

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours**Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
 Answering the question in **Part-A** is Compulsory,
 Three Questions should be answered from **Part-B**

PART-A

1. (a) Solve the D.E $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$
- (b) Solve the D.E $(D^2 - a^2) y = e^{-ax} + \sin ax$
- (c) Find the Laplace transform of $\frac{e^{at} - e^{bt}}{t}$
- (d) Find $J \left(\frac{u, v}{x, y} \right)$ if $u = e^x$ & $v = e^y$
- (e) Form the PDE by eliminating the arbitrary function $f(x+y+z, xy-z^2) = 0$
- (f) Solve the PDE by variable separable method $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$

[4+4+3+3+4+4]

PART-B

2. (a) Solve the D.E $(D^2 + a^2) y = \sec ax$
- (b) A mass 'm' suspended from one end of a spring is subjected to force $f = f_0 \sin at$ in the direction of its length. The force f is measured positive vertically down words and time $t = 0$, m is at rest. If the spring constant is k , then find the displacement of m at time t .
[8+8]
3. (a) Solve the D.E $x(3ydx + 2xdy) + 8y^4(ydx + xdy) = 0$
- (b) A body is heated to 105°C and placed in a air at 15°C . After 1 hour its temperature is 60°C . How much time is required for it to cool 37°C .
[8+8]
4. (a) Find the Laplace transform of (i) $L\{t \cdot e^{-t} \sin t\}$ (ii) $L\{\sinh at \cdot \sin at\}$
- (b) Find $L^{-1} \left(\frac{s}{s^4 + 4a^4} \right)$
[8+8]
5. (a) Expand $e^{2x} \sin 3y$ in a Taylor's series about $(0,0)$
- (b) Find the maxima and minima of $x^3 y^2 (1-x-y)$
[8+8]
6. (a) Solve the PDE $z(z^2 + xy)(px - qy) = x^4$
- (b) Solve the PDE $(D^2 - DD^1)z = \cos x \cos 2y$
[8+8]
7. The ends A and B of rod 20cm long have the temperature at 30°C and 80°C until steady state prevail. The temperature of the ends are changed at 40°C and 60°C respectively. Find the temperature distribution in the rod at time t .

[16]

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PART-A

1. (a) Solve the D.E $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x} (\log y)^2$
- (b) Solve the D.E $(D^2+a^2) y = e^{ax} + \cos ax$
- (c) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$
- (d) Find $J \left(\frac{u,v}{x,y} \right)$ if $u = e^{x+y}$ & $v = e^{-x+y}$
- (e) Form the PDE by eliminating the arbitrary function $f(xy+yz+zx, x+y+z) = 0$
- (f) Solve the PDE by variable separable method $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$

[4+4+3+3+4+4]

PART-B

2. (a) Solve the D.E $(D^2+a^2) y = \tan ax$.
- (b) A mass 4.9 kg is suspended from one end of a spring. A pull of 10 kg will stretch it to 5cm, The mass is pull down 6 cm below the static equilibrium position and then released. then find the displacement of mass at time t. [8+8]
3. (a) Solve the D.E $xy(ydx + xdy) + x^2y^2(2ydx - xdy) = 0$
- (b) The rate of at which the bacteria multiply is proportional to the instantaneous number present .If the original number doubles in 2 hrs, in how many hours will it triple. [8+8]
4. (a) Find the Laplace transform of periodic function $f(t) = \begin{cases} t/a & 0 \leq t \leq a \\ (2a-t)/a & a \leq t \leq 2a \end{cases}$
- (b) Find $L^{-1} \left(\frac{s}{(s^2 + a^2)^2} \right)$ [8+8]
5. (a) Using Taylor's series expand $e^x \cdot \cos y$ near $(1, \pi/4)$
- (b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $z^2+x^2+y^2=1$ using Lagrange's multiplier method. [8+8]

6. (a) Solve the PDE $(x^2+y^2+yz)p+(x^2+y^2-xz)q = z(x+y)$
(b) Solve the PDE $(D^3-2D^2 D^1)z = 2e^{2x}+3x^2y$.

[8+8]

7. A rod 100 cm long, with insulated sides has kept the temperature at 0^0c and 100^0c until steady state prevail. The two ends are suddenly insulated and kept so. Find the temperature distribution in the rod .

[16]



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PART-A

1. (a) Solve the D.E $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$
 - (b) Solve the D.E $(D^2+4)y = x e^{2x}$
 - (c) Evaluate $\int_0^{\infty} \frac{\sin t}{t} dt$
 - (d) Find $J \left(\frac{u, v, w}{x, y, z} \right)$ if $u = x + y + z, uv = y + z, uvw = z$
 - (e) Solve the PDE $xp - yq = y^2 - x^2$
 - (f) Solve the PDE by variable separable method $4 \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 3z$ and $z(0, y) = e^{-5y}$
- [4+4+3+3+4+4]

