TEST PAPER 5

Total Questions: 60 **Time allotted 75 minutes**

Q.1) A person at the top of a hill observes that the angles of depression of two consecutive kilometer stones on a road leading to the foot of hill are 30^0 and 60^0. The height of the hill is

(a) $\frac{\sqrt{3}}{2}$ km (b) $\frac{\sqrt{5}}{2}$ km

(c) $\frac{\sqrt{6}}{2}$ km (d) $\frac{\sqrt{7}}{2}$ km

Q.2) The value of $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$ is

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$

(c) $\frac{16}{7}$ (d) $\frac{17}{6}$

Q.3) If for a triangle ABC,
$1+\cos 2A+\cos 2B+\cos 2C=0$, then the triangle must be

(a) equilateral (b) isosceles

(c) right-angled (d) obtuse-angled

Q.4) The lines $4x+4y=1, 8x-3y=2$ and $y=0$ are

(a) sides of an isosceles triangle (b) concurrent

(c) parallel (d) sides of an equilateral triangle

Q.5) The line $3x+4y-24=0$ cuts the x-axis at a and y-axis at B. The incentre of the triangle OAB, where O is the origin, is at

(a) (2, 2) (b) (3, 3)

(c) (4, 3) (d) (3, 4)

Q.6) The centre of the circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{3}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},\frac{1}{2}\right)$

(c) $\left(-\frac{1}{\sqrt{2}},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},-\sqrt{2}\right)$

Q.7) Match List I (*Equation of circles*) with List II (*Their centres*) and select the correct answer using the codes given below the lists:

	List I (Equation of circles)		List II (Their centres)
A.	$(x+2)^2+(y+1)^2=1$	1.	(1, –2)
B.	$(x-1)^2+(y+2)^2=1$	2.	(–1, 2)
C.	$(x+1)^2+(y-2)^2=4$	3.	(2, 1)
D.	$(x-2)^2+(y-1)^2=4$	4.	(–2, –1)

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	2	1	3
(c)	3	1	2	4
(d)	3	2	1	4

Q.8) The eccentricity of hyperbola $25x^2 - 9y^2 = 144$ is

(a) $\frac{\sqrt{34}}{12}$ (b) $\frac{\sqrt{34}}{3}$

(c) $\frac{6}{\sqrt{34}}$ (d) $\frac{3}{\sqrt{34}}$

Q.9) The distance from the major axis of any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its corresponding point on the auxiliary circle are in the ratio

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$

(c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

Q.10) The distance of a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ from the centre is 2. The eccentric angle of the point is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) π

Q.11) The focus of the curve $y^2 + 4x - 6y + 13 = 0$ is

(a) (2, 3) (b) (–2, 3)

(c) (2, –3) (d) (–2, –3)

Q.12) If a line makes angles of 60^0 and 45^0 with the positive direction of the axes of x and y respectively, then the angle made by the line with positive direction of the z-axis is equal to

(a) 60^0 (b) 120^0

(c) either 60^0 or 120^0 (d) neither 60^0 nor 120^0

Q.13) Match List I (*Equation of curves*) with List II (*Their types*) and select the correct answer using the codes given below the lists:

	List I (Equation of curves)		List II (Their types)
A.	$x^2 + x + 1 = y$	1.	Circle
B.	$x^2 + y^2 + 2x + 2y - 6 = 0$	2.	Parabola
C.	$2x^2 + 3y^2 + 4x + 6y = 0$	3.	Ellipse
D.	$3x^2 - 2y^2 + 6x - 4y = 0$	4.	Hyperbola
		5.	Pair of straight lines

Codes:

	A	B	C	D
(a)	2	3	1	4

(b)	4	3	1	5
(c)	2	1	3	4
(d)	4	1	3	5

Q.14) The direction cosines of the line perpendicular to the line, with direction ratios (1, -2, -2) and (0, 2, 1) are

(a) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ (d) $\left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

Q.15) The length of the normal from the origin to the plane $x+2y-2x=9$ is equal to

(a) 2 (b) 3

(c) 4 (d) 5

Q.16) If the angle between the lines joining the end points of minor axis of an ellipse with its foci is $\frac{\pi}{2}$,then the eccentricity of the ellipse is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\left(2\sqrt{2}\right)}$

Q.17) The lines $2x=3y=-z$ and $6x=-y=-4z$

(a) area parallel (b) are perpendicular

(c) interest at an angle of 45^0 (d) interest at an angle of 60^0

Q.18) The distance between the parallel planes $4x-2y+4z+9=0$ and $8x-4y+8z+21=0$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) $\frac{7}{4}$

