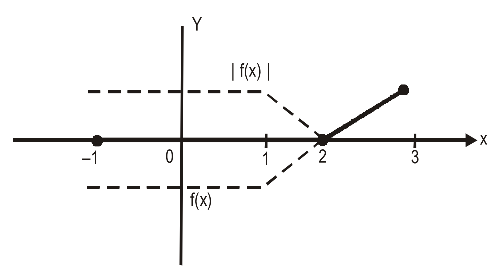


Clearly at x = 1, 2 the curve has two gradients hence non differentiable. **Ans.(2)**

3. as $f(-|x|) = \begin{cases} f(x) & x < 0 \\ f(0) & x = 0 \\ f(-x) & x > 0 \end{cases}$

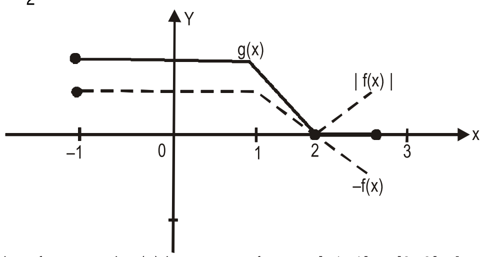
$\therefore$ for x < 0 f(– | x |) = f(x) = –1
for x = 0 f (– | x |) = f(0) = –1
and for x > 0 f(–| x |) = f(–x) = –1
hence f(–| x |) is a constant function. **Ans.(3)**

4. Transforming f(x) to $g(x) = \frac{1}{2}$ (| f (x) | + f(x))



Evidently for x $\in$ [–1, 2] g (x) = 0 hence g(x) is constant in [–1, 2]. **Ans.(3)**

5. $g(x) = \frac{1}{2}(|f(x)| - f(x))$



Evident from graph g(x) is constant for $x \in [-1, 1] \cup [2, 3]$. **Ans.(4)**

6. $g(x) = \frac{|f(x)|}{f(x)}$ i.e., $\frac{|f(x)|}{f(x)} = 1$ for $0 < f(x) < \infty$

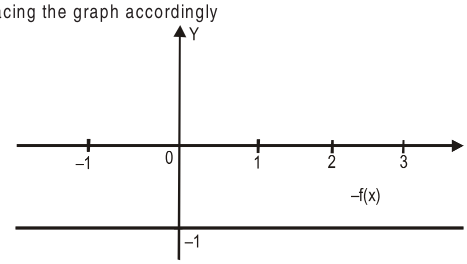
$\frac{|f(x)|}{f(x)} = -1$ for $-\infty < f(x) < 0$

from the graph of f(x), $0 < f(x) < \infty \ \forall \ x \in (2, 3)$. **Ans.(2)**

7. We know $\frac{|x|}{x} = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ +1 & x > 0 \end{cases}$

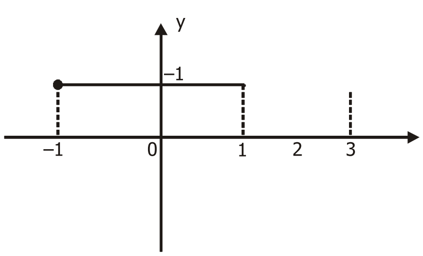
$\therefore \ g(x) = f\left(\frac{|x|}{x}\right) = \begin{cases} f(-1) & x < 0 \\ f(0) & x = 0 \\ f(+1) & x > 0 \end{cases}$

Tracing the graph accordingly



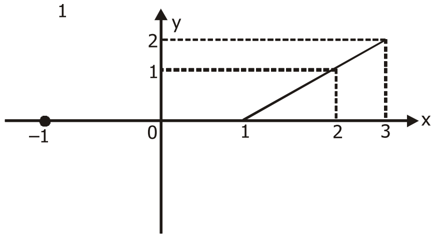
evidently g(x) is a constant function. **Ans.(3)**

8. Graph of f(|x|) is as shown



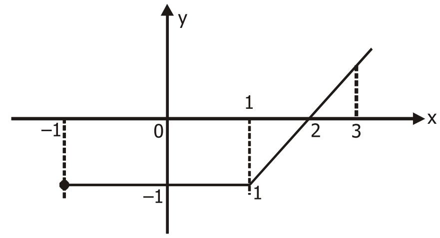
Obviously $|f(|x|)| = x$. Therefore no value of x. **Ans.(4)**

9. Graph of $|f(x) + 1| > 0$ is as shown



Clearly $|f(x) + 1| > 0$ for $x > 1$. **Ans.(3)**

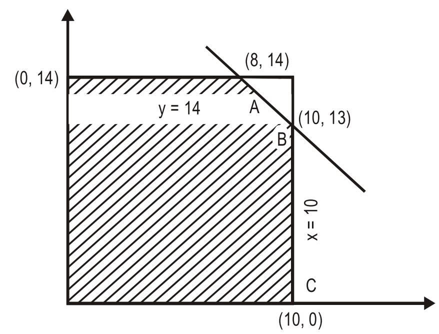
10. graph of f(|x|) is as shown



Clearly $f(|x|) < 0$ for $x < 2$. **Ans.(1)**

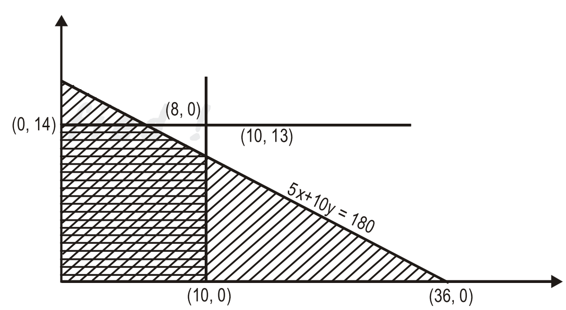
Set VI

1. Obviously $x \le 10$, $y \le 14$ and $5x + 10y \le 180$. **Ans.(1)**
2. $3x + 5y$. **Ans.(3)**
3. Hence required feasible region is given by ABCD, and vertices are (8, 14), (10, 13), (10, 0) and (0, 14)



Ans.(3)

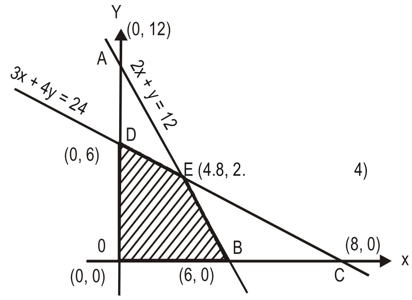
4. Max z = 3 (10) + 5(13) = 95.



Ans.(3)

5. Obviously $x + y \le (80 \times 60 = 480)$ and $2x + y \le (10 \times 60 = 600)$. **Ans.(1)**
6. $2x + 2y$. **Ans.(3)**
7. **Ans.(3)**

8. Step (1) The equations are $2x + y = 12$
$3x + 4y = 24$ and $x = 0, y = 0$



Step (2) The feasible region is OBED.
Step (3) The coordinates of corners of feasible region are O(0, 0), B(6, 0), E(4.8, 2.4) and D(0, 6).
Step (4) $P(O) = 3 \times 0 + 2 \times 0 = 0$
$P(B) = 3 \times 6 + 2 \times 0 = 18$
$P(E) = 3 \times 4.8 + 2 \times 2.4 = 19.2$ and
$P(D) = 3 \times 0 + 2 \times 6 = 12$

$\therefore$ The maximum value of P is 19.2 and it occurs at the vertex E (4.8, 2.4). **Ans.(2)**

9. The lines of regression are $y = ax + b$ and $x = cy + d$. Since the line of regression passes through $(\bar{x}, \bar{y})$ We have $\bar{y} = a\bar{x} + b$ and $\bar{x} = c\bar{y} + d$

Now on solving these equations, we get

$\bar{x} = \frac{bc+d}{1-ac}, \bar{y} = \frac{ad+b}{1-ac}$. **Ans.(2)**

10. Since lines of regression pass through $(\bar{x}, \bar{y})$, hence the equation will be

$$4\bar{x} + 3\bar{y} + 7 = 0$$

$$3\bar{x} + 4\bar{y} + 8 = 0$$

On solving the above equations, we get the required answer

$\bar{x} = -\frac{4}{7}$ and $\bar{y} = -\frac{11}{7}$. **Ans.(1)**

SET VII

1. $$\frac{1}{1+x^{a-b}+x^{a-c}} + \frac{1}{1+x^{b-c}+x^{b-a}} + \frac{1}{1+x^{c-a}+x^{c-b}}$$

$$= \frac{1}{1+\frac{x^{-b}}{x^{-a}}+\frac{x^{-c}}{x^{-a}}} + \frac{1}{1+\frac{x^{-c}}{x^{-b}}+\frac{x^{-a}}{x^{-b}}} + \frac{1}{1+\frac{x^{-a}}{x^{-c}}+\frac{x^{-b}}{x^{-c}}}$$

$$= \frac{x^{-a}}{x^{-a}+x^{-b}+x^{-c}} + \frac{x^{-b}}{x^{-b}+x^{-c}+x^{-a}} + \frac{x^{-c}}{x^{-a}+x^{-b}+x^{-c}}$$

$= \frac{x^{-a}+x^{-b}+x^{-c}}{x^{-a}+x^{-b}+x^{-c}} = 1$. **Ans.(1)**

2. $$\left(\frac{a^p}{a^q}\right)^{p+q}\left(\frac{a^q}{a^r}\right)^{q+r}\left(\frac{a^r}{a^p}\right)^{r+p}$$

$= [a^{p-q}]^{p+q}.\,[a^{q-r}]^{q-r}[a^{r-p}]^{r+p}$

$= a^{p^2-q^2}.a^{q^2-r^2}.a^{r^2-p^2}$

$a^0 = 1$. **Ans.(2)**

3. $a^x = b^y = c^z = k \Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}$

$abc = 1 \Rightarrow k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = 1 = k^{\circ} \quad \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

$\Rightarrow xy + yz + zx = 0$. **Ans.(1)**

4. $(2.381)^x = (0.2381)^y = 10^z = k$ say

$\Rightarrow 2.381 = k^{\frac{1}{x}}, 0.2381 = k^{\frac{1}{y}} 10 = k^{\frac{1}{z}}$

$2.381 = 0.2381 \times 10$

$\Rightarrow k^{\frac{1}{x}} = k^{\frac{1}{y}} \times k^{\frac{1}{z}} \quad \Rightarrow k^{\frac{1}{x}} = k^{\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow \frac{1}{x} = \frac{1}{y} + \frac{1}{z} \ \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{z} \ \Rightarrow z\left(\frac{1}{x} - \frac{1}{y}\right) = 1$. **Ans.(1)**

5. $x = 2^{20}$

$\log x = -20 \log 2 = -20x\,(.30103)$

$= -6.02060 = (\bar{7}).97940$

The characteristic is $(\bar{7})$.

The number of zeros after decimal when we take antilogarithms is 6. The first significant figure is in 7th place. **Ans.(4)**

6. Since $\log_l x, \log_m x, \log_n x$ are in A.P., we have,

$2 \log_m x = \log_l x + \log_n x$

or $\frac{2}{\log_x m} = \frac{1}{\log_x l} + \frac{1}{\log_x n} = \frac{\log_x l + \log_x n}{\log_x l.\log_x n}$

$2 \log_x n = \frac{\log_x nl.\log_x m}{\log_x l}$

$\Rightarrow 2 \log_x n = \log_x nl.\log_x m.\log_l x$

$(\log_x nl) \log_l m$

$\therefore n^2 = ln.\log_l m$. **Ans.(3)**

7. We have $= \frac{\sqrt{7}}{\sqrt{(16+6\sqrt{7}) - \sqrt{(16-6\sqrt{7})}}}$

$$= \frac{\sqrt{7}}{\sqrt{(9+7+2.3\sqrt{7}) - \sqrt{(9+7-2.3.\sqrt{7})}}}$$

$$= \frac{\sqrt{7}}{\sqrt{(3+\sqrt{7})^2 - \sqrt{(3-\sqrt{7})^2}}}$$

$$= \frac{\sqrt{7}}{(3+\sqrt{7})-(3-\sqrt{7})} = \frac{\sqrt{7}}{2\sqrt{7}} = \frac{1}{2}$$

= a rational number. **Ans.(4)**

8. $$G.E. = \frac{4+3\sqrt{2}}{4\times4\sqrt{3} - 8\sqrt{2} + 10\sqrt{2} - 8\times2\sqrt{3} + 5\times2\sqrt{2}}$$

$$= \frac{4+3\sqrt{2}}{12\sqrt{2}} \times \frac{12\sqrt{2}}{12\sqrt{2}} = \frac{2\sqrt{2}+3}{12} = \frac{1}{4} + \frac{1}{6}\sqrt{2} = a + b\sqrt{2}$$

Hence $a = \frac{1}{4}; b = \frac{1}{6}$. **Ans.(4)**

9. $$G.E\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}-\sqrt{9+5-2\sqrt{9\times5}}}}$$

$$= \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+(3-\sqrt{5})}}$$

$$= \sqrt{2+\sqrt{5}-\sqrt{9-4\sqrt{5}}} \quad = \sqrt{2+\sqrt{5}-\sqrt{5+4-2\sqrt{5\times4}}}$$

