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Numbers

1. A number is divisible by 2, if its unit's place digit is 0, 2, 4, or 8

2. A number is divisible by 3, if the sum of its digits is divisible by 3

3. A number is divisible by 4, if the number formed by its last two digits is divisible by 4

4. A number is divisible by 8, if the number formed by its last three digits is divisible by 8

5. A number is divisible by 9, if the sum of its digits is divisible by 9

6. A number is divisible by 11, if, starting from the RHS,
   (Sum of its digits at the odd place) – (Sum of its digits at even place) is equal to 0 or 11x

7. $(a + b)^2 = a^2 + 2ab + b^2$

8. $(a - b)^2 = a^2 - 2ab + b^2$

9. $(a + b)^2 - (a - b)^2 = 4ab$

10. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

11. $(a^2 – b^2) = (a + b)(a - b)$

12. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

13. $(a^3 – b^3) = (a - b)(a^2 + ab + b^2)$

14. Results on Division:

    Dividend = Quotient × Divisor + Remainder

15. An Arithmetic Progression (A. P.) with first term 'a' and Common Difference 'd' is given by:

    [a], [(a + d)], [(a + 2d)], … … …, [a + (n - 1)d]

    n^{th} term, Tn = a + (n - 1)d
    Sum of first 'n' terms, Sn
    = n/2 (First Term + Last Term)

16. A Geometric Progression (G. P.) with first term 'a' and Common Ratio 'r' is given by:

    $a, ar, ar^2, ar^3, \ldots \ldots \ldots, ar^{n-1}$

    n^{th} term, Tn $= ar^{n-1}$
    Sum of first 'n' terms Sn $= [a(1 - r^n)] / [1 - r]$

17. $(1 + 2 + 3 + \ldots \ldots \ldots + n) = [n(n + 1)] / 2$

18. $(1^2 + 2^2 + 3^2 + \ldots \ldots \ldots + n^2) = [n(n + 1)(2n + 1)] / 6$

19. $(1^3 + 2^3 + 3^3 + \ldots \ldots \ldots + n^3) = [n^2(n + 1)^2] / 4$

H.C.F & L.C.M of Numbers

20. Product of two numbers = Their H. C. F. × Their L. C. M.

Surds & Indices

21. $a^m \times a^n = a^{(m+n)}$

22. $a^m / a^n = a^{(m-n)}$

23. $(ab)^m = a^m b^m$

24. $(a / b)^m = a^n / b^n$

25. $a^0 = 1$

26. $\sqrt[n]{a} = a^{1/n}$

27. $(\sqrt[n]{a})^n = (a^{1/n})^n$
$= a$

28. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

29. $\sqrt[n]{\frac{a}{b}} = \sqrt[n]{a} / \sqrt[n]{b}$

30. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

31. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Percentage

32. To express x% as a fraction, we have x% = x / 100

33. To express a / b as a percent, we have a / b = (a / b × 100) %

34. If 'A' is R% more than 'B', then 'B' is ***less*** than 'A' by

OR

If the price of a commodity increases by R%, then the ***reduction*** in consumption, not to increase the expenditure is

$\{100R / [100 + R]\}$ %

35. If 'A' is R% less than 'B', then 'B' is ***more*** than 'A' by

OR

If the price of a commodity decreases by R%, then the ***increase*** in consumption, not to increase the expenditure is

$\{100R / [100 - R]\}$ %

36. If the population of a town is 'P' in a year, then its population after 'N' years is

$P\,(1 + R/100)^N$

37. If the population of a town is 'P' in a year, then its population 'N' years ago is

$$P / [(1 + R/100)^N]$$

Profit & Loss

38. If the value of a machine is 'P' in a year, then its value after 'N' years at a depreciation of 'R' p.c.p.a is

$$P (1 - R/100)^N$$

39. If the value of a machine is 'P' in a year, then its value 'N' years ago at a depreciation of 'R' p.c.p.a is

$$P / [(1 - R/100)^N]$$

40. Selling Price = [(100 + Gain%) × Cost Price] / 100

= [(100 - Loss%) × Cost Price] / 100

Ratio & Proportion

41. The equality of two ratios is called a proportion. If a : b = c : d, we write a : b :: c : d and we say that a, b, c, d are in proportion.
In a proportion, the *first and fourth* terms are known as *extremes*, while the *second and third* are known as *means*.