Q.19) Match List I (Equations of spheres) with List II (Their centres) and select the correct answer using the codes given below the lists:

	List I (Equation of spheres)		List II (Their centres)
A.	$x^2+y^2+z^2+3x-3y+3z-49=0$	1.	(2, 3, 1)
B.	$x^2+y^2+z^2-4x-6y-2z+9=0$	2.	$\left(-\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}\right)$
C.	$2x^2+2y^2+2z^2-x-y-z=0$	3.	(1, 1, 1)
D.	$x^2+y^2+z^2-2x-2y-2z=0$	4.	$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	3	1	4	2
(c)	2	1	4	3
(d)	3	4	1	2

Q.20) The radius of the sphere $3x^2+3y^2+3z^2-8x+4y-15=0$ is equal to

(a) 2 (b) 3
(c) 4 (d) 5

Q.21) The condition for the lines $x=az+b, y=cz+d$; and $x=a_1z+b_1 y=c_1z+d_1$ to be perpendicular is

(a) $ac_1+a_1c+1=0$ (b) $aa_1+cc_1+1=0$
(c) $ac_1+a_1c-1=0$ (d) $aa_1+cc_1-1=0$

Q.22) If n forces $\overrightarrow{PA_1}, \overrightarrow{PA_2}, ..., \overrightarrow{PA_n}$ diverge from point P and n other forces $\overrightarrow{A_1Q}, \overrightarrow{A_2Q}, ..., \overrightarrow{A_nQ}$ converge to a point Q, then the resultant of the $2n$ forces is represented, in magnitude and direction by

(a) $n\overrightarrow{PQ}$ (b) $n\overrightarrow{QP}$
(c) $2n\overrightarrow{PQ}$ (d) $n^2\overrightarrow{PQ}$

Q.23) If $\left(\vec{a}-\vec{b}\right),\left(\vec{a}+\vec{b}\right)=0$, then

(a) $\vec{a}$ and $\vec{b}$ are perpendicular (b) $\vec{a}$ and $\vec{b}$ are parallel
(c) $\left|\vec{a}\right|=\left|\vec{b}\right|$ (d) $\vec{a}=2\vec{b}$

Q.24) If $\vec{A}+t\vec{B}$ is perpendicular to $\vec{C}$, where $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}, \vec{B}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}+\hat{j}$ then t is equal to

(a) 4 (b) 5
(c) 6 (d) 7

Q.25) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ where a_1, a_2, a_3 are scalar quantities and $\hat{i}, \hat{j}, \hat{k}$, unit vectors in three mutually perpendicular directions, then $\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{k}\right|^2$ is equal to

(a) $a_1^2+a_3^2$ (b) $a_1^2+a_2^2+a_3^2$
(c) $2a_1^2+a_2^2+a_3^2$ (d) $a_1^2+2a_2^2+a_3^2$

Q.26) The magnitude of the projection of the vector $\vec{A}=\hat{i}-2\hat{j}+\hat{k}$ on the vector $\vec{B}=4\hat{i}-4\hat{j}+7\hat{k}$ lies between

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 4 and 5

Q.27) The moment about the point A(3, -1, 3) of a force $\vec{F}=2\hat{i}+\hat{j}+4\hat{k}$ through the point B (5, 1, 4) is

(a) $3\hat{i}+2\hat{j}-\hat{k}$ (b) $7\hat{i}-6\hat{j}-2\hat{k}$
(c) $5\hat{i}+\hat{j}+3\hat{k}$ (d) $\hat{i}+2\hat{j}-\hat{k}$

Q.28) If $\left|\vec{a}\times\vec{b}\right|+\left|\vec{a}.\vec{b}\right|^2=144$ and $\left|\vec{a}\right|=4$, then $\left|\vec{b}\right|$ is equal to

(a) 3 (b) 8
(c) 12 (d) 16

Q.29) If the work done by a force $\hat{i}+\hat{j}+8\hat{k}$ along a given vector in the xy-plane is 8 units and the magnitude of the given vector is $4\sqrt{3}$, then the given vector is represented as

(a) $\left(4+2\sqrt{2}\right)\hat{i}+\left(4-2\sqrt{2}\right)\hat{j}$ (b) $\left(4i+4\sqrt{2}j\right)$

(c) $\left(4\sqrt{2}i + 4j\right)$ (d) $\left(4 + 2\sqrt{2}\right)\left(\hat{i} + \hat{j}\right)$

Q.30) Consider the following statements:
1. 5^{th} decile and 50^{th} percentile are the same.
2. 2^{nd} quartile and 50^{th} decile are the same.
3. 2^{nd} quartile and 50^{th} percentile are the same.
4. 2^{nd} quartile, 50^{th} decile and 50^{th} percentile are the same.