$= \sqrt{2+\sqrt{5}-(\sqrt{5}-2)} = \sqrt{4} = 2$. **Ans.(3)**

10. $f(x) = lx^4 + mx^3 + 2x^2 + 4$ is exactly divisible by

$x^2 - x - 2 = (x + 1)(x - 2)$

Hence $f(-1) = 0$, $f(2) = 0$

$\Rightarrow l - m + 2 + 4 = 0$

$\Rightarrow l - m = -6$ ....(1)

$16l + 8m + 8 + 4 = 0$

$\Rightarrow 16l + 8m = -12$

$l - m = -6$ ....(1)

$\Rightarrow 4l + 2m = -3$ ....(2)

$\Rightarrow 2l - 2m = -12$

Adding $6l = -15$

$\therefore l = -\frac{5}{2}$

$m = l + 6$

$= -\frac{5}{2} + 6 = \frac{7}{2}$. **Ans.(2)**

SET VIII

1. Given $U = [2\ \ -3\ \ 4] \Rightarrow V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow U.V = [2\ \ -3\ \ 4]\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$= [6 - 6 + 4] = [4] \Rightarrow XY = [0\ 2\ 3]\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = [0 + 4 + 12] = [16].$

Then U.V + X.Y = 16 + 4 = [20]. **Ans. (4)**

2. Given that the matrix A is of order $= 2 \times 3$ and matrix B is of order $= 3 \times 2$. Therefore AB matrix is of order $= 2 \times 2$ and BA matrix is of order $= 3 \times 3$. The matrix AB and BA both are defined. **Ans.(1)**

3. We have $F(\alpha)\,F(-\alpha)$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I,$$

$\therefore F(-\alpha) = [F(\alpha)]^{-1}$. **Ans.(1)**

4. Given $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow |A| = abc.$

$$\therefore A^{-1} = \frac{1}{abc}\begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}.$$ **Ans.(3)**

5. If matrix is invertible then $|A| \neq 0 \Rightarrow \begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \neq 0$

or $\lambda(0 - 1) + 1(-6 + 1) + 4(-3 - 0) \neq 0$ or $-\lambda - 5 - 12 \neq 0$

or $-\lambda \neq 17$ $\lambda \neq -17$.**Ans.(2)**

6. $|A| = \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{vmatrix} = 1.$ Adj $A = \begin{vmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{vmatrix} = 1.$

$A^{-1} = \frac{Adj.A}{|A|}$. **Ans.(1)**

7. We have $\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$. The system admits a unique solution if and only if the coefficient matrix is of rank 3, i.e.

if $\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} = 15(5 - \lambda) \neq 0$. Thus for a unique solution $\lambda \neq 5$ and μ may have any value. if $\lambda = 5$, the system will have no solution for those values of μ for which the matrices $A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 5 \end{bmatrix}$ and $K = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & 5 & \mu \end{bmatrix}$ are not of the same rank. But A is of rank 2 and K is not of rank 2 unless $\mu = 9$. Thus if $\lambda = 5$ and $\mu \neq 9$, the system will have no solution. If $\lambda = 5$ and $\mu = 9$, the system will have an infinite number of solutions. **Ans.(2)**

8. We have $AA' = \begin{bmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix} \times \begin{bmatrix} -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$

$$= \begin{bmatrix} 4/9+1/9+4/9 & -4/9+2/9+2/9 & -2/9-2/9+4/9 \\ -4/9+2/9+2/9 & 4/9+4/9+1/9 & 2/9-4/9+2/9 \\ -2/9-2/9+4/9 & 2/9-4/9+2/9 & 1/9+4/9+4/9 \end{bmatrix} = I$$

Hence the matrix is orthogonal. **Ans.(1)**

9. A matrix is a Hermitian matrix if $\overline{A} = A'$, which could be easily verified for the given matrix. **Ans.(1)**

10. $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 \\ -2 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix}$.[Applying $\frac{1}{3}(C_1), \frac{1}{2}(C_3)$]

$\sim \begin{bmatrix} 1 & 0 & 2 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ [Applying $C_2 \to C_1 + C_2$, $C_3 \to C_1 + C_3$]. Obviously the 3rd order minor zero. But there exists a second order non-zero minor i.e., $\begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} \neq 0$. Hence rank of given matrix is 2. **Ans.(3)**

SET IX

1. Comparing given series with $1 + nx + \frac{n(n-1)}{2!}x^2 + ...,$

we get $nx = \frac{3}{2^3}$ and $\frac{n(n-1)}{2!}x^2 = \frac{1.3\,(3^3)}{12.2^6}$.

Simplifying $n = \frac{-1}{2}$ and $x = \frac{-3}{4}$,

so the sum of the given series $= \left(1 - \frac{3}{4}\right)^{-1/2} = \left(\frac{1}{4}\right)^{-1/2} = 2$. **Ans.(1)**

2. Given $y = 2x + 3x^2 + 4x^3 + ...$ or $1 + y = 1 + 2x + 3x^2 + 4x^3 + ...$

$\Rightarrow 1 + y = (1 - x)^{-2} \Rightarrow \frac{1}{\sqrt{1+y}} = 1 - x$ or $x = 1 - \frac{1}{\sqrt{1+y}}$. **Ans.(C)**

3. The expression can be divided into two parts.

$$\left(1-\frac{n}{1}\frac{1}{(1+nx)}+\frac{n(n-1)}{2}\frac{1}{(1+nx)^2}-\frac{n(n-1)(n-2)}{1.2.3(1+nx)^3}+......\right)$$

$$+\left(\frac{-nx}{1+nx}+\frac{n(n-1)x}{(1+nx)^2}-\frac{n(n-1)(n-2)}{1.2}\frac{x}{(1+nx)^3}+....\right)$$

$$=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{(n-1)}{1(1+nx)}+\frac{(n-1)(n-2)}{1.2(1+nx)^2}+.....\right)$$

$$=\left(\frac{nx}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}$$

$$=\left(\frac{nx}{1+nx}\right)^n-\left(\frac{nx}{1+nx}\right)\left(\frac{1}{1+nx}\right)^{n-1}=0.$$ **Ans.(D)**

4. Comparing with $(1+y)^n = 1+ny+\frac{n(n-1)}{2!}y^2+...$

$$ny=\frac{1}{3}x,\ \frac{n(n-1)}{2!}y^2=\frac{4}{3.6}x^2.\ \therefore\ \frac{(n-1)}{2n}=2 \text{ or } n=-\frac{1}{3}.$$

$\therefore$ $y = -x$, sum $= (1-x)^{-1/3}$.

Aliter : Since the given series contains all positive terms.
Hence from the options it will be expansion of $(1-x)^{-1/3}$ only. **Ans.(C)**

5. $$1+\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2^2}+...=1+\frac{1}{2}\left(\frac{1}{2}\right)+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}\left(\frac{1}{2}\right)^2+...$$

$$=\left(1-\frac{1}{2}\right)^{-1/2}=2^{1/2}=\sqrt{2}$$ **Ans.(D)**

6. Let t_n be the nth term of the series 4 + 11 + 22 + 37 + 56 + .. Since the differences of the successive terms in this series are in AP. So, let
$t_n = an^2 + bn + c$
Putting n =1, 2, 3 we get
$a + b + c = 4$, $4a + 2b + c = 11$ and $9a + 3b + c = 22$.
Solving these equations, we obtain a = 2, b = 1 and c = 1.
$\therefore$ $t_n = 2n^2 + n + 1$, n = 1, 2, ...

So, sum of the series = $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}$

$$=2\sum_{n=1}^{\infty}\frac{n^2}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}+\sum_{n!}^{\infty}+\sum_{n=1}^{\infty}\frac{1}{n!}$$

$= 2(2e) + e + (e-1) = 6e - 1$. **Ans.(B)**

7. We have $y+\frac{y^3}{3}+\frac{y^5}{5}+.....\infty=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+.....\infty\right)$

$$\Rightarrow\ \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)=\log\left(\frac{1+x}{1-x}\right)\Rightarrow\log\left(\frac{1+y}{1-y}\right)=\log\left(\frac{1+x}{1-x}\right)^2$$

$$\Rightarrow\ \frac{1+y}{1-y}=\frac{(1+x)^2}{(1-x)^2}\Rightarrow\frac{2}{2y}=\frac{(1+x)^2+(1-x)^2}{(1+x)^2-(1-x)^2}$$

$$\Rightarrow\ \frac{1}{y}=\frac{2(1+x^2)}{1+x^2}\Rightarrow y=\frac{2x}{1+x^2}\Rightarrow x^2y=2x-y.$$ **Ans.(C)**

8. Clearly, the given series is an arithmeticogeometric series whose corresponding G.P. is $1, -x, x^2, -x^3$, ... The common ratio of this G.P. is $-x$. Let S_∞ denote the sum of the given infinite series. Then,

$S_\infty = 1 - 3x + 5x^2 - 7x^3 + ...\ \infty$ .... (i)

$\Rightarrow (-x)\,S_\infty = -x + 3x^2 - 5x^3 + ...\ \infty$ .... (ii)

Subtracting (ii) from (i), we get

$(1 + x)\,S_\infty = 1 + [-2x + 2x^2 - 2x^3 + ...\ \infty]$

$$=1+\left[\frac{-2x}{1-(-x)}\right]=1-\frac{2x}{1+x}=\frac{1-x}{1+x}\Rightarrow S_\infty=\frac{1-x}{(1+x)^2}.$$ **Ans.(C)**

9. The given series is not an arithmetico-geometric series, because 1^2, 5^2, 9^2, 13^2, ... is not a G.P. However, their successive differences (5^2-1^2), (9^2-5^2), (13^2-9^2), ..., i.e. 24, 56, 88, ... from an A.P. So, the process of obtaining the sum of an infinite arithmeticogeometric series will be repeated twice as given below

Let $S_\infty = 1^2 + 5^2x + 9^2x^2 + 13x^3 + ...\ \infty$ .... (i)

Multiplying (i) by x, we get

$xS_\infty = 1^2x + 5^2x^2 + 9^2x^3 + ...\infty$ .... (ii)

Subtracting (ii) from (i), we get

$(1-x)\,S_\infty = 1^2 + (5^2-1^2)\,x + (9^2-5^2)\,x^2 + (13^2-9^2)\,x^3 + ...\infty$

$(1-x)\,S_\infty = 1 + 24x + 56x^2 + 88x^3 + ...\ \infty$ ..... (iii)

This is an arithmetico-geometric series in which the common ratio of the corresponding G.P. is x. Multiplying (iii) by x, we get

$x\,(1-x)\,S_\infty = x + 24x^2 + 56x^3 + ...\infty$ ..... (iv)

Subtracting (iv) from (iii), we get

$(1-x)\,S_\infty - x\,(1-x)\,S_\infty = 1 + 23x + 32x^2 + 32x^3 + ...\infty$

$$\Rightarrow (1-x)^2\,S_\infty = 1 + 23x + \frac{32x^2}{1-x}$$

$$\Rightarrow S_\infty=\frac{1}{(1-x)^2}+\frac{23x}{(1-x)^2}+\frac{32x^2}{(1-x)^3}=\frac{1+22x+9x^2}{(1-x)^3}.$$ **Ans.(D)**

10. We have,
$1 + (1 + a)\,b + (1 + a + a^2)\,b^2 + (1 + a + a^2 + a^3)\,b^3 + ...$ to ∞

$$\sum_{n=1}^{\infty}(1+a+a^2+...+a^{n-1})\ b^{n-1}=\sum_{n=1}^{\infty}\left(\frac{1-a^n}{1-a}\right)b^{n-1}$$

$$\sum_{n=1}^{\infty}\frac{b^{n-1}}{1-a}-\sum_{n=1}^{\infty}\frac{a^nb^{n-1}}{1-a}=\frac{1}{1-a}\ \sum_{n=1}^{\infty}b^{n-1}-\frac{a}{1-a}\ \sum_{n=1}^{\infty}(ab)^{n-1}$$

$$=\frac{1}{1-a}[(1+b+b^2+...\infty)]-\frac{a}{1-a}[(1+ab+(ab)^2+...\infty)]$$

$$=\frac{1}{1-a}\cdot\frac{1}{1-b}-\frac{a}{(1-a)(1-ab)}=\frac{1}{(1-ab)(1-b)}.$$ **Ans.(C)**

SET X

1. Centre is $(-4, -5)$ and passes through $(2, 3)$. **Ans.(2)**

2. Given $2\sqrt{g^2} = 10 \Rightarrow g = 5$ and $2\sqrt{f^2} = 24 \Rightarrow f = 12$.

Therefore, radius is $\sqrt{(5^2+12^2)}=13$. **Ans.(4)**

3. Let its centre be (h, k), then h − k = 1... (i). Also radius a = 3. Equation is $(x-h)^2 + (y-k)^2 = 9$.
Also it passes through (7, 3) i.e., $(7-h)^2 + (3-k)^2 = 9$ ... (ii). We get h and k form (i) and (ii) solving simultaneously as (4, 3). Equation is
$x^2 + y^2 - 8x - 6y + 16 = 0$. **Ans.(1)**
Trick : Check from options.