42. Product of extremes = Product of means

43. Mean proportion between a and b is $\sqrt{ab}$

44. The *compounded ratio* of the ratios (a : b), (c : d), (e : f) is (ace : bdf)

45. $a^2 : b^2$ is a duplicate ratio of a : b

46. $\sqrt{a} : \sqrt{b}$ is a sub-duplicate ration of a : b

47. $a^3 : b^3$ is a triplicate ratio of a : b

48. $a^{1/3} : b^{1/3}$ is a sub-triplicate ratio of a : b

49. If a / b = c / d, then, (a + b) / b = (c + d) / d, which is called the *componendo*.

50. If a / b = c / d, then, (a - b) / b = (c - d) / d, which is called the *dividendo*.

51. If a / b = c / d, then, (a + b) / (a - b) = (c + d) / (c - d), which is called the *componendo & dividendo*.

52. Variation: We say that x is directly proportional to y if x = ky for some constant k and we write, x α y.

53. Also, we say that x is inversely proportional to y if x = k / y for some constant k and we write x α 1 / y.

Partnership

54. If a number of partners have invested in a business and it has a profit, then

    Share Of Partner = (Total_Profit × Part_Share / Total_Share)

Chain Rule

55. The cost of articles is directly proportional to the number of articles.

56. The work done is directly proportional to the number of men working at it.

57. The time (number of days) required to complete a job is inversely proportional to the number of hours per day allocated to the job.

58. Time taken to cover a distance is inversely proportional to the speed of the car.

Time & Work

59. If A can do a piece of work in n days, then A's 1 day's work = $1/n$.

60. If A's 1 day's work = $1/n$, then A can finish the work in n days.

61. If A is thrice as good a workman as B, then:
    Ratio of work done by A and B = 3 : 1,
    Ratio of times taken by A & B to finish a work = 1 : 3

Pipes & Cisterns

62. If a pipe can fill a tank in 'x' hours and another pipe can empty the full tank in 'y' hours (where y > x), then on opening both the pipes, the net part of the tank filled in 1 hour is

    $(1/x - 1/y)$

Time And Distance

63. Suppose a man covers a distance at 'x' kmph and an equal distance at 'y' kmph, then average speed during his whole journey is

    $[2xy / (x + y)]$ kmph

Trains

64. Lengths of trains are 'x' km and 'y' km, moving at 'u' kmph and 'v' kmph (where, u > v) in the same direction, then the time taken y the over-taker train to cross the slower train is

    $[(x + y) / (u - v)]$ hrs

65. Time taken to cross each other is

    $[(x + y) / (u + v)]$ hrs

66. If two trains start at the same time from two points *A* and *B* towards each other and after crossing they take *a* and *b* hours in reaching *B* and *A* respectively.

Then, A's speed : B's speed = $(\sqrt{b} : \sqrt{a})$.

67. x kmph = (x × 5/18) m/sec.

68. y metres/sec = (y × 18/5) km/hr.

Boats & Streams

69. If the speed of a boat in still water is u km/hr and the speed of the stream is v hm/hr, then:

    Speed downstream = (u + v) km/hr.
    Speed upstream = (u - v) km/hr.

70. If the speed downstream is a km/hr and the speed upstream is b km/hr, then:

    Speed in still water = ½ (a + b) km/hr.
    Rate of stream = ½ (a - b) km/hr.

Alligation or Mixture

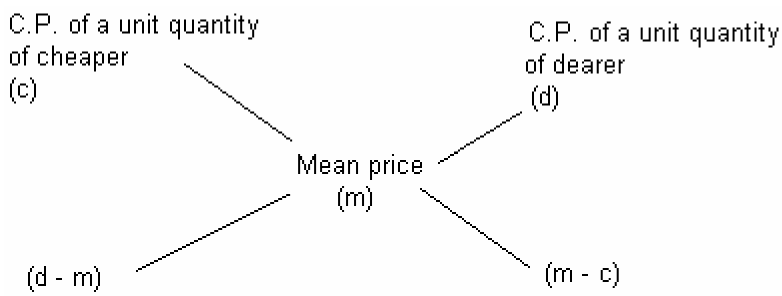
71. *Alligation*: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture at a given price.

