Code No: 09A1BS01 **R09** **Set No. 2**

I B.Tech Examinations,Dec.-Jan., 2011-2012
MATHEMATICS-I
Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE, MIM, MIE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the directional derivative of $\frac{1}{r}$ in the direction of $\bar{r} = xi + yj + 3k$ at $(1, 1, 2)$
   (b) Evaluate $\iint\limits_S \bar{F}.\bar{n}ds \ where \ \bar{F} = zi + xj + 3y^2zk$ and S is the surface of the cylinder $x^2 + y^2 = 1$ moulded in the first octant between z = 0 and z =2 [8+7]

2. (a) Find the length of the curve $y = \log \frac{(e^x-1)}{(e^x+1)}$ from x=1; x=2
   (b) Evaluate $\int\limits_0^{4a} \int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2}dxdy$ [8+7]

3. (a) Verify Rolle's theorem for $f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$
   (b) Find the Maximum or minimum values of $f = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ [8+7]

4. (a) Find L$[te^{-t}\cosh t]$.
   (b) Find inverse Laplace transforms of $\frac{5s-2}{s^2(s+2)(s-1)}$ [8+7]

5. (a) Solve the differential equation $(D^2 - 2D + 4)y = e^{2x}Cosx$
   (b) Solve the differential equation $(D^3 - 3D^2 + 3D - 1)y = Sinx + x^3$ [8+7]

6. (a) Find the radius of curvature for the curve $r^2 = a^2 \cos 2\theta$
   (b) Prove that the evolute of the curve x = $a\left(\cos t + \log\tan\frac{t}{2}\right), \quad y = a\sin t$ is the catanary $y = a\cosh {}^{x}\!/_{a}$. [7+8]

7. (a) Form the differential equation by eliminating arbitrary constants y = Ae^x+Be^{-x}
   (b) Solve the differential equation $(e^y + 1)\cos xdx + e^y \sin xdy = 0$
   (c) Find the curve in which the perpendicular upon the tangent from the foot of the ordinate of the point of contact is constant and equal to a. [4+6+5]

8. (a) Test the convergence of the series $\frac{(n!)^2x^{2n}}{(2n)!}$
   (b) Test the convergence of the series $\sum \frac{\left(\sqrt{5}-1\right)^n}{n^2+1}$ [7+8]

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Code No: 09A1BS01 **R09** **Set No. 4**

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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sqrt{n+1}-\sqrt{n-1}$
   (b) Test the convergence of the series $u_n = \frac{(n+1)^n.x^n}{n^{n+1}}$
   (c) For what values of the following series is convergent $-x+\frac{x^2}{2^2}-\frac{x^2}{2^2}+\frac{x^3}{3^2}-\frac{x^4}{4^2}+...$ [5+5+5]

2. (a) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$
   (b) Find the inverse Laplace transform of $\log\left(1+\frac{16}{s^2}\right)$ [7+8]

3. (a) Expand $(1+x)^x$ in powers of x.
   (b) Find the maximum and minimum values of $x+y+z$ subject to $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ [7+8]

4. (a) Form the differential equation by eliminating arbitrary constants
   $x^2 + y^2 - 2a\,y = a^2$
   (b) Solve the differential equation $\frac{dy}{dx} = \frac{x^2+xy}{x^2+y^2}$
   (c) Find the orthogonal Trajectories of the family of curves $x^{2/3}+y^{2/3}=a^{2/3}$ [4+5+6]

5. (a) Find the length of the cycloid x = a (θ + sinθ) y = a (1-cosθ) between two consecutive cusps. Show that the length of the arc of the cycloid between the points $\theta = 0$ and $\theta = 2\Psi$ is given by s = 4asin ψ. Show further that for this curve $s = \sqrt{8ay}$.
   (b) Evaluate the double integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dy\,dx$ [7+8]

6. (a) Prove that, if the center of curvature of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at one end of the minor axis lies at the other end then the eccentricity of the ellipse is $\frac{1}{\sqrt{2.,}}$
   (b) Find the envelope of a family of circles $x^2+y^2-2ax\cos\alpha-2ay\sin\alpha=c^2$ where α is the parameter. [8+7]

7. (a) Solve the differential equation $(D^2+9)y=\cos 3x+\sin 2x$
   (b) A mass of 4 lbs suspended from a light elastic string of natural length 3 feet extends it to a distance 2 ft. One end of the string is fixed and a mass of 2 lbs is attached to other. The mass is held so that the string is just un stretched and is then let go. Find the amplitude, period and the maximum velocity of the S.H.M. [8+7]

8. Verify stoke's theorem for $F = y^2i + yj - zxk$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2 and\ z \geq 0$ [15]

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Code No: 09A1BS01 **R09** **Set No. 1**