4. Let x = a, x = b, y = c and y = d be the sides of the square. The length of each diagonal of the square is equal to the diameter of the circle i.e., $2\sqrt{(1+4+93)}$. Let l be the length of each side of the square.

Then $2l^2 = (\text{Diagonal})^2 \Rightarrow 2l^2=\left[2\cdot\sqrt{(1+4+93)}\right]^2\Rightarrow l=14$. Therefore each side of the square is at distance 7 from the centre (1, −2) of the given circle. This implies that a = −6, b = 8, c = −9, d = 5. Therefore the vertices of the square are (−6, −9), (−6, 5), (8, −9), (8, 5). **Ans.(1)**

5. It represent a circle, if $a = b \Rightarrow 3/k = 4 \Rightarrow k = 3/4$. **Ans.(1)**

6. Since the circle passes through (0, 0), hence c = 0.

Also $2\sqrt{(g^2-c)}=2\Rightarrow g=1$ and $2\sqrt{(f^2-c)}=2\Rightarrow f=1$. Hence radius is $\sqrt{2}$ and centre is (−1, −1). Therefore, the required equation is
$x^2 + y^2 + 2x + 2y = 0$. **Ans.(3)**
Trick: The centre of circle lies in III quadrant, which is there only in (3).

7. Since the circle touches x-axis at (3, 0) its centre is (3,k) and radius is k. Hence the equation of circle is $(x-3)^2 + (y-k)^2 = k^2$. Since it passes through (1, 4), therefore $k^2 = 4 + (k-4)^2 \Rightarrow k = \frac{5}{2}$.

Hence required equation of circle is

$$(x-3)^2 + \left(y-\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2 \Rightarrow x^2 + y^2 - 6x - 5y + 9 = 0 .$$ **Ans.(1)**

Trick : Only (1) passes through (1, 4).

8. Let centre be (h, k), then $\sqrt{\{(h-2)^2+(k-3)^2\}} = \sqrt{\{(h-4)^2+(k-5)^2\}}$ ... (i) and $k - 4h + 3 = 0$... (ii) .From (i), we get $-4h - 6k + 8h + 10k$ $= 16 + 25 - 4 - 9$ or $4h + 4k - 28 = 0$ or $h + k - 7 = 0$ ... (iii).

From (iii) and (ii), we get (h, k) as (2, 5). Hence centre is (2, 5) and radius is 2. Hence equation of circle is $x^2 + y^2 - 4x - 10y + 25 = 0$. **Ans.(2)**

9. The equation of family of circles through (2, –2) and (–1, –1) is

$$(x-2)(x+1) + (y+2)(y+1) + \lambda\left(\frac{y+2}{-2+1} - \frac{x-2}{2+1}\right) = 0 .$$

Now for point (5, 2) to lie on it, we should have λ given by

$$3.6 + 4.3 + \lambda\left(\frac{4}{-1} - 1\right) = 0 \Rightarrow \lambda = \frac{30}{5} = 6 .$$

Hence equation is $(x-2)(x+1) + (y+2)(y+1) + 6\left(\frac{y+2}{-1} - \frac{x-2}{3}\right) = 0$

or $x^2 + y^2 - 3x - 3y - 8 = 0$. **Ans.(2)**

Trick : Here the circle $x^2 + y^2 - 3x - 3y - 8 = 0$ is satisfied by (2, –2), (–1 –1) and (5, 2). Therefore students are advised to check such type of problems conversely.

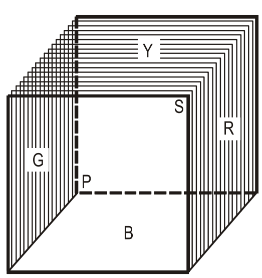
10. Infinite, as this is a family of coaxial circles. **Ans.(4)**

SET XI

1. **Ans.(3)**
2. **Ans.(2)**
3. **Ans.(4)**
4. **Ans.(1)**
5. **Ans.(1)**

For Q.6 to Q.10 :

On the basis of the given details, the cube will be painted as indicated in the following figure.



Here 'Y' stands for Yellow ; 'R' for Red ; 'B' for Brown; 'G' for Green; 'P' for Pink and 'S' for Silver. The colour of each face is indicated at the centre of each face.

6. The face opposite to Red is Green. **Ans.(2)**
7. The upper face is painted yellow. **Ans.(3)**
8. Clearly, the faces adjacent to Green are Pink, Silver, Yellow and Brown. **Ans.(4)**
9. Clearly, the face opposite to silver is Pink. **Ans.(1)**
10. The faces adjacent to Red face are Silver, Pink, Brown and Yellow. **Ans.(2)**

SET XII

1. Some doctors and some actors are males. But, doctor and actor are entirely different. **Ans.(1)**
2. Both Rose and Lotus are flowers. But, Rose and Lotus are entirely different. **Ans.(2)**
3. Father, Mother and Child are entirely different. **Ans.(3)**
4. Some ornaments are made of gold and some of silver. Gold and Silver are entirely different. **Ans.(1)**
5. Clearly, time taken $= \frac{\text{sumof lengthof two trains}}{\text{totalspeed of two trains}}$

$= \frac{L_1 + L_2}{V_1 + V_2}$ = (L_1" L_2) @ (V_1" V_2). **Ans.(2)**

6. Total fare = B + 15% of B + 2% of B + 200

$= B + \frac{B \times 15}{100} + \frac{B \times 2}{100} + 200$

= B" (B * 15) @ 100" (B * 2) @ 100"200. **Ans.(2)**

7. Profit percentage $= \frac{S-(C+L+T)}{C+L+T} \times 100$

= {S'(C"L"T)} @ (C" L" T) * 100. **Ans.(3)**

8. Clearly, total marks = (T – 2) × 2 + $\frac{4K}{3}$ + 5 × 2

= (T'2) * 2"4 * K @ 3"5 * 2. **Ans.(5)**

9. Marks out of 150 in first periodical = P.

Marks out of 100 in first periodical $= \left(\frac{P}{150} \times 100\right)$

Marks out of 180 in second periodical = T.

Marks out of 100 in second periodical = $\left(\frac{T}{180} \times 100\right)$

Marks out of 400 in final examination = M.

Marks out of 100 in final examination $= \left(\frac{M}{400} \times 100\right)$.

$\therefore$ Total marks

$$= \left[10\% \text{ of} \left(\frac{P}{150} \times 100\right)\right] + \left[+15\% \text{of} \left(\frac{T}{180} \times 100\right)\right] + \left[75\% \text{ of} \left(\frac{M}{400} \times 100\right)\right]$$

$$= \left[\frac{10}{100} \text{of} \left(\frac{P}{150} \times 100\right)\right] + \left[\frac{15}{100} \text{of} \left(\frac{T}{180} \times 100\right)\right] + \left[\frac{75}{100} \text{of} \left(\frac{M}{400} \times 100\right)\right]$$

$$= \left(\frac{P}{150} \times 10\right) + \left(\frac{T}{180} \times 15\right) + \left(\frac{M}{400} \times 75\right)$$

= P @ 150 * 10)" (T @ 180 * 15)" M @ 400 * 75). **Ans.(2)**

10. Let 'x' be the number of males in Mota Hazri.

	Chota Hazri	Mota Hazri
Males	x – 4522	x
Females	2(x – 4522)	x + 4020

$x = 4020 - 2(x - 4522) = 2910 \Rightarrow x = 10154$

$\therefore$ Number of males in Chota Hazri = 10154 – 4522 = 5632. **Ans.(3)**

SET XIII

1. The series is : –0!, –1!, –2!, –3!. **Ans.(5)**
2. The series is: × 1 – 1, × 2 + 2, × 3 – 3, × 4 + 4.... Replace (2) with (4). **Ans.(2)**
3. The series is : $\times 1 - 1^2, \times 2 - 1^2, \times 3 - 1^2, \times 4 - 1^2$,....

Replace (3) with (4). **Ans.(3)**

4. The series is : $-11^2, -9^2, -7^2\ -5^2,....$
Replace (1) with (4). **Ans.(1)**

5. The series is : $1, 1^2, 1^3, 2, 2^2, 2^3,....$
Replace (4) with (5). **Ans.(4)**

6. The series is $+ 2^2, + 4^2, + 6^2$ ..... **Ans.(4)**

7. The series is × 2 − 1, × 3 +3, × 4 − 3, × 5 + 5 ...... **Ans.(2)**

8. The series is × 0.5, × 1, × 1.5, × 2. **Ans.(1)**

9. The series is ÷ 2 + 4, ÷ 2 + 4...... **Ans.(2)**

10. The series is × 8 + 1, × 7 + 2, × 6 + 3 ..... **Ans.(1)**

SET XIV

1. Let E_1 = the event of success of the first student.
E_2 = the event of success of the second student.
E_3 = the event of success of the third student.
Let $A = E_1 \cap E_2 \cap E'_3$ = the event that the first and second student succeed and the third fails
$B = E'_1 \cap E_2 \cap E_3$, $C = E_1 \cap E'_2 \cap E_3$
$D = E_1 \cap E_2 \cap E_3$

According to question, $P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{1}{5}$

$$\therefore P(E'_1) = \frac{2}{3}, P(E'_2) = \frac{3}{4}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{1}{5}$$

$$\therefore P(E'_1) = \frac{2}{3}, P(E'_2) = \frac{3}{4}, P(E'_3) = \frac{4}{5}$$

Clearly, $A \cup B \cup C \cup D$ = the event of success of at least two student.
Since A, B, C, D are mutually exclusive events and E_1, E_2, E_3 are independent events.

∴ required probability,

$P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$
$= P(E_1 \cap E_2 \cap E'_3) + P(E_1 \cap E_2 \cap E_3)$
$+ P(E_1 \cap E'_2 \cap E_3) + P(E'_1 \cap E_2 \cap E_3)$
$= P(E_1).P(E_2).P(E'_3) + P(E'_1) . P(E_2) . P(E_3)$
$+ P(E_1) . P(E'_2) . P(E_3) + P(E_1).P(E_2) . P(E_3)$

$$= \frac{1}{3}.\frac{1}{4}\left(1-\frac{1}{5}\right)+\left(1-\frac{1}{3}\right).\frac{1}{4}.\frac{1}{5}+\frac{1}{3}.\left(1-\frac{1}{4}\right).\frac{1}{5}+\frac{1}{3}.\frac{1}{4}.\frac{1}{5}$$

$$= \frac{4}{60}+\frac{2}{60}+\frac{3}{60}+\frac{1}{60}=\frac{10}{60}=\frac{1}{6}.$$ **Ans.(1)**

2. Let A = the event of first student solving the problem
B = the event of second student solving the problem
C = the event of third student solving the problem
Let $E = A \cup B \cup C$ = the event of the problem being solved by at least one student.
= the event that the problem will be solved
According to question,

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

$$\therefore P(A') = 1-\frac{1}{2}=\frac{1}{2}, P(B') = 1-\frac{1}{3}=\frac{2}{3} \text{ and } P(C') = 1-\frac{1}{4}=\frac{3}{4}$$

Since A, B, C are independent events
∴ required probability,
$P(E) = P(A \cup B \cup C) = 1 - P(A') . P(B') . P(C')$

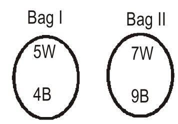
$$= 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = 1-\frac{1}{4}=\frac{3}{4}.$$

Second method :
$P(E) = P(A \cup B \cup C)$
$= P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$
$= P(A) + P(B) + P(C) - P(A) . P(B) - P(B) . P(C) - P(C) .P(A)$
$+ P(A) . P(B) . P(C)$

$$= \frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{2}.\frac{1}{3}-\frac{1}{3}.\frac{1}{4}-\frac{1}{4}.\frac{1}{2}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4} = \frac{12+8+6-4-2-3+1}{24}=\frac{18}{24}=\frac{3}{4}.$$

Ans.(3)

3.