72. *Mean Price*: The cost price of a quantity of the mixture is called the mean price.

73. Rule of Alligation: If two ingredients are mixed, then:

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \left(\frac{\text{(C. P. of dearer)} - \text{(Mean price)}}{\text{(Mean price)} - \text{(C. P. of cheaper)}}\right)$$

74. We represent the above formula as under:



75. .: (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - c)

Simple Interest

76. Let Principle = P, Rate = R% per annum and Time = T years. Then,

    a. S.I. = (P × R × T) / 100
    b. P = (100 × S.I.) / (R × T),
    c. R = (100 × S.I.) / (P × T),
    d. T = (100 × S.I.) / (P × R).

Compound Interest

77. Let Principle = P, Rate = R% per annum and Time = T years. Then,

I. When interest is compounded Annually,

Amount = $P\,(1 + R/100)^N$

II. When interest is compounded Half-yearly:

Amount = $P\,(1 + R/2/100)^{2N}$

III. When interest is compounded Quarterly:

Amount = $P\,(1 + R/4/100)^{4N}$

78. When interest is compounded Annually, but the time is in fraction, say $3\frac{7}{8}$ years.

Then, Amount = $P\,(1 + R/100)^3 \times (1 + \frac{7}{8}R/100)$

79. When Rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd, and 3rd year respectively,

Then, Amount = $P\,(1 + R_1/100)\,(1 + R_2/100)\,(1 + R_3/100)$

80. Present worth of Rs. x due n years hence is given by:

Present Worth = $x / (1 + R/100)^n$

Logarithms

81. Logarithm: If a is a positive real number, other than 1 and $a^m = x$, then we write $m = \log_a x$ and say that the value of log x to the base a is m.

82. Properties of Logarithms:

a. $\log_a (xy) = \log_a x + \log_a y$

b. $\log_a (x/y) = \log_a x - \log_a y$

c. $\log_x x = 1$ (i.e. Log of any number to its own base is 1)

d. $\log_a 1 = 0$ (i.e. Log of 1 to any base is 0)

e. $\log_a (x^p) = p \log_a x$

f. $\log_a x = 1 / \log_x a$

g. $\log_a x = \log_b x / \log_b a$
$= \log x / \log a$ (Change of base rule)

h. When base is not mentioned, it is taken as 10
i. Logarithms to the base 10 are known as common logarithms
j. The logarithm of a number contains two parts, namely characteristic and mantissa. The integral part is known as characteristic and the decimal part is known as mantissa.

I. Case 1: When the number is greater than 1.
In this case, the characteristic is one less than the number of digits in the left of decimal point in the given number.

II. Case 2: When the number is less than 1.
In this case, the characteristic is one more than the number of zeroes between the decimal point and the first significant digit of the number and it is negative.

e.g.

Number	Characteristic
234.56	2
23.456	1
2.34	0
0.234	-1
0.0234	-2
0.00234	-3

III. For mantissa, we look through the log table.

IV. *Antilog*: If **log x = y**, then **antilog y = x**.

Area

83. Rectangle:

    a. Area of a rectangle = (length × breadth)

    b. Perimeter of a rectangle = 2 (length + breadth)

84. Square:

    a. Area of square = $(\text{side})^2$

    b. Area of a square = ½ $(\text{diagonal})^2$

85. Area of 4 walls of a room
    = 2 (length + breadth) × height

86. Triangle:

    a. Area of a triangle = ½ × base × height

    b. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where

       s = ½ (a + b + c), and a, b, c are the sides of the triangle

    c. Area of an equilateral triangle = $\sqrt{3}$ / 4 × $(\text{side})^2$

    d. Radius of incircle of an equilateral triangle of side a = a / 2$\sqrt{3}$

    e. Radius of circumcircle of an equilateral triangle of side a = a / $\sqrt{3}$

87. Parallelogram/Rhombus/Trapezium:

    a. Area of a parallelogram = Base × Height

b. Area of a rhombus = ½ × (Product of diagonals)

c. The halves of diagonals and a side of a rhombus form a right angled triangle with side as the hypotenuse.

d. Area of trapezium = ½ × (sum of parallel sides) × (distance between them)