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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find $L\left[3\cos .3t\cos 4t\right]$
   (b) Find the inverse Laplace transform of log $\left(1+\frac{16}{s^2}\right)$ [7+8]

2. (a) Test the convergence of the series $\frac{3^2}{6^2}+\frac{3^2.5^2}{6^2.8^2}+\frac{3^2.5^2.7^2}{6^2.8^2.10^2}+......$
   (b) Test whether the following series is absolutely convergent or conditionally convergent $\frac{1}{5\sqrt{2}}-\frac{1}{5\sqrt{3}}+\frac{1}{5\sqrt{4}}.....(-1)^n\frac{1}{5\sqrt{n}}$ [7+8]

3. (a) $\bar{F}$ = grad $(x^3+y^3+z^3-3xyz)$ Find curl $\bar{F}$
   (b) If F = 4xzi - y^2 j + yz k evaluate $\iint\limits_S F.nds$ where S is the surface of the cube bounded by x=0, x = 1, y= 0, y = 1, z = 0, z = 1. [8+7]

4. (a) Find the volume of the solid generated by cycloid x = a (θ + sinθ), y = a (1+cosθ), when it is revolved about its base.
   (b) Evaluate $\int_0^{\log z}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}dzdydx$ [8+7]

5. (a) For the curve $x=a\sin 2\theta\left(1+\cos 2\theta\right), y=a\cos 2\theta\left(1-\cos 2\theta\right)$ find the radius of curvature at the point $\theta=\frac{\pi}{3}$
   (b) Trace the curve $x=a\left(\theta+\sin\theta\right)$, $y=a\left(1+\cos\theta\right)$ [8+7]

6. (a) Form the differential equation by eliminating arbitrary constants $y^2=(x-C)^3$
   (b) Solve the differential equation $y(xy+e^x)dx-e^xdy=0$
   (c) Find the orthogonal Trajectories of the family of curves $x^2+y^2=ax$ [4+6+5]

7. (a) Expand $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ in powers of x.
   (b) Find a point with in a triangle such that the sum of the squares of its distances from the three vertices is a minimum. [7+8]

8. (a) Solve the differential equation $(D^2-4)y=2\cos^2 x$
   (b) A particle is executing S.H.M, with amplitude 5 meters and time 4 seconds. Find the time required by the particle in passing between points which are at distances 4 and 2 meters from the centre of force and are on the same side of it. [8+7]

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Code No: 09A1BS01 **R09** **Set No. 3**

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MATHEMATICS-I
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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Verify lagranges mean value theorem for $f(x) = (x)(x-2)(x-3)\, in\, (0,4)$
   (b) If $f(x,y) = \tan^{-1} xy$ compute $f(.9, -1.2)$ approx. [8+7]

2. (a) Find L $[t \sin 3t \cos 2t]$
   (b) Find inverse Laplace transforms of $\cot^{-1}(s)$ [8+7]

3. (a) Find a unit normal vector to the surface $x^3+y^3+z^3 = 3$ at the point $(1,-2,1)$
   (b) Applying, Green's theorem evaluate $\int (y - \sin x)\, dx + \cos x dy$, where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}\ and\ y = \frac{2x}{\pi}$ [8+7]

4. (a) Prove that the surface of the solid generated by the revolution of the tractrix. $x = a\cos\theta + \frac{a}{2}\log\tan^2\frac{\theta}{2}\ and\, y = a\sin\theta$ about its asymptote in equal to the surface of a sphere of radius a.
   (b) Evaluate $\int\limits_0^a \int\limits_0^{\sqrt{a^2-y^2}} (x^2+y^2)\, dxdy$ by changing into polar coordinates. [8+7]

5. (a) Solve the differential equation $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = xe^x \sin x$
   (b) A mass of 200 gm is tied at the end of a spring which extends to 4 cms under a force of 196,000 dynes. The spring is pulled 5 cms and released. Find the displacement, t seconds after release, if there is a damping force of 2000 dynes per cm per second. [8+7]

6. (a) If ρ_1 and ρ_2 are radii of curvature at the extremities of any chord of the cardioids $r = a(1+\cos\theta)$, which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$
   (b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$. Where $a^2 + b^2 = c^2$ [8+7]

7. (a) Test the convergence of the series $\frac{x^{2n}}{(n+2)^{3/2}}$
   (b) Test the convergence of the series $\sum \frac{2^2.4^2.6^2....(2n)^2}{3^2.4^2.5^2.....(2n-1)^2}$ [7+8]

8. (a) Form the differential equation by eliminating arbitrary constants
   xy = Ae^x+Be^{-x}+x^2
   (b) Solve the differential equation $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$
   (c) If 30% of a radio active substance disappears in 10 days, how long will it take for 90% of it to disappear? [4+6+5]

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