Let A = the event that the ball transferred from first bag to the second bag is white.
B = the event that the ball transferred from the first bag to the second bag is black
C = the event that a white ball is drawn from the second bag after transfer to one ball from first bag to the second bag.
Let $E_1 = A \cap C$ and $E_2 = B \cap C$
Let $E = E_1 \cup E_2$
Now, $P(E) = P(E_1) + P(E_2)$ [∵ E_1 and E_2 are mutually exclusive events]
$= P(A \cap C) + P(B \cap C)$
$= P(A) . P(C/A) + P(B) . P(C/B)$

$$= \frac{5}{9}.\frac{8}{17}+\frac{4}{9}.\frac{7}{17}=\frac{40}{153}+\frac{28}{153}=\frac{68}{151}=\frac{4}{9}.$$ **Ans.(3)**

4. Since a cube has 6 faces and a tetrahedron of x^k, where k < 5 in the expansion of
$E = (x + x^2 + x^3 + x^4 + x^5 + x^6) (x + x^2 + x^3 + x^4)$
Coeff. of x^k in E = 0, where k = 0 or k = 1
Coeff. of x^2 in E = 1
Coeff. of x^3 in E = 2
Coeff. of x^4 in E = 3
∴ Sum of coeff. of (where k < 5) in E = (1 + 2 + 3) = 6
∴ Favourable number of cases = (24 − 6) = 18

Hence, required probability $= \frac{18}{24}=\frac{3}{4}.$ **Ans.(2)**

5. The sample space S = {1, 2, 3, 4, 5, 6} × {1, 2, 3, 4, 5, 6}
∴ n(S) = 36 and let E be the event getting the sum of digits on dice equal to 7, then E = {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}
∴ n(E) = 6
p = Probability of getting the sum 7

$$p = \frac{6}{36}=\frac{1}{6},$$

∴ q = 1 − p

$$= 1-\frac{1}{6} = \frac{5}{6}$$

∴ Probability of not throwing the sum 7 in first m trials = q^m

$$\therefore P(\text{at least one 7 in m throws}) = 1 - q^m = 1-\left(\frac{5}{6}\right)^m$$

According to the question, $1-\left(\frac{5}{6}\right)^m > 0.95 \Rightarrow \left(\frac{5}{6}\right)^m < 1-0.95$

$$\Rightarrow \left(\frac{5}{6}\right)^m < 0.05 \Rightarrow \left(\frac{5}{6}\right)^m < \frac{1}{20}$$

Taking logarithm,
$\Rightarrow m \{\log_{10}5 - \log_{10}6\} < \log_{10} 1 - \log_{10}20$
$\Rightarrow m \{1- \log_{10} 2 - \log_{10}2 - \log_{10}3\} < 0 - \log_{10}2 - \log_{10}10$

$\Rightarrow m\ \{1 - 2\log_{10}2 - \log_{10}3\} < -\log_{10}2 - 1$

$\Rightarrow m\ \{1 - 0.6020 - 0.4771\} < -\ 0.3010 - 1$

$\Rightarrow -\ 0.079\ m < -\ 1.3010$

$\Rightarrow m > \dfrac{1.3010}{0.079} = 16.44$

$\therefore$ m > 16.44. **Ans.(3)**

6. Let T : A speaks the truth and F : A does not speak the truth.

$\therefore P[T]\ \dfrac{1}{3}, P[F] = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

Let E denote the event that A makes a statement. We have to find P[T | E]. By Baye's formula,

$$P[T|E] = \frac{P[T]P[E|T]}{P[T]P[E|T] + P[F]P[E|F]}, \qquad .... (1)$$

where P[E | T] is the probability that D speaks truth to C and C speaks truth to B and B speaks truth OR D speaks truth to C and C speaks falsehood to B and B speaks falsehood OR D speaks falsehood to C and C speaks truth to B and B speaks falsehood OR D speaks falsehood to C and C speaks falsehood to B and B speaks truth.

We are given that each of the four people speak the truth (falsehood) with probability $\dfrac{1}{3}\left(\dfrac{2}{3}\right)$.

$$\therefore P[E|T] = \frac{1}{3}.\frac{1}{3}.\frac{1}{3} + \frac{1}{3}.\frac{2}{3}.\frac{2}{3} + \frac{2}{3}.\frac{1}{3}.\frac{2}{3} + \frac{2}{3}.\frac{2}{3}.\frac{1}{3} = \frac{13}{27}.$$

$$\therefore P[E|F] = 1 - P[E|T] = 1 - \frac{13}{27} = \frac{14}{27}.$$

Hence, by (1), $P[T|E] = \dfrac{\frac{1}{3}.\frac{13}{27}}{\frac{1}{3}.\frac{13}{27} + \frac{2}{3}.\frac{14}{27}} = \dfrac{13}{41}$. **Ans.(1)**

7. Given, mean np = 2

and variance, npq = 1 $\Rightarrow$ 2q = 1 $\Rightarrow$ q = 1/2

$\therefore$ p = 1 − q = 1 − 1/2 = $\dfrac{1}{2}$

$\therefore$ n = 4

$\therefore$ The binomial distribution is $\{(1.2) + (1.2)\}^4$

Now, P(X > 1) = 1 − {P (X = 0) + P(X = 1)}

$= 1 - (1/2)^4 - {}^4C_1.(1/2^4)$

$= 1 - {}^nC_0\, p^0 q^n = 1 - q^n = 1 - \left(\dfrac{5}{6}\right)^n$

$$\therefore 1 - \left(\frac{5}{6}\right)^n > 0.95 \Rightarrow \left(\frac{5}{6}\right)^n < (1 - 0.95) = 0.05 = \frac{1}{20}.$$

$\therefore n[\log_{10}5 - \log_{10}6] < \log_{10}1 - \log_{10}20$

or $n[\log_{10}10 - \log_{10}2 - \log_{10}2 - \log_{10}3] < -\log_{10}2 - \log_{10}10$

or $n[1 - 2\log_{10}2 - \log_{10}3] < -1 - \log_{10}2$

or $-0.0791\ n < -1.3010$

$\therefore n > \dfrac{1.3010}{0.0791} = 16.44$

Hence, the least number of trials = 17. **Ans.(1)**

8. The sample space associated with the random experiment is given by

S = {TTTT, TTTH, TTHT, THTT, HTTT, TTHH, THTH, THHT, HTHT, HTTH, HHTT, THHH, HTHH, HTHH, HHTH, HHHT, HHHH}

Now, {X = 0} = {TTTT, TTTH, TTHH, THHH, HHHH}

{X = 2} = {HTHT}

You can check up that the remaining outcomes belong to {X = 1}. Thus, P(X = 0) = 5/16, P(X = 1) = 10/16 and P(X = 2) = 1/16. Therefore, the probability distribution of X is given by

x :	0	1,	2
P(X = x) :	$\frac{5}{16}$	$\frac{10}{16}$	$\frac{1}{16}$

Now mean of X = E(X) = $0\left(\dfrac{5}{16}\right) + 1\left(\dfrac{10}{16}\right) + 2\left(\dfrac{1}{16}\right) = \dfrac{12}{16} = \dfrac{3}{4}$

and $E(X^2) = 0^2\left(\dfrac{5}{16}\right) + 1^2\left(\dfrac{10}{16}\right) + 2^2\left(\dfrac{1}{16}\right) = \dfrac{14}{16} = \dfrac{7}{8}$

Therefore var (X) = $E(X^2) - E(X)^2 = \dfrac{7}{8} = \left(\dfrac{3}{4}\right)^2 = \dfrac{5}{16}$. **Ans.(1)**

9. For a particular game, let A_i (B_i) denote the number of heads obtained by A(B) is i when he tosses two (three) fair coins. A will win a particular gme under one of the following mutually exclusive ways: (i) A_1 and B_0 occur, (ii) A_2 and B_0 occur; (iii) A_2 and B_1 occur. Therefore, P(wins a particular game)

$= P[(A_1 \cap B_0) \cup (A_2 \cap B_0) \cup (A_2 \cap B_1)$

$= P(A_1 \cap B_0) + P(A_2 \cap B_0) + P(A_2 \cap B_1)$

$= P(A_1)\,P(B_0) + P(A_2)\,P(B_0) + P(A_2)\,P(B_1)$

$$= \left(\frac{2}{4}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{8}\right) = \frac{2+1+3}{32} = \frac{6}{32} = \frac{3}{16}$$

Now, A and B tie a particular game under the following mutually exclusive ways :

(i) A_0 and B_0 occur; (ii) A_1 and B_1 occurs;

(ii) A_1 and B_1 occurs; (i) A_2 and B_2 occur. Thus, P(A and B tie a particular game)

$= P[(A_0 \cap B_0) \cup (A_1 \cap B_1) \cup (A_2 \cap B_2)]$

$= P(A_0 \cap B_0) + P(A_1 \cap B_1) + P(A_2 \cap B_2)$

$= P(A_0)\,P(B_0) + P(A_1)\,P(B_1) + P(A_2)\,P(B_2)$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{8}\right) + \left(\frac{2}{4}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{8}\right) = \frac{1+6+3}{32} = \frac{5}{16}$$

Thus, P(A wins the game)

$$= \frac{3}{16} + \frac{5}{16} \times \frac{3}{16} + \left(\frac{5}{16}\right)^2 \times \frac{3}{16} + ... = \frac{3/16}{1-(5/16)} = \frac{3}{11}$$ **Ans.(2)**

10. Here random experiment is : formation of a 9 digit number with the digit 1, 2, 3,... 9 when no digit is repeated.

Let S = the sample space

and E = the event that the number formed is divisible by 11.

Thus n (S) = total number of 9 digit numbers formed with the digits 1, 2, 3, ..., 9 when no digit is repeated. $= \lfloor 9$

Let the number formed be x and x = $a_1a_2 .... a_9$.

where a_i = the digit at the i th place form left.

We know that a number is divisible by 11 if a − b is divisible by 11,

where a is the sum of the digits at odd places and b is the sum of the digits at even places.

If x is divisible by 11, then

$(a_1, a_3 + ... + a_9) - (a_2 + a_4 + ... + a_8) = 11k$, where $k \in I$ ...(1)

Also here $a_1 + a_2 + a_3 + ... + a_9$ = sum of 1, 2 , 3, ..., 9

$= \dfrac{9 \times 10}{2} = 45$ .... (2)

(2) − (1) $\Rightarrow 2(a_2 + a_4 + a_6 + a_8) = 45 - 11\,k$ .... (3)

Since L.H.S. of (3) is an even number, therefore k must be an odd number.

Also $a_2 + a_4 + a_6 + a_8 \geq 1 + 2 + 3 + 4 = 10$ .... (4)

and $a_2 + a_4 + a_6 + a_8 \leq 9 + 8 + 7 + 6 = 30$ .... (5)

From (3), (4) and (5), we have

$$10 \leq \frac{45 - 11k}{2} \leq 30 \Rightarrow 20 \leq 45 - 11k \leq 60$$

$\Rightarrow -25 \le 11k \le 15 \Rightarrow \frac{25}{11} \ge K \ge -\frac{15}{11} \quad \Rightarrow -\frac{15}{11} \le k \le \frac{25}{11} \Rightarrow k = 1, 1\ [\because k \text{ is odd}]$

$\therefore$ From (3), $a_2 + a_4 + a_6 + a_8 = 28, 28, 17$ .... (6)

Groups of four different integers out of 1, 2, 3, ... 9, whose sum is 28 are:

(i) {9, 8, 7, 4} (ii) {9, 8, 6, 5}

These digits should be put at even places

Groups of four different integers out of 1, 2, 3, ..., 9 whose sum is 17 are :

(i) {1, 2, 6, 8} (iv) {1, 2, 5, 9}
(vii) {1, 3, 6, 7}

(ii) {1, 3, 5, 8} (v) {1, 3, 4, 9}
(viii) {1, 4, 5, 7} .... (8)

(iii) {2, 3, 5, 7} (vi) {2, 3, 4, 8}
(ix) {2, 4, 5, 6}

These digits should be put at even places.

$\therefore$ From (7) and (8), $n(E) = 2\lfloor 4 \lfloor 5 + 9 \lfloor 4 \lfloor 5 = 11 \lfloor 4 \lfloor 5$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{11\lfloor 4\lfloor 5}{\lfloor 9} = \frac{11}{126}$. **Ans.(1)**

SET XV

1. We have first term $T_1 = 121 = 10^2 + 2 \times 10 + 1$

Second term $T_2 = 12321 = 10^4 + 2 \times 10^3 + 3 \times 10^2 + 2 \times 10 + 1$

Third term

$T_3 = 1234321 = 10^6 + 2 \times 10^5 + 3 \times 10^4 + 4 \times 10^3 + 3 \times 10^2 + 2 \times 10 + 1$

....

nth term $T_n = 1\ 2\ 3\ .....\ n\ (n+1)\ n\ .....4321$

$= 1 \times 10^{2n} + 2 \times 10^{n-1} + 3 \times 10^{2n-2} + ... + n \times 10^{n+1} + (n+1) \times 10^n$

$+ n \times 10^{n-1} + (n-1) \times 10^{n-2} + ... + 3 \times 10^2 + 2 \times 10 + 1$

$= 10^{2n}\left(1 + 2\left(\frac{1}{10}\right) + 3\left(\frac{1}{10}\right)^2 + ... + n\left(\frac{1}{10}\right)^{n-1}\right) + (1 + 2 \times 10 + 3 \times 10$

$+ ..... + n \times 10^{n-1} + (n+1) \times 10^n)$

$= 10^{2n} S_1 + S_2$ (say) ....(1)

$\therefore\ S_1 = 1 + 2\left(\frac{1}{10}\right) + 3\left(\frac{1}{10}\right)^2 + ... + (n-1)\left(\frac{1}{10}\right)^{n-2} + n\left(\frac{1}{10}\right)^{n-1}$

$\therefore\ \frac{1}{10}S_1 = 0 + \left(\frac{1}{10}\right) + 2\left(\frac{1}{10}\right)^2 + ... + (n-1)\left(\frac{1}{10}\right)^{n-1} + n\left(\frac{1}{10}\right)^n$

Subtracting, we get

$\frac{9}{10}S_1 = 1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + ... + \left(\frac{1}{10}\right)^{n-1} - n\left(\frac{1}{10}\right)^n$

$= \frac{1.\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{\left(1 - \frac{1}{10}\right)} - n\left(\frac{1}{10}\right)^n \Rightarrow S_1 = \frac{100}{81}\left(1 - \frac{1}{10^n}\right) - \frac{10n}{9.10^n}$

$\Rightarrow S_1 = \frac{10^2}{81}\left(1 - \frac{1}{10^n}\right) - \frac{90n}{81.10^n}$ .... (2)

and $\because S_2 = 1 + 2(10) + 3(10)^2 + ... + n(10)^{n-1} + (n+1)(10)^n$

$\therefore 10S_2 = (10) + 2(10)^2 + ... + n(10)^n + (n+1)10^{n+1}$

Subtracting, we get

$-9S_2 = 1 + 10 + (10)^2 + ... + (10)^n - (n+1)(10)^{n+1}$

$= \frac{10^{n+1} - 1}{10 - 1} - (n+1)10^{n+1}$

$\therefore\ S_2 = \frac{1 - 10^{n+1}}{81} + \frac{(n+1)\,10^{n+1}}{9}$ .... (3)

Substituting the values of S_1 and S_2 from (2) and (3) in (1), we get

$T_n = 10^{2n}.\frac{10^2}{81}\left(1 - \frac{1}{10^n}\right) - \frac{90n.10^{2n}}{81.10^n} + \frac{1 - 10^{n+1}}{81} + \frac{(n+1)10^{n+1}}{9}$

$= \frac{1}{81}[10^{2n+2} - 10^{n+2} + 9n\,10^{n+1} + 1 - 10^{n+1} + 9(n+1)10^{n+1}]$

$= \frac{1}{81}[10^{2n+2} - 10.10^{n+1} + 1 + 8.10^{n+1}]$

$= \frac{1}{81}[10^{2n+2} - 2.10^{n+1} + 1] = \left(\frac{10^{n+1} - 1}{9}\right)^2$

Since sum of digits of $10^{n+1} - 1$ is divisible by 9.