88. Circle/Arc/Sector, where R is the radius of the circle:

a. Area of a circle $= \pi R^2$

b. Circumference of a circle $= 2\pi R$

c. Length of an arc $= \Theta/360 \times 2\pi R$

d. Area of a sector $= \frac{1}{2}(\text{arc} \times R)$

$= \Theta/360 \times \pi R^2$

Volume & Surface Area

89. Cuboid:

Let length = l, breadth = b & height = h units Then,

a. Volume $= (l \times b \times h)$ cu units

b. Surface Area $= 2(lb + bh + hl)$ sq. units

c. Diagonal $= \sqrt{l^2 + b^2 + h^2}$ units

90. Cube:

Let each edge of a cube be of length a. Then,

a. Volume $= a^3$ cu units

b. Surface Area $= 6a^2$ sq. units

c. Diagonal $= (\sqrt{3} \times a)$ units

91. Cylinder:

Let radius of base = r & height (or length) = h. Then,

a. Volume $= (\pi r^2 h)$ cu. units

b. Curved Surface Area $= (2\pi rh)$ sq. units

c. Total Surface Area $= 2\pi r(r + h)$ sq. units

92. Cone:

Let radius of base = r & height = h. Then,

a. Slant height, l = $\sqrt{h^2 + r^2}$ units

b. Volume = ($\frac{1}{3} \pi r^2 h$) cu. units

c. Curved Surface Area = ($\pi r l$) sq. units

d. Total Surface Area = $\pi r(r + l)$ sq. units

93. Sphere:

Let the radius of the sphere be r. Then,

a. Volume = ($4/3\ \pi r^3$) cu. units

b. Surface Area = ($4\pi r^2$) sq. units

94. Hemi-sphere:

Let the radius of the sphere be r. Then,

a. Volume = ($2/3\ \pi r^3$) cu. units

b. Curved Surface Area = ($2\pi r^2$) sq. units

c. Total Surface Area = ($3\pi r^2$) sq. units

Stocks & Shares

95. *Brokerage*: The broker's charge is called brokerage.

96. When stock is purchased, brokerage is added to the cost price.

97. When the stock is sold, brokerage is subtracted from the selling price.

98. The selling price of a Rs. 100 stock is said to be:

a. **at par**, if S.P. is Rs. 100 exactly;
b. **above par** (or **at premium**), if S.P. is more than Rs. 100;
c. **below par** (or **at discount**), if S.P. is less than Rs. 100.

99. By '*a Rs. 800, 9% stock at 95*', we mean a stock whose face value is Rs. 800, annual interest is 9% of the face value and the market price of a Rs. 100 stock is Rs. 95.

True Discount

100. Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that:

Sum due = *Rs. 156 due 4 years hence*;
Present Worth (P.W.) = Rs. 100;
True Discount (T.D.) = Rs. (156 - 100)
= (Sum due) – (P.W.)

101. T.D. = Interest on P.W.

102. Amount = (P.W.) + (T.D.)

103. Interest is reckoned on R.W. and true discount is reckoned on the amount

104. Let rate = *R*% per annum & time = *T* years. Then,

a. P.W. = (100 × Amount) / (100 + [R × T])
= (100 × T.D.) / (R × T)

b. T.D. = (P.W.) × R × T / 100
= ([Amount] × R × T) / (100 + [R × T])

c. Sum = ([S.I.] × [T.D.]) / ([S.I.] – [T.D.])

d. (S.I.) – (T.D.) = S.I. on T.D.

e. When the sum is put at compound interest, then

P.W. = Amount / $(1 + R/100)^T$

Banker's Discount

105. Banker's Discount (B.D.) is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.

106. Banker's Gain (B.G.) = (B.D.) – (T.D.) for the unexpired time

107. When the date of the bill is not given, grace days are not to be added

108. B.D. = S.I. on bill for unexpired time

109. B.G. = (B.D.) – (T.D.)
= S.I. on T.D.
= $(T.D.)^2$ / P.W.

110. T.D. = $\sqrt{P.W. \times B.G.}$

111. B.D. = (Amount × Rate × Time) / 100

112. T.D. = (Amount × Rate × Time) / (100 + [Rate × Time])

113. Amount = (B.D. × T.D.) / (B.D. – T.D.)

114. T.D. = (B.G. × 100) / (Rate × Time)