Which of these is/are correct?
(a) Only 1 (b) 2 and 3
(c) 1, 2 and 3 (d) 1, 2, 3 and 4

Q.31) The figure formed by joining the mid-points of the upper horizontal sides of each rectangle of a histogram is called
(a) frequency curve (b) frequency polygon
(c) more than ogive (d) less than ogive

Q.32) Average age of a teacher and three students is 20 years. If all the three boys are of same age and the difference between the ages of a boy and the teacher is 20 years, then the age of the teacher is equal to
(a) 25 years (b) 30 years
(c) 35 years (d) 45 years

Q.33) In an examination, the standard of passing was 40%. Out of 9 students who appeared, 4 failed and the remaining got 80%, 57%, 51%, 68% and 79% marks. The median of the percentage marks is equal to
(a) 51% (b) 57%
(c) 68% (d) 79%

Q.34) The median of 19 observations is 30. Two more observations are made and their values are 8 and 32. The median of the 21 observations taken together is equal to
(a) 28 (b) 30
(c) 32 (d) 34

Q.35) If X follows binomial distribution with mean 3 and variance 2, then P (X $\geq$ 8) is equal to
(a) $\frac{17}{3^9}$ (b) $\frac{18}{3^9}$
(c) $\frac{19}{3^9}$ (d) $\frac{20}{3^9}$

Q.36) The most probable number of heads in 80 tosses and a biased coin, given that the probability of a head in a single toss is $\frac{3}{5}$, is
(a) 48 (b) 32
(c) 16 (d) 12

Q.37) In tossing a coin twice, let E and F denote occurrence of head on first toss and second toss respectively, P (E $\cup$ F) is equal to
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

Q.38) The probability of having a king and a queen when the two cards are drawn at random from a pact of 52 cards is

(a) $\frac{16}{663}$ (b) $\frac{8}{663}$

(c) $\frac{4}{663}$ (d) $\frac{2}{663}$

Q.39) A card is drawn from an ordinary pack and a gambler bets that it is either a spade or an ace. The odds against his winning are

(a) 9 : 4 (b) 9 : 5

(c) 9 : 6 (d) 9 : 8

Q.40) Thirteen cards are drawn simultaneously from a deck of 52. If aces count 1, face cards 10 and others according to denomination, then expectation of the total score on 13 cards is

(a) 85 (b) $\frac{85}{13}$

(c) $\frac{4}{13}$ (d) 4

Q.41) The probability that a leap year, selected at random, will contain 53 Sundays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

Directions: *The following 4 (four) items consists of two statements, one labeled as Assertion (‘) and the other labeled as Reason (R). You are to examine these two statements carefully and decide if the Assertion (A) and the Reason (R) are individually true if so, whether the Reason R is the correct explanation for the given Assertion A. Select you answers to these items using the codes given below and mark your answer sheet accordingly.*

Codes:

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true and R is not the correct explanation of “A.
(c) A is true but R is false.
(d) A is false and R is true.

Q.42) Assertion (A) : The vector product of a Force $\vec{F}$ and displacement $\vec{r}$ is equal to the work done.
Reason (R) : Work done is not a vector.

Q.43) Assertion (A) : $\lim_{x\to a}\frac{\tan x}{x}=1$

Reason (R) : If $\lim_{x\to a} f(x)=l, \lim_{x\to a} g(x)=m$, then $\lim_{x\to a}\{f(x)g(x)\}=lm.$

Q.44) Assertion (A) : $f(x)=\begin{cases} x^2\sin\frac{1}{x}, x\neq 0 \\ 0, x=0 \end{cases}$ is continuous at $x = 0$

Reason (R) : Both $h(x) = x^2$ and $g(x)=\begin{cases} \sin\frac{1}{x}, x\neq 0 \\ 0, x=0 \end{cases}$ are continuous at $x = 0$

Q.45) Assertion (A) : $|x|$ is not continuous at $x = 0$.

Reason (R) : $|x|$ is not differentiable at $x = 0$.