$\therefore\ \frac{10^{n+1} - 1}{9}$ is a positive integer

Thus T_n is a perfect square. **Ans.(1)**

2. Give $x_1, x_2, x_3, ..., x_n$ are in H.P.

$\therefore\ \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, ..., \frac{1}{x_n}$ are in A.P.

Let D be the common difference of the A.P. then

$\frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = ... \frac{1}{x_n} - \frac{1}{x_{n-1}} = D$

$\therefore \Rightarrow \frac{x_1 - x_2}{x_1 x_2} = \frac{x_2 - x_3}{x_2 x_3} = ... = \frac{x_{n-1} - x_n}{x_{n-1}\,x_n} = D$

$\Rightarrow\ x_1 x_2 = \frac{x_1 - x_2}{D}, x_2 x_3 = \frac{x_2 - x_3}{D}, ..., x_{n-1}\,x_n = \frac{x_{n-1} - x_n}{D}$

Adding all such expressions we get

$\Rightarrow x_1x_2 + x_2x_3 = ... + x_{n-1}\,x_n = \frac{x_1 - x_n}{D}$

$\Rightarrow x_1x_2 + x_2x_3 + ... + x_{n-1}\,x_n = \frac{x_1 x_n}{D}\left(\frac{1}{x_n} - \frac{1}{x_1}\right)$

$= \frac{x_1 x_n}{D}\left(\frac{1}{x_1} + (n-1)\,D - \frac{1}{x_1}\right) = \frac{x_1 x_n}{D}[(n-1)D]$

$= (n-1)\,x_1 x_n$

Hence $x_1 + x_2 + x_2 x_3 + ... + x_{n-1}\,x_n = (n-1)\,x_1 x_n$. **Ans.(1)**

3. Let p and (p + 1) be removed number from 1, 2, ..., n then sum of remaining numbers $= \frac{n(n+1)}{2} - (2p+1)$

From given condition $\frac{105}{4} = \frac{\frac{n(n+1)}{2} - (2p+1)}{n-2}$

$\Rightarrow 2n^2 - 103n - 8p + 206 = 0$

Since n and p are integers so n must be even let n = 2r

we get $p = \frac{4r^2 + 103(1-r)}{4}$

Since p is an integer then (1 – r) must be divisible by 4. Let r = 1 + 4t, we get $n = 2 + 8t$ and $p = 16t^2 - 95t + 1$, Now $1 \le p < n$

$\Rightarrow 1 \le 16r^2 - 95t + 1 < 8t + 2$

$\Rightarrow t = 6 \Rightarrow n = 50$ and $p = 7$

Hence removed numbers are 7 and 8. **Ans.(2)**

4. Let 1st term of the rth group is T_r, and the 1st terms of all rows are 1. 2. 4, 8,.....respectively.

$\therefore T_r = 1.2^{r-1} = 2^{r-1}$

Hence the sum of the numbers the rth group is

$$= \frac{2^{r-1}}{2}\{2.2^{r-1} + 2^{r-1} - 1).1\}$$

($\because$ no. of terms in rth group is 2^{r-1})

$= 2^{r-2}\{2^r + 2^{r-1} - 1\}$

$\therefore$ Sum of the numbers in the nth group is $2^{n-2}[2^n + 2^{n-1} - 1]$. **Ans.(4)**

5. General term can be written as $T_n = \frac{n^2}{500 + 3n^3}$

then $\frac{dT_n}{d_n} = \frac{n(1000 - 3n^3)}{(500 + 3n^3)^2}$

For max. or min. $T_n \Rightarrow \frac{dT_n}{d_n} = 0$

$\therefore \; n = \left(\frac{1000}{3}\right)^{1/3}$ Now $6 < \left(\frac{1000}{3}\right)^{1/3} < 7$

Hence T_7 is largest term. So largest term in the given sequence is $\frac{49}{1529}$

Ans.(3)

6. We have a, b, c are in A.P.

$\Rightarrow 2b = a + c$ .... (1)

α, β, γ are in H.P.

$\Rightarrow \; \beta = \frac{2\alpha\gamma}{\alpha + \gamma}$ ....(2)

$a\alpha, b\beta, c\gamma$ are in G.P.

$\Rightarrow b^2\beta^2 = a\alpha\, c\gamma$ .... (3)

Substituting the values of b and β from (1) and (2), in (3) we get

$$\Rightarrow \left(\frac{a+c}{2}\right)^2 \left(\frac{2\alpha\gamma}{\alpha+\gamma}\right)^2 = a\alpha c\gamma$$

$$\Rightarrow \frac{a^2 + c^2 + 2ac}{ac} = \frac{\alpha^2 + \gamma^2 + 2\alpha\gamma}{\alpha\gamma}$$

$$\Rightarrow \frac{a^2 + c^2}{ac} + 2 = \frac{\alpha^2 + \gamma^2}{\alpha\gamma} + 2 \Rightarrow \frac{a^2 + c^2}{ac} = \frac{\alpha^2 + \gamma^2}{\alpha\gamma}$$

$\Rightarrow \alpha\gamma a^2 + \alpha\gamma c^2 = ac\alpha^2 + ac\gamma^2$

$\Rightarrow a\alpha(a\gamma - c\alpha) - c\gamma(a\gamma - c\alpha) = 0$

$\Rightarrow (a\gamma - c\alpha)(a\alpha - c\gamma) = 0$

$a\alpha - c\gamma \neq 0$ ($\because a\alpha, c\gamma$ are distinct given)

$\therefore \; a\gamma - c\alpha = 0$

$\Rightarrow a\gamma = c\alpha$ ....(4)

using this in (3), $b^2\beta^2 = a^2\gamma^2$

$\Rightarrow b\beta = a\gamma$ ....(5)

from (4) and (5), $a\gamma = b\beta = c\alpha$

$\Rightarrow \frac{a}{(1/\gamma)} = \frac{b}{(1/\beta)} = \frac{c}{(1/\alpha)} \Rightarrow a : b : c = \frac{1}{\gamma} : \frac{1}{\beta} : \frac{1}{\alpha}$. **Ans.(2)**

7. Given $f(x) = x^3 + 3x - 9$

$\therefore \quad f'(x) = 3x^2 + 3$

Hence $f'(x) > 0$ in $[-5, 3]$

$\therefore \; f(-5) = (-5)^3 + 3(-5) - 9 = -149$

and $f(3) = 3^3 + 3.3 - 9 = 27$

Hence least value of f (x) is –149 and greatest value of f (x) is 27.

Let a, ar, ar^2, ...be a G.P. with common ratio $|r| < 1$

($\because$ given infinitely G.P.)

and also given $S_\infty = 27$

$\frac{a}{1-r} = 27$ ....(1)

and $a - ar = f'(0)$

$\Rightarrow a(1 - r) = f'(0) = 3 \; \{\because f'(0) = 3\}$

$\therefore a(1 - r) = 3$ .... (2)

from (1) and (2), we get $(1 - r)^2 = \frac{1}{9}$

$\Rightarrow \; 1 - r = \pm\frac{1}{3} \;\therefore\; r = 1 \pm \frac{1}{3}$

r = 4/3, 2/3 ($\because |r| < 1$)

$r \neq 4/3$. Hence r = 2/3. **Ans.(4)**

8. $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ [$\because a < 1$]

$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$ [$\because b < 1$]

$z = \sum_{n=0}^{\infty} (ab)^n = \frac{1}{1-ab}$ [$\because ab < 1$ since $a < 2, b < 1$]

$\therefore 1 - a = \frac{1}{x} \Rightarrow a = 1 - \frac{1}{x} = \frac{x-1}{x}$

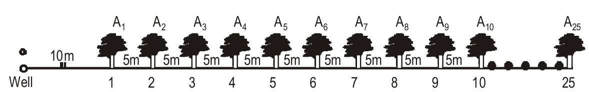
$1 - b = \frac{1}{y} = \frac{y-1}{y}$

$1 - ab = \frac{1}{z} \Rightarrow ab = 1 - \frac{1}{z} = \frac{z-1}{z}$

$\frac{x-1}{x} . \frac{y-1}{y} = \frac{z-1}{z} \Rightarrow \frac{xy - (x+y) + 1}{xy} = \frac{z-1}{z}$

$\Rightarrow \; xyz - (x + y)z + z = xyz - xy$

$\Rightarrow \; xy + z = (x + y)z = xz + yz$. **Ans.(1)**

9. 

Distance covered by gardener for water to second tree

$D_2 = A_10 + 0A_2 = 10 + 15 = 25$ metre

Distance covered by gardener for water to third tree $D_3 = A_20 + 0A_3$

= 15 + 20 = 35 metre

Distance covered by gardener for water to fourth tree

$D_4 = A_30 + 0a_4 = 20 + 25 = 45$ metre

Hence distance covered by gardener for water to all the trees

D = 10 + 25 + 35 + 45 + ... to 25 terms

= 10 + (25 + 35 + 45 + ... to 24 terms)

$= 10 + \frac{24}{2}\{2.25 + (24 - 1)10\}$

= 10 + 12 {50 + 230}

= 10 + 12.280 = 10 + 3360

= 3370 metre. **Ans.(3)**

10. Let the three digit be a, ar, ar^2 then according to hypothesis

$100a + 10ar + ar^2 + 792 = 100ar^2 + 10ar + a$

$\Rightarrow a(r^2 - 1) = 8$ ....(1)

and a, ar + 2, ar^2 are in A.P.

then $2(ar + 2) = a + ar^2$

$\Rightarrow a(r^2 - 2r + 1) = 4$ ....(2)

Dividing (1) by (2),

then $\dfrac{a(r^2-1)}{a(r^2-2r+1)} = \dfrac{8}{4}$

$\Rightarrow \dfrac{(r+1)(r-1)}{(r-1)^2} = 2 \;\Rightarrow\; \dfrac{r+1}{r-1} = 2$

$\therefore$ r = 3 from (1), a = 1

thus digits are 1, 3, 9 and so the required number is 931. **Ans.(1)**

SET XVI

1. Denoting A_1, B_1, A_2 and B_2 for their taking out the ball, a chart is made to denote the winner

		A_1	B_1	A_2	B_2	No. of ways
1.	**points** : number on the ball sum	1 Even (1 of 3) Even	1 Even (1 of 2) Even	0 odd (1 of 3) odd	2 odd (1 of 2) Even	${}^3C_1 \times {}^2C_1 \times {}^3C_1 \times {}^2C_1 = 36$
2.	**points** : number on the ball sum	1 odd (1 of 3) odd	1 odd (1 of 2) Even	0 Even (1 of 3) Even	2 Even (1 of 2) Even	${}^3C_1 \times {}^2C_1 \times {}^3C_1 \times {}^2C_1 = 36$
3.	**points** : number on the ball sum	1 Even (1 of 3) Even	2 odd (1 of 3) odd	0 odd (1 of 2) Even	–	${}^3C_1 \times {}^3C_1 \times {}^2C_1 = 18$
4.	**points** : number on the ball sum	1 Even (1 of 3) Even	1 Even (1 of 2) Even	2 Even (1 of 1) Even	–	${}^3C_1 \times {}^2C_1 \times {}^1C_1 = 6$

Total number of ways in which the game can be won when A starts the game

= 36 + 36 + 18 + 6 = 96. **Ans.(3)**

2. Let A = $\{a_1, a_2, a_3, \ldots, a_n\}$

The two elements P and Q such that $P \cap Q$ can be chosen out of n is ${}^nC_2$ ways a general element of A must satisfy one of the following possibilities : (Here general element be $a_i (1 \le i \le n)$)

(i) $a_i \in P$ and $a_i \in Q$

(ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$

(iv) $a_i \notin P$ and $a_i \notin Q$

Let $a_1, a_2 \in P \cap Q$

there is only one choice each of them (i.e., (i) choice), and three choices (ii), (iii) and (iv) for each of remaining (n – 2) elements.