PART-B

2. (a) Solve the D.E $(D^2+a^2)y = x \sin ax$
- (b) In an L-C-R circuit, the charge q on a plate of a condenser is given by $Lq'' + Rq' + q/c = E \sin pt$. If initially the current and charge are zero. Then find current in the circuit. [8+8]
3. (a) Solve the D.E $(x^2 + y^2)dx - 2xy dy = 0$
- (b) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. [8+8]
4. (a) Find the Laplace transform of periodic function $f(t) = \begin{cases} \sin at & 0 \leq t \leq \pi/a \\ -\sin at & \pi/a \leq t \leq 2\pi/a \end{cases}$
- (b) Find $L^{-1} \left(\frac{s}{(s^2 + a^2)(s^2 + b^2)} \right)$ using convolution theorem. [8+8]
5. (a) Expand $e^x \log(1+y)$ in a Taylor's series about $(0,0)$
- (b) Find the point on the plane of
 (i) $2x+3y-z = 5$ (ii) $3x-4y+5z = 26$ which is nearest to the origin. [8+8]

6. (a) Solve the PDE $(x^2-y^2-yz)p+(x^2-y^2-xz)q = z(x-y)$
(b) Solve the PDE $(D^2 - 4DD^1 + D^{1^2})z = e^{2x+y}$

[8+8]

7. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ subject to
 $u(0, y) = 0$, $u(l, y) = 0$
 $u(x, 0) = 0$ ($0 < x < l$)
 $u(x, l) = x(l - x)$ ($0 < x < l$)

[16]



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PART-A

1. (a) Solve the D.E $xy(1+xy^2)\frac{dy}{dx}=1$
 - (b) Solve the D.E $(D^2+4D+4)y=e^{-2x}+x^2$
 - (c) Evaluate $\int_0^{\infty} e^{-3t}t \sin t dt$
 - (d) Find $J\left(\frac{u,v,w}{x,y,z}\right)$ if $u=yz/x, v=xz/y, w=xy/z$
 - (e) Solve the PDE $z(p^2+q^2+1)=1$
 - (f) Solve the PDE by variable separable method $3\frac{\partial z}{\partial x}+2\frac{\partial z}{\partial y}=0$ and $z(x,0)=4e^{-x}$
- [4+4+3+3+4+4]

PART-B

2. (a) Solve the D.E $(D^2+a^2)y=\operatorname{cosec}ax$.
 - (b) In an L-C-R circuit, the current 'i' is given by $Li^{11}+Ri^1+1/c=pE \cos pt$. Then find current in the circuit 'i' when (i) $cR^2 > 4L$ (ii) $cR^2 < 4L$
- [8+8]
3. (a) Solve the D.E $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$
 - (b) Find the orthogonal trajectory of $r^n = a^n \sin n\theta$
- [8+8]
4. (a) Find the Laplace transform of periodic function $f(t) = \begin{cases} \cos at & 0 \leq t \leq \pi/a \\ -\cos at & \pi/a \leq t \leq 2\pi/a \end{cases}$
 - (b) Find $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ using convolution theorem.
- [8+8]
5. (a) Expand $e^x \cdot \sin y$ in powers of x & y
 - (b) Find the Extrema of (i) $a^2-x^2-y^2$ (ii) x^3y^2-xy
- [8+8]

6. (a) Solve the PDE $(mz-ny)p+(nx-lz)q = (ly-mx)$
(b) Solve the PDE $(D^2 + DD^1 - 6D^{1^2})z = \cos(2x + y)$

[8+8]

7. Solve the wave equation $c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ subject to
 $y(0,t) = 0$, $y(l,t) = 0$
 $y(x,0) = f(x)$ ($0 < x < l$)
 $\frac{\partial y}{\partial t}(x,0) = g(x)$ ($0 < x < l$)

Also find the solution (i) if $f(x) \neq 0$, $g(x) = 0$ (ii) $f(x) = 0$, $g(x) \neq 0$

[16]



Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) If 30% of a radioactive substance disappear in 10 days, how long will it take for 90% of it to disappear?
(b) Solve the D.E $(\cos^3 x)y' + y \cos x = \sin x$ [8+7]
2. (a) Solve the D.E $(D^2-4)y = e^{2x} + \sin 2x$
(b) Solve the D.E $(D^2-4D+2)y = x^2 e^{2x} + \cos 2x$ [8+7]
3. (a) Verify whether $u = \frac{x+y}{1-xy}$ & $v = \tan^{-1}(x) + \tan^{-1}(y)$ are functionally depended or independent.
(b) Find Taylor series expansion for $\tan^{-1}(y/x)$ about (1,1) [8+7]
4. (a) Trace the curve $xy^2 = a^2(x-a)$ ($a > 0$)
(b) Trace the curve $r = a(1 + \cos \theta)$ [8+7]
5. (a) Find the perimeter of the curve $r = a(\cos \theta + \sin \theta)$
(b) Find the volume of the solid generated by revolution of $x = a \cos^3 \theta$, $y = \sin^3 \theta$ about its x-axis. [8+7]
6. (a) By change of order of integration evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2) dx dy$
(b) Evaluate $\iiint xyz dx dy dz$ over a positive octant of a sphere with centre zero and radius a. [8+7]
7. (a) Find the directional derivative of $f = x^3 y^2 z$ at (1,2,3) along the direction of $9\vec{i} + 3\vec{j} + \vec{k}$
(b) Prove that $\text{curl}(\text{curl} f) = \text{grad div} f - \nabla^2 f$ [8+7]
8. Verify Stokes theorem for $f = y^2 \vec{i} + yz \vec{j} - zx \vec{k}$ and S is the upper half of the surface $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$. [15]