Q.46) The domain of the function $\sqrt{\frac{1}{\sqrt{x}}\sqrt{x+1}}$ is

(a) $0 < x < \left\{\frac{\sqrt{3}-1}{2}\right\}$ (b) $0 < x \le \left\{\frac{\sqrt{5}-1}{2}\right\}$

(c) $\left\{\frac{-1-\sqrt{5}}{2}\right\} < x < 0$ (d) $0 \le x < \left\{\frac{\sqrt{5}-1}{2}\right\}$

Q.47) The inverse of the function $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is equal

(a) $\log_{10}(2-x)$ (b) $\frac{1}{2}\log_{10}(2x-1)$

(c) $\frac{1}{4}\log_{10}\frac{2x}{2-x}$ (d) $\frac{1}{4}\log_{10}\frac{1+x}{1-x}$

Q.48) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to

(a) 1 (b) 0

(c) e (d) $e^{1/3}$

Q.49) If $\lim_{x\to 0}\frac{a^x - x^a}{x^a - a^a} = -1$, then a is equal to

(a) -1 (b) 0

(c) 1 (d) 2

Q.50) If $f(x) = |x-3| + |x-4|$, then in the interval [0, 5], the function $f(x)$ is

(a) differentiable at $x = 3$

(b) differentiable at $x = 4$

(c) not differentiable at $x = 3$ and $x = 4$

(d) not continuous in the interval [0, 5]

Q.51) If $y = 3x^2 + 2$ and if x changes from 10 to 10.1, then the approximate change in y will be

(a) 4 (b) 5

(c) 6 (d) 8

Q.52) If $y = \tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ and $z = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{dy}{dz}$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Q.53) If $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log_e\sqrt{-x^2}$, then $\left(\frac{d^2y}{dx^2}\right)_{x=0}$ is equal to

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) 2

Q.54) A flower-bed in the form of a circular sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible surface area, then the radius of the circle is

(a) 25 m (b) 20 m

(c) 10 m (d) 5 m

Q.55) $\int \sin^3 x \cos x dx$ is equal to

(a) $\frac{\cos^4 x}{4} + c$ (b) $\sin^4 x + c$

(c) $\frac{\sin^4 x}{4} + c$ (d) $\frac{\sin 4x}{4} + c$

Q.56) The area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum is

(a) $\frac{8a^2}{3}$ (b) $\frac{4a^2}{3}$

(c) $\frac{2a^2}{3}$ (d) $\frac{a^2}{3}$

Q.57) $\int e^{\log(\tan x)} dx$ is equal to

(a) log tan $x + c$ (b) log sec $x + c$

(c) tan $x + c$ (d) $e^{\tan x} + c$

Q.58) $\int a^x dx$ is equal to

(a) $\frac{a^{x+1}}{x+1} + c$ (b) $a^x \log x + c$

(c) $a^x \log a + c$ (d) $\frac{a^x}{\log a} + c$

Q.59) If $A_n = \int_0^{\pi/4} \tan^n x \, dx \left(n \geq 2\right)$, than $A_n + A_{n-2}$ is equal to

(a) $\frac{1}{n} + \frac{1}{n-1}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n}$ (d) $\frac{1}{n-1}$

Q.60) The value of $\int_{-1}^{1} x|x| dx$ is equal to

(a) 0 (b) 2

(c) -2 (d) $\propto$

ANSWER KEYS

1.	(a)	13.	(c)	25.	(d)	37.	(c)	49.	(a)
2.	(d)	14.	(a)	26.	(b)	38.	(b)	50.	(d)
3.	(c)	15.	(b)	27.	(b)	39.	(a)	51.	(c)
4.	(b)	16.	(b)	28.	(a)	40.	(a)	52.	(c)
5.	(a)	17.	(b)	29.	(a)	41.	(b)	53.	(b)
6.	(d)	18.	(a)	30.	(d)	42.	(b)	54.	(c)
7.	(a)	19.	(c)	31.	(b)	43.	(b)	55.	(c)
8.	(b)	20.	*	32.	(c)	44.	(d)	56.	(a)
9.	(b)	21.	(b)	33.	(a)	45.	(b)	57.	(b)
10.	(b)	22.	(a)	34.	(b)	46.	(a)	58.	(d)
11.	(b)	23.	(c)	35.	(c)	47.	(d)	59.	(d)
12.	(c)	24.	(b)	36.	(a)	48.	(d)	60.	(a)