$\therefore$ Number of ways of remaining elements

$= 3^{n-2}$

Hence number of ways in which $P \cap Q$ contains exactly two elements

$= {}^nC_2 \times 3^{n-2}$. **Ans.(2)**

3. The numbers will be five digit beginning with 2, 3, 4 or 5.

So, the ten thousands places can be filled in 4 ways.

Each of thousands, hundreds and tens places can be filled in 10 ways.

So the first four place can be filled in 4 × 10 × 10 × 10 ways.

After filling these the sum digits used is either even or odd.

$\therefore$ the last place can be filled in 5 ways.

($\because$ if the sum of the digits is even, one of the digits 0, 2, 4, 6, 8 will be used and if the sum of the digits is odd, one of digits 1, 3, 5, 7, 9 will be used).

$\therefore$ the required numbers = 4 × 10 × 10 × 10 × 5 = 20,000. **Ans.(1)**

4. The required number of ways = The number of ways in which 3n different things can be divided in 3 equal groups

= The number of ways to distribute 3n different things equally among three persons

$= \dfrac{3n!}{3!\,(n!)^3} = \dfrac{3n!}{6\,(n!)^3}$. **Ans.(3)**

5. Each of the digits 1, 2, 3 or 4 occurs in Unit's place in

$4.4.4 = 4^3$ nos.

($\because$ 4 choices each for Ten's, Hundred's & Thousand's places, as repetitions allowed).

$\therefore$ Sum of the values of Units in all the non.

$= 4^3(1 + 2 + 3 + 4) . 1 = 640.$

Similarly each of 1, 2, 3 or 4 occurs in Ten's place in

$4.\ 4.\ 4 = 4^3$ nos.

($\because$ 4 choices each for Unit's H.'s Thou.'s places)

$\therefore$ Sum of the values of Tens in all the nos.

$= 4^3 (1 + 2 + 3 + 4). 10 = 6400$

Similarly, Sum of the values of Hundreds in all the nos.

$= 4^3 (1 + 2 + 3 + 4). 100 = 64000$

Sum of the values of Thousands in all the nos. $= 4^3 (1 + 2 + 3 + 4)$.

1000 = 640000 [Total = 711040]. **Ans.(1)**

6. Aggregate of marks = 50 × 3 + 100 = 250

$\therefore$ 60% of the aggregate = 150

Now the number of ways of getting 150 marks in aggregate = coefficient of x^{150} in

$(x^0 + x^1 + x^2 + \ldots + x^{50})^3 (x^0 + x^1 + x^2 + \ldots + x^{100})$

= coefficient of x^{150} in $(1 - x^{51})^3 (1 - x^{101}) (1 - x)^{-4}$

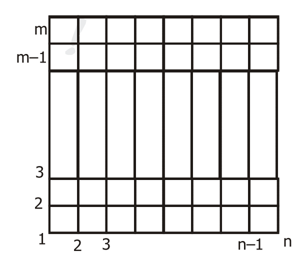
= coefficient of x^{150} in $(1 - 3x^{51} + 3x^{102} - x^{153}) (1 - x^{101}) (1 - x)^{-4}$

= coefficient of x^{150} in $(1 - 3x^{51} - x^{101} + 3x^{102}) (1 + {}^4C_1 x + {}^5C_2 x^2 + \ldots)$

$= {}^{153}C_{150} - 3.{}^{102}C_{99} - {}^{52}C_{49} + 3.{}^{51}C_{48}$

= 110556. **Ans.(3)**

7. To form 1 × 1 squares, we will have to select two consecutive horizontal lines and two consecutive vertical lines. This can be done in (m – 1) (n – 1) ways.



To form 2 × 2 squares, we will have to select two horizontal lines having distance between them 2. Similarly two vertical lines, having distance between them 2. This can be done in (m – 2) (n – 2) ways.

$\therefore$ Total no. of ways

= (m – 1) (n – 1) + (m – 2) (n– 2) + ... (m – m + 1) (n – m + 1)

$= \displaystyle\sum_{r=1}^{m-1} (m-r)(n-r) = \sum_{k=1}^{m-1} \left(mn - r(m+n) + r^2\right)$

$= mn\,(m-1) - (m+n)\,\dfrac{1}{2}\,m(m-1) + \dfrac{1}{6}\,m\,(m-1)\,(2m-1)$

$= \dfrac{1}{6}\,m(m-1)\,(3n - m - 1)$. **Ans.(2)**

8. No. of ways in which 2 letters be rightly placed and 3 letters are wrongly placed are

$= {}^5C_2 \,.\, Q_3 = 10 \,.\, 3!\left(\frac{1}{2!}-\frac{1}{3!}\right) = 10\times 2 = 20$. **Ans.(4)**

9. The total number of seats

= 1 grand father + 5 sons and daughters + 8 grand children

= 14

The grand children with to occupy the 4 seats on either side of the table

= 4 ! ways

= 24 ways

and grand father can occupy a seat in (5 – 1) ways = 4 ways

(Since 4 gaps between 5 sons and daughters) and the remaining seat can be occupied in 5 ! ways

= 120 ways (5 seats for sons and daughters)

Hence required number of ways, By the principle of multiplication law

= 24 × 4 × 120

= 11520. **Ans.(3)**

10. Since $x \geq 1$, then $y \geq 2$ $(\therefore x < y)$

If y = n then n take the values from 1 to n – 1 and z can take the values from 0 to n–1 (i.e., n values) thus for each values of y $(2 \leq y \leq 9)$, x and z take n (n – 1) values.

Hence the three digit numbers are of the form xyz

$$= \sum_{n=1}^{9} n(n-1) \quad \{\because \ \Sigma 1.(1-1) = 0\}$$

$$= \sum_{n=1}^{9} n^2 - \sum_{n=1}^{9} n$$

$$= \frac{9(9+1)(18+1)}{6} - \frac{9.(9+1)}{2}$$

= 285 – 45 = 240. **Ans.(1)**

SET XVII

1. Let $A = \{a_1, a_2, \ldots, a_n\}$

Let S be the sample space and E_1 be the event that $P_i \cap P_j = \phi$ for $i \neq j$ and E_2 be the event that $P_1 \cap P_2 \cap \ldots\ldots \cap P_m = \phi$.

$\therefore$ Number of subsets of A = 2^n

$\therefore$ each $P_1, P_2, \ldots\ldots, P_m$ can be selected in 2^n ways.

$\therefore$ n(S) = total number of selections of $P_1, P_2, \ldots\ldots, P_m$

$= (2^n)^m$

$= 2^{mn}$

When $P_i \cap P_j = \phi$ for $i \neq j$, element of A either does not belong to may of subsets, or it belongs to at most one of them. Therefore, there are m + 1 choices for each element

$\therefore$ n $(E_1) = (m+1)^n$.

$\therefore$ Required probability, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{(m+1)^n}{2^{mn}}$. **Ans.(1)**

2. Given P(A) = a, ...(1)

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = b$

or $P(\bar{A})P(\bar{B})P(\bar{C}) = b$

$\{1 - P(A)\}\{1 - P(B)\}\ \underline{\{1 - P(C)\}} = b$ ....(2)

$P(A \cap B \cap C) = c$

or $1 - P(A \cap B \cap C) = c$

or 1 – P(A)P(B) P(C) = c ....(3)

and $P(\bar{A})P(\bar{B})P(C) = p$

[1 – P(A)][1 – P(B)]P(C) = p .....(4)

Let P(A) = x, P(B) = y and P(C) = z

then (1), (2), (3) and (4) will be reduced to

x = a, (1 – x) (1 – y) (1 – z) = b, 1 – xyz = c

z (1 – x) (1 – y) = p

From these equation is,

$$\frac{(1-x)(1-y)(1-z)}{z(1-x)(1-y)} = \frac{b}{p}$$

$$\Rightarrow \frac{1-z}{z} = \frac{b}{p} \quad \Rightarrow \frac{1-z}{z} + 1 = \frac{b}{p} + 1 \quad \Rightarrow \frac{1}{z} = \frac{b+p}{p}$$

$\therefore$ z = p/(b + p)

Since z (1 – x) (1 – y) = p

$$\therefore 1 - y = \frac{p}{z(1-x)}$$

$$= \frac{p}{\frac{p}{(b+p)}(1-a)}$$

$$\text{or } y = 1 - \frac{p(b+p)}{p(1-a)} = \frac{1-a-b-p}{(1-a)}$$

Putting these values of x, y and z in 1 – xyz = c, we get

$$1 - a.\ \frac{(1-a-b-p)}{(1-a)} . \frac{p}{(b+p)} = c$$

$\Rightarrow$ (1 – a) (b + p) – a (1 – a – b – p) p = c(1 – a)(b + p)

$\Rightarrow$ (1 – a) (b + p) – a(1 – a – b – p)p = c (1 – a) (b + p)

$\Rightarrow ap^2 + [ab - (1-a)(a+c-1)]p + b(1-a)(1-c) = 0$. **Ans.(2)**

3. Let 3n consecutive integers (start with the integer m) are

m, m +1, m + 2, ..., m + 3n – 1

Now we write these 3n numbers in 3 rows as follows

m, m + 3, m + 6, ... m + 3n – 3

m + 1, m + 4, m + 7, ..., m + 3n – 2

m + 2, m + 5, m + 8, ..., m + 3n – 1

The total number of ways of choosing 3 integers out of 3n is

$${}^{3n}C_3 = \frac{3n(3n-1)(3n-2)}{12.3}$$

$$= \frac{n(3n-1)(3n-2)}{2}$$

The sum of the three numbers shall be divisible by 3 if and only if either all the three numbers are from the same row or all the three numbers are from different rows. Therefore, the number of favourable ways is

$$3({}^nC_3) + ({}^nC_1)({}^nC_1)({}^nC_1) = \frac{3n(n-1)(n-2)}{12.3} + n^3$$

$$= \frac{3n^3 - 3n^2 + 2n}{2}$$

$\therefore$ The required probability

$$= \frac{\text{Favourable ways}}{\text{Total ways}} = \frac{\frac{3n^3 - 3n^2 + 2n}{2}}{\frac{n(3n-1)(3n-2)}{2}}$$

$\frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$. **Ans.(1)**

4. Let E be the event of any one cutting a spade in one cut, and let S be the sample space then

$n(E) = {}^{13}C_1$

and $n(S) = {}^{52}C_1$

$$\therefore P(E) = p = \frac{n(E)}{n(S)} = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$$

$$\Rightarrow P(E) = p = \frac{1}{4}$$

$$\therefore P(\overline{E}) = q = 1 - p = \frac{3}{4}$$

The probability of A winning (when A starts the game)

$= p + qqqqp + (qqqq)^2.\ p + ...$ to ∞

$= p + q^4p + q^8p + ...\ \infty$

$$= \frac{p}{1-q^4} \quad \text{(sum of infinite G.P.)}$$

$$= \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^4}$$

$$= \frac{64}{175}$$

$\therefore$ Expectation of A = Rs.350 × probability

$= \text{Rs. } 350 \times \frac{64}{175}$

= Rs.128.

