

MATHEMATICS PAPER IIA - MARCH 2010

ALGEBRA AND PROBABILITY

TIME : 3hrs

Max. Marks.75

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS.

10X2 =20

Noe : Attempt all questions. Each question carries 2 marks.

1. If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$  then find the value of  $\frac{1}{\alpha} + \frac{1}{\beta}$
2. If  $-1, 2$  and  $\alpha$  are the roots of  $2x^3 + x^2 - 7x - 6 = 0$  then find ' $\alpha$ '.
3. If  $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$  find  $2A + B^T$
4. Find the determinant of  $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$
5. If  ${}^{(n+1)}P_5 : {}^nP_6 = 2 : 7$  find  $n$
6. Find  ${}^{10}C_5 + 2 \cdot {}^{10}C_4 + {}^{10}C_3$
7. Find the coefficient of  $x^{-7}$  in the expansion of  $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^{11}$
8. Find  $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$
9. If  $A, B$  are two events with  $P(A) = 0.5, P(B) = 0.4$  and  $P(A \cap B) = 0.3$ , then find probability of neither  $A$  nor  $B$  occurs.
10. A Poisson variable satisfies  $P(X = 1) = P(X = 2)$ . Find  $P(X = 5)$

## SECTION B

### SHORT ANSWER TYPE QUESTIONS.

5X4 =20

Note : Answer any FIVE questions. Each question carries 4 marks.

11. Determine the range of the expression  $\frac{x^2 + x + 1}{x^2 - x + 1}$ ,  $x \in \mathbb{R}$
12. If  $I \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then show that  $(aI + bE)^3 = a^3I + 3a^2bE$ .
13. Find the sum of all four digit numbers that can be formed using the digits 1,2,4,5,6 with out repetition.
14. Find the number of ways of forming a committee of 5 persons from a group of 5 Indians and 4 Russians such that there are atleast 3 Indians in the committee.
15. Resolve  $\frac{x}{(x-1)(x-2)}$  in to partial fractions.
16. Show that  $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \dots = 1 - \log_e 2$
17. If A, B, C are three independent events of an experiment such that  $P(A \cap B^c \cap C^c) = \frac{1}{4}$ ,  $P(A^c \cap B \cap C^c) = \frac{1}{8}$ ,  $P(A^c \cap B^c \cap C^c) = \frac{1}{4}$ , then find P(A), P(B) and P(C).

## SECTION C

### LONG ANSWER TYPE QUESTIONS.

5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Solve that equation  $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ .
19. Show that  $\det \begin{bmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{bmatrix} = 2(a+b+c)^3$
20. Solve the equations  $x + y + z = 6$ ,  $x - y + z = 2$ ,  $2x - y + 3z = 9$  by Gauss Jordan method.

21. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of

$$(x+a)^n \text{ then prove that i) } P^2 - Q^2 = (x^2 - a^2)^n \quad \text{ii) } 4PQ = (x+a)^{2n} - (x-a)^{2n}$$

22. Find the sum of the infinite series  $\frac{1}{4} + \frac{5}{4 \cdot 8} + \frac{5 \cdot 7}{4 \cdot 8 \cdot 12} + \dots \infty$

23. A, B, C are 3 news papers from a city, 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% both A and C, 4% read both B and C and 2% read all the three. Find the percentage of the population who read atleast one news paper and find the percentage of population who read the newspaper A only.

24. A random variable X has the following probability distribution

$X = x_i$	0	1	2	3	4	5
$P(X = x_i)$	0	k	2k	3k	4k	5k

Find k, mean and variance of X.

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