The probability of B winning

$= qp + qqqq\,(qp) + (qqqq)^2\,(qp) + ...\ \infty$

$$= \frac{qp}{1-q^4}$$

$$= \frac{\frac{3}{4}\times\frac{1}{4}}{1-\left(\frac{3}{4}\right)^4}$$

$$= \frac{48}{175}$$

$\therefore$ Expectation of B = Rs.350 × probability

$= \text{Rs.}350 \times \frac{48}{175}$

= Rs.96

the probability of C winning $= qq\,p + qqqq\,(qq\,p) + (qqqq)^2\,qq\,p + ...\ \infty$

$$= \frac{qq\,p}{1-qqqq}$$

$$= \frac{\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}}{1-\left(\frac{3}{4}\right)^4}$$

$$= \frac{36}{175}$$

$\therefore$ Expectation of C = Rs.350 × probability

$= \text{Rs.}350 \times \frac{36}{175}$

= Rs.72

Expectation of D = Rs.350 – (Sum of the expectations A, B, C)

= Rs.350 – (Rs.128 + Rs.96 + Rs.72)

= Rs.54. **Ans.(4)**

5. Let Ei be the event that the integer 2i is drawn and A be the even that an even number is drawn, then (where i = 1, 2, 3, ...n)

$A = E_1 \cup E_2 \cup ... \cup E_n$

$\therefore P(A) = P(E_1 \cup E_2 \cup .... \cup E_n)$

$= P(E_1) + P(E_2) + ... + P(E_n)$ ....(1)

$\{\because E_1, E_2, ..., E_n$ are mutually exclusive$\}$

But given

$P(E_i) \propto \log 2i$

$P(E_i) = c \log 2i$, where c is a constant

$\therefore P(A) = c \log 2 + c \log 4 + c \log 6 + ... + c \log 2n$ [from (1)]

$= c\,[\log 2 + \log 4 + \log 6 + ... + \log 2n]$

$= c \log (2.4.6....2n)$

$= c \log \{2^n.(1.2.3.....n)\}$

$= c \log (2^n.n!)$

$= c \log 2^n + c \log n!$

$= c\,(n \log 2 + \log n!)$

and let B be the event that integer 2 is chosen

also $B = E_2$

$\therefore A \cap B = E_2$ $\{\because E_2 \subseteq A\}$

$\therefore P(A \cap B) = P(E_2) = c \log 2$

$\therefore$ Required probability,

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(E_2)}{P(A)}$$

$$= \frac{c \log 2}{c\{n \log 2 + \log n!\}} = \frac{\log 2}{(n \log 2 + \log n!)}$$. **Ans.(1)**

6. Given that $P(A) = \alpha$, $P(B/A) = P(B'/A') = 1 - \alpha$

thus $P(A') = 1 - P(A)$

$= 1 - \alpha$

and $P(B/A') = 1 - P(B'/A')$

$= 1 - (1 - \alpha)$

$= \alpha$

$$\therefore P(A'/B') = \frac{P(A' \cap B)}{P(B)}$$

$$= \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{P(B) - P(A)P(B/A)}{P(B)}$$

$$= \frac{P(B) - P(A)P(B/A)}{P(B)} \quad \left\{\because P(B/A) = \frac{P(A \cap B)}{P(A)}\right\}$$

$$= \frac{P(B) - \alpha(1-\alpha)}{P(B)} \quad ....(2)$$

But $P(B) = P(A).P(B/A) + P(A').P(B/A')$

$= \alpha.(1 - \alpha) + (1 - \alpha).\alpha$ {from (1)}

$= 2\alpha\,(1 - \alpha)$ .....(3)

Putting the value of P(B) from (3) in (2), then

$$P(A'/B) = \frac{2\alpha(1-\alpha) - \alpha(1-\alpha)}{2\alpha(1-\alpha)}$$

$$= \frac{\alpha(1-\alpha)}{2\alpha(1-\alpha)} = \frac{1}{2}$$

which is independent of α. **Ans.(1)**

7. Let $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ be the seven digits and the remaining two be a_8 and a_9.

Let $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = 9k, k \in I.$ .....(1)

Also $a_1 + a_2 + a_3 + a_4 + ...+ a_9 = 1 + 2 + 3 + 4 + ... + 9$

$= \frac{9 \times 10}{2}$

$= 45$ ....(2)

Subtracting (1) from (2), then

$a_8 + a_9 = 45 - 9k$ ...(3)

Since $a_1 + a_2 + a_3 + ... + a_9$ and $a_1 + a_2 + ... + a_7$ are divisible by 9 if and only if $a_8 + a_9$ is divisible by 9. Let S be the sample space and E be the event that the sum of the digits a_8 and a_9 is divisible by 9.

$\because a_8 + a_9 = 45 - 9k$

Maximum value of $a_8 + a_9 = 17$ and minimum value of $a_8 + a_9 = 3$

$3 \le 45 - 9k \le 17$

$\Rightarrow -42 \le -9k \le -28$

$\Rightarrow \frac{42}{9} \ge k \ge \frac{28}{9}$ or $\frac{28}{9} \le k \le \frac{42}{9}$

Hence k = 4 ($\because$ k is positive integer)

$\therefore$ from (3)

$a_8 + a_9 = 45 - 9 \times 4$

$\therefore a_8 + a_9 = 9$

Now possible pair of (a_8, a_9) can be

{(1, 8), (2, 7), (3, 6), (4, 5)}

$\therefore$ E = {(1, 8), (2, 7), (3, 6), (4, 5)}

$n(E) = 4$ & $n(S) = {}^9C_2 = 36$

$\therefore$ Required probability $P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$. **Ans.(3)**

8. Let A, B, C be three independent events having probabilities p, q and r respectively.

Then according to the question, we have

P (only the first occurs) $= P(A \cap \overline{B} \cap \overline{C})$ {A, B, C are independent}

$P(A)P(\overline{B})P(\overline{C})$

$= p(1 - q)(1 - r) = a$ ...(1)

P (only the second occurs) = $P(\overline{A} \cap B \cap \overline{C})$

$P(\overline{A})P(B)P(\overline{C})$

$= (1 - p)q(1 - r) = b$ ...(2)

and P (only the third occurs) = $P(\overline{A} \cap \overline{B} \cap C)$

$= P(\overline{A})P(\overline{B})P(C)$

$= (1 - p)(1 - q)r = c$ ...(3)

Multiplying (1), (2) & (3), then

$pqr\{(1 - p)(1 - q)(1 - r)\}^2 = abc$

or $\frac{abc}{pqr} = [(1-p)(1-q)(1-r)^2 = x^2 \text{ (say)}]$ ....(4)

$\therefore (1 - p)(1 - q)(1 - r) = x$ ....(5)

Dividing (1) by (5), then

$\frac{p}{1-p} = \frac{a}{x}$

or $px = a - ap$

$\therefore p = \frac{a}{(a+x)}$

similarly $q = \frac{b}{(b+x)}$ and $r = \frac{c}{c+x}$

Replacing the values of p, q and r in (4), then

$$\left\{\left(1 - \frac{a}{a+x}\right)\left(1 - \frac{c}{b+x}\right)\left(1 - \frac{c}{c+x}\right)\right\}^2 = x^2$$

$$\Rightarrow \frac{(x^3)^2}{(a+x)^2(b+x)^2(c+x)^2} = x^2 \Rightarrow \frac{x^3}{(a+x)(b+x)(c+x)} = x$$

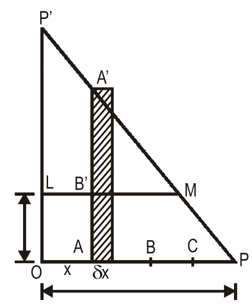
or $(a+x)(b+x)(c+x) = x^2$

Hence x is a root of the equation $(a + x)(b + x)(c + x) = x^2$. **Ans.(1)**

9. The points are as likely to fall in the order O, A, C, P as in the order O, C, A, P. We may therefore suppose that C is to the right of A.

Draw OP' at right angles to OP and equal to it. Complete the figure as in the diagram, where OL = AB' = b.

If δx is small, the number of cases in which the distance of A from O lies between x and $x + \delta x$ and C is in AP, is represented by δx. AP i.e. by the area of the shaded rectangle.



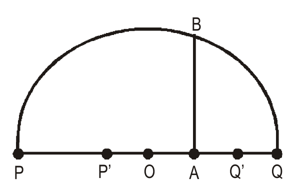
Of these, the favourable cases are those in

which C lies in BP, and their number is represented by the upper part of the shaded rectangle cut off by LM. Hence the total number of cases is represented by area of the triangle of OPP', and the total number of favourable cases by the area of the triangle LMP',

$\therefore$ the required chance = $\frac{\Delta LMP'}{\Delta OPP'} = \left(\frac{a-b}{a}\right)^2$. **Ans.(2)**

10. Let PQ be a diameter of a circle with centre O and radius a. Take a point A at random in PQ.

Let AP = x, AQ = y, then x + y = 2a, and all values of x between 0 and 2a are equally likely.



Draw the ordinate AB, then $AB^2 = AP, AQ = xy$

If P', Q' are the mid points of OP, OQ, the

ordinates at these points are equal to a $\sqrt{\frac{3}{4}}$

Hence AB > a $\sqrt{\frac{3}{4}}$ if and only if, A lies in

P' Q'.

Hence the chance that xy > $\frac{3}{4}$ > a^2 is

$\frac{1}{2}$. **Ans.(1)**

SET XVIII

1. Two numbers out of the set S can be chooseh is ${}^{190}C_2$ ways.

Now if one of the two numbers is zero, then $x^2 + y^2$ is a perfect square. This can be done in 189 ways.

The set {(3, 4), (6, 8), (9,12) ... (141, 188)} has elements whose sum of the squares is a perfect square.

⇒ This can be done in 47 ways same with the set

{(7, 24), ...,(49, 168)}	7 ways
{(8, 15), ...,(64, 180}	8 ways
{(5, 13), ...,(70,182)}	14 ways
{(9, 40), ...,(36, l60)}	4 ways
{(11,60), ...,(33,180)}	3 ways
{(16, 63), ...,(48, 189)}	3 ways
{(15, 112), (17, 144), (19, 180)}	3 ways

Total such subsets are 278.

Hence the required probability is $\frac{278 \times 2}{190 \times 189} = \frac{278}{17955}$. **Ans.(1)**

2. Now let the orthocentre be (x, y)

$\Rightarrow (x-2)^2 + (y-3)^2 = (5-2)^2 + (5-3)^2$

$\Rightarrow x^2 + y^2 - 4x - 6y = 0$

$\Rightarrow x = 2 \pm \sqrt{13-(y-3)^2}$

⇒ y can take the values as 1, 2, 3, 4, 5, 6

⇒ the required probability is $\frac{6}{10} = \frac{3}{5}$

Note that HD = DE here, where H is the orthocentre. **Ans.(1)**

3. The composition of the balls in the red box and in the green box; and the sum suggested in the problem may be one of the following

Red box		Green box		Sum of Green in Red and Red in Green
Red	Green	Green	Red	
0	5	3	6	11
1	4	4	5	9
2	3	5	4	7
3	2	6	3	5
4	1	7	2	3
5	0	8	1	1

Of these the 2nd and the last correspond to the sum being NOT a prime number. Hence, the required probability

$= \frac{{}^6C_1 \times {}^8C_4 + {}^6C_5 \times {}^8C_0}{{}^{14}C_5} = \frac{420+6}{2002} = \frac{213}{1001}$. **Ans.(2)**

4. Let A denote the event that the target is hit when x shells are fired at point I.

we have $P(E_1) = \frac{8}{9}, P(E_2) = \frac{1}{9}$.

$\Rightarrow P(A/E_1) = 1 - \left(\frac{1}{2}\right)^x$ and $P(A/E_2) = 1 - \left(\frac{1}{2}\right)^{21-x}$

$\Rightarrow P(A) = \frac{8}{9}\left[1 - \left(\frac{1}{2}\right)^x\right] + \frac{1}{9}\left[1 - \left(\frac{1}{2}\right)^{21-x}\right]$

$\Rightarrow \frac{dp(A)}{dx} = \frac{8}{9}\left[\left(\frac{1}{2}\right)^x \log 2\right] + \frac{1}{9}\left[-\left(\frac{1}{2}\right)^{21-x} \log 2\right]$

Now we must have $\frac{dp(A)}{dx} = 0$

$\Rightarrow x = 12$, also $\frac{d^2p(A)}{dx^2} < 0$

Hence P(A) is maximum where x = 12. **Ans.(4)**

5. The required probability

= 1 – (probability of the event that the roots of $x^2 + px + q = 0$ are non real.)

The roots of $x^2 + px + q = 0$ will be non-real if and only if $p^2 - 4q < 0$, i.e., $p^2 < 4q$.

We enumerate the possible values of p and q for which this can happen in table.

q	p	Number of pairs of p, q
1	1,	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6
Total		**38**

Thus, the number of possible pairs = 38. Also, the total number of possible pair is 10 × 10 = 100.

∴ the required probability = 1 – 38/100 = 1 – 0.38 = 0.62. **Ans.(3)**

6. There are 11 ways to choose x and 11 ways to choose y If 5 be the sample space then

n(s) = Total number of choosing x and y

= 11 x 11

= 121 The number of different values of y for a given value of x can be determined as follows

when x = 0, we have $|0 - y| \le 5$

$\Rightarrow |y| \le 5$

$\Rightarrow -5 \le y \le 5$

$\Rightarrow 0 \le y \le 5$ {because $y \ge 0$}

gives six values of y, i.e., {0, 1, 2, 3, 4, 5)

when x = 1, we have $|1 - y| \le 5$

$\Rightarrow -5 \le 1 - y \le 5$

$\Rightarrow 5 \ge y - \ge -5$

$\Rightarrow 6 \ge y \ge -4$

$\Rightarrow 0 \le y \le 6$ {because $y \ge 0$}

gives seven values of y, i.e., {0, 1, 2, 3, 4, 5, 6}

When x = 2, we have $|2 - y| \le 5$

$\Rightarrow -5 \le 2 - y \le 5$

$\Rightarrow 5 \ge -2 + y \ge -5$

$\Rightarrow 7 \ge y \ge -3$

$\therefore 0 \le y \le 1$ (since $y \ge 0$)

gives 8 values of y, i.e., {0, 1, 2, 3, 4, 5, 6, 7} similarly we can show that when x equals 3, 4, 5, 6, 7, 8, 9, 10 there are 9, 10, 11, 10, 9, 8, 7, 6; y-values respectively. Let E be the event of favourable cases then

n(E) = 6 + 7 + 8 + 9 + 10 + 11 + 10 + 9 + 8 + 7 + 6 = 91

Hence required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{91}{121}$. **Ans.(4)**

7. The smaple space is S = {–0.50, – 0.49, –0.48, ..., –0.01, 0.00, 0.01, ... 0.49}

Let E be the event that the round off error is at least 10 paise, then E' is the event that a round off error is at most a paise.

$\therefore$ E' = {–0.09, – 0,08, ..., –0.01, 0.00, 0.01, ..., 0.09}

$\therefore$ n(E') = 19 and n(S) = 100

$\therefore\ P(E') = \frac{n(E')}{n(S')} = \frac{19}{100}$

$\therefore$ required probability, P(E) = 1 – P(E') = $1 - \frac{19}{100} = \frac{81}{100}$. **Ans.(3)**

8. Let S be the sample space and E be the event of getting a large number than the previous number.

$\therefore$ n(S) = 6 × 6 × 6 = 216

Now we count the number of favourable ways. Obviously, the second number has to be greater than 1. If the second number is i(i > 1), then the number of favourable ways = (i – 1) × (6 × i)

n(E) = Total number of favourable ways

$= \sum_{i=1}^{6} (i-1)x(6-i)$

= 0 + 1 × 4 + 2 × 3 + 3 × 2 + 4 × 1 + 5 × 0

= 4 + 6 + 6 + 4 = 20

Therefore, the required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{20}{216} = \frac{5}{54}$. **Ans.(3)**

9. Given equation $x + \frac{100}{x} > 50$

$\Rightarrow x^2 - 50x + 100 > 0$

$\Rightarrow (x - 25)^2 > 525$

$\Rightarrow x - 25 < -\sqrt{525}$ or $x - 25 > \sqrt{525}$

$\Rightarrow x < 25 - \sqrt{525}$ or $x > 25 + \sqrt{525}$

As x is a positive integer and $\sqrt{525}$ = 22.91, we must have

$x \le 2$ or $x \ge 48$

Let E be the event for favourable cases and S be the sample space.

E = {1, 2, 48, 49, 50, ...,100}

n(E) = 55

and n(S) = 100

Hence the required probability $P(E) = \frac{n(E)}{n(S)} = \frac{55}{100} = \frac{11}{20}$. **Ans.(1)**

10. Let E_1 : the toss result in a head, E_2 : the toss result in a tail.

A : noted number is 7 or 8.

We have $P(E_1) = 1/2$, $P(E_2) = 1/2$

Also, $P(A \mid E_1) = P(7) + P(8) = \frac{6}{36} + \frac{5}{36} = \frac{11}{36}$

and $P(A \mid E_2) = 2/11$.

Using the total probability rule,

$P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2)$

$= \left(\frac{1}{2}\right)\left(\frac{11}{36}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{11}\right) = \frac{121+72}{792} = \frac{193}{792}$. **Ans.(2)**

SET XIX

1. Number of code words ending with an even integer. In this case, the code word can have any of the numbers 2, 4, 6, 8 at the extreme right position. So, the extreme right position can be filled in 4 ways. Now, next left position can be filled by two English alphabets in $^{26}P_2$ ways.

Hence, the total number of code words which end with an even integer

= 4 × 8 × $^{26}P_2$ = 4 × 8 × 650 = 20800. **Ans.(4)**

2. Since SALIM occupies the second position and the two girls RITA and SITA are always adjacent to each other. So, none of these two girls can occupy the first seat. Thus, first seat can be occupied by any one of the remaining two students in 2 ways. Second seat can be occupied by SALIM in only one way. Now, in

the remaining three seats SITA and RITA can be seated in the following four ways :

	I	II	III	IV	V
1.	X	SALIM	SITA	RITA	X
2.	X	SALIM	RITA	SITA	X
3.	X	SALIM	X	SITA	RITA
4.	X	SALIM	X	RITA	SITA

Now, only one seat is left which can be occupied by the 5th student in one way. Hence, the number of required type of arrangements

= 2 × 4 × 1 = 8. **Ans.(3)**

3. Let the two classes be C_1 and C_2 and the four rows be R_1, R_2, R_3, R_4. There are 16 students in each class. So, there are 32 students. According to the given conditions there are two different ways in which 32 students can be seated :

	R_1	R_2	R_3	R_4
I	C_1	C_2	C_1	C_2
II	C_2	C_1	C_2	C_1

Since the seating arrangement can be completed by using any one of these two ways. So, by the fundamental principle of addition, Total no. of seating arrangements = No. of arrangement in I case + No. of arrangements in II case. Now, 16 students of class C_1 can be seated in 16 chairs in $^{16}P_{16}$ = 16! ways. And, 16 students of class C_2 can be seated in 16 chairs in $^{16}P_{16}$ = 16! ways. Hence, the total no. of seating arrangements = (16! × 16!) + (16! × 16!) = 2 (16! × 16!). **Ans.(1)**

4. In the first group, one question can be selected or can be rejected; so three questions can be dealt with in 2 × 2 × 2 ways, but this includes the case when all three questions have been left; so they can be selected in $2^3 - 1 = 7$ ways. Similarly four questions of the second group can be selected in $2^4 - 1 = 15$ ways. Thus all seven questions can be selected in 15 × 7 = 105 ways; but this includes the case when all questions have been solved; hence leaving that case, total number of ways required is 105 – 1 = 104. **Ans.(2)**

5. We have the following two possibilities :

(I) When Chemistry part I is borrowed. In this case the boy may borrow Chemistry Part II. So, he has to select now two books out of the remaining 7 books of his interest. This can be done in $^7C_2$ ways.

(II) When Chemistry part I is not borrowed : In this case the boy does not want to borrow Chemistry Part II. So, he has to select three books from the remaining 6 books.

This can be done in $^6C_3$ ways. Hence, the required number of ways = $^7C_2 + {^6C_3}$ = 21 + 20 = 41. **Ans.(3)**

6. The selection of 6 balls, consisting of at least two balls of each colour from 5 red and 6 white balls can be done as :

(a) 2 red balls, 4 white balls $^5C_2 \times {}^6C_4$.

(b) 3 red balls, 3 white balls $^5C_3 \times {}^6C_2$.

(c) 4 red balls, 2 white balls $^5C_4 \times {}^6C_2$.

Since the selection can be one of (a), (b), (c).

Hence No. of ways = $^5C_2 \times {}^6C_4 + {}^5C_3 \times {}^6C_3 + {}^5C_4 \times {}^6C_2 = 425$. **Ans.(3)**

7. 52 families have at most 2 children, while 35 families have more than 2 children. The selection of 20 families of which at least 18 families must have at most 2 children can be made as under:

(I) 18 families out of 52 and 2 families out of 35 or

(II) 19 families out of 52 and 1 family out of 35 or

(III) 20 families out of 52.

No. of ways $^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20} \times {}^{35}C_0$. **Ans.(4)**

8. Let the number of green balls be x. Then the number of red balls is 2x.

Let the number of blue balls be y.

Then, $x + 2x + y = 10 \Rightarrow y = 10 - 3x$.

Clearly, x can take values 0, 1, 2, 3. The corresponding values of y are 10, 7, 4 and 1. Thus, the possibilities are (0, 10, 0), (2, 7, 1), (4, 4, 2) and (6, 1, 3) where (r, b, g) denotes the number of red, blue and green balls. Hence no. of ways = 4. **Ans.(3)**

9. We have $x \ge 1$, $y \ge 2$, $z \ge 3$ and $t \ge 0$, where x, y, z, t are integers

$x \ge 1$, $y \ge 2$, $z \ge 3$ and $t \ge 0$.

Let $u = x - 1$, $\nu = y - 2$, $w = z - 3$. Then,

$x \ge 1 \Rightarrow u \ge 0$, $y \ge 2 \Rightarrow \nu \ge 0$, $z \ge 3 \Rightarrow w \ge 0$. Thus,

we have $u + 1 + \nu + 2 + w + 3 + t = 29$, where $u \ge 0$, $\nu \ge 0$, $w \ge 0$, $t \ge 0$

$u + \nu + w + t = 23$

The total number of solutions of this equation is

$^{23+4-1}C_{4-1} = {}^{26}C_3 = 2600$. **Ans.(2)**

10. The number of triangles = Total number of triangles

– No. of triangles having one side common with the octagon

– No. of triangles having two side common with the octagon

$= {}^8C_3 - {}^8C_1 \times {}^4C_1 - 8 = 16$. **Ans.(3)**

SET XX

1. For each question in Part A, the student has three choices:

(i) The student does not attempt the question;

(ii) The student attempts the first part of the question; and

(iii) The student attempts the alternative part of the question.

Therefore, the total number of choices is 3^5. But this includes a choice in which the student does not attempt any question in Part A. Therefore, the total number of choices is $3^5 - 1 = 243 - 1 = 242$. Similarly, we can show there are that there are $2^4 - 1 = 16 - 1 = 15$ choices for Part B. Hence, the number of ways in which the student can attempt the question paper is (242) (15) = 3630. **Ans.(4)**

2. There are 32 places for the teeth in the mouth. For each place, we have two choices, either there is a tooth or there is no tooth at that place. Therefore, the number of ways to fill up 32 places is 2^{32}. As there is no person without a tooth, the maximum population of the country in which no two persons have identical set of teeth is $2^{32} - 1$. **Ans.(1)**

3. When repetitions are allowed, three letters from the English alphabet can be chosen in $26 \times 26 \times 26 = 26^3$ ways, and a three digit number for the car can be chosen in 999 ways. Thus, the number of plates in this case is $(26^3)(999)$. **Ans.(3)**

4. We can arrange r persons on m chairs on a particular side in ^mP_r ways and s persons on m chairs on the other side in ^mP_s ways. We can arrange (2m – r – s) persons on the remaining (2m – r – s) chairs in $^{2m-r-s}P_{2m-r-s}$ ways. Thus, number of ways of arranging the persons subject to the given conditions is $(^mP_r)(^mP_s)\,(^{2m-r-s}P_{2m-r-s})$. **Ans.(4)**

5. The total number of seats required at the table is 1 + m + 2n. The grand children can occupy the n seats on either side of the table in $(^{2n}P_{2n})$ ways. The grandfather can occupy a seat in $^{m-1}P_1$ ways. Therefore remaining seats can be pied in ^mp_m ways. Therefore, the required numbers of ways is

$(^{2n}P_{2n})\,(^mP_m)\,(^{m-1}P_1) = (2n)!\,m!\,(m-1)$. **Ans.(1)**

6. Each of the digits 1, 2, 3, 4 occurs in Unit's place in

$3.\,2.\,1 = 3! = 6$ ways

($\because$ 3 choices for Tens, 2 for Hundred's 1 for Thousands's places as no repition).

$\therefore$ Sum of the values of Units in all the nos.

$= 3!\,(1 + 2 + 3 + 4)\,.\,1 = 60$

Sum of values of Tens in all the nos.

$= 3!\,(1 + 2 + 3 + 4)\,.\,10 = 60$

Sum of the values of Hundreds in all the nos. $3!(1 + 2 + 3 + 4)\,.\,100 = 6000$

Sum of the values of Thousands in all the nos.

$= 3!\,(1 + 2 + 3 + 4).\,1000 = 60000$ [Total = 66660]. **Ans.(3)**

7. No. of ways in which all the letters can be put into wring envelopes is

$$Q_5 = 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 120\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{20}\right) = 44$$. **Ans.(1)**

8. If we keep the toys of Bhawna and Quincy together, the problem is to find the number of ways in which Bhawna can take 4 toys out of 11 toys, not including the number of ways in which she takes her original 4 toys.

This can be done in $^{11}C_4 - 1 = 330 - 1 = 329$ ways. **Ans.(3)**

9. The number of ways to select any number of mangoes

$^5C_0 + {}^5C_1 + {}^5C_2 + \ldots\ldots = {}^5C_5 = 2^5$

The number of ways to select any number of apples

$= {}^4C_0 + {}^4C_1 + \ldots.\ {}^4C_4 = 2^4$.

$\therefore$ the required number of ways to select fruits

$= 2^5 \times 2^4 - 1$ (excluding the way in which 0 mangoes and 0 apples are selected $= 2^9 - 1$. **Ans.(2)**

10. The digits which can be recognised as digits when they are inverted are 0, 1, 2, 5, 6, 8 and 9.

Since a number cannot begin with zero all the numbers having 0 at unit's place should be discarded. For otherwise when read upside down the number will begin with 0. We now list the different possibilities in the following table.

Number of digits	Total number of numbers
1	7
2	$6 \times 6 = 6^2$
3	$6 \times 7 \times 6 = 6^2.7$
4	$6 \times 7 \times 7 \times 6 = 6^2.7^2$
5	$6 \times 7 \times 7 \times 7 \times 6 = 6^2.7^3$
6	$6 \times 7 \times 7 \times 7 \times 7 \times 6 = 6^2.7^4$

Thus, the number of required numbers

$= 7 + 6^2 + 6^2.7 + \ldots..+ 6^2.\,7^4$

$= 7 + 6^2\,\dfrac{(7^5 - 1)}{7 - 1} = 7 + 6\,(7^5 - 1) = 6\,.7^5 + 1 = 100843$. **Ans.